

# Electronic Designer's Shock and Vibration Guide for Airborne Applications

A GOVERNMENT RESEARCH REPORT

## U.S. DEPARTMENT OF COMMERCE

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**ELECTRONIC DESIGNER'S SHOCK  
AND VIBRATION GUIDE  
FOR AIRBORNE APPLICATIONS**

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RCA SERVICE COMPANY  
A DIVISION OF RADIO CORPORATION OF AMERICA*

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**WRIGHT AIR DEVELOPMENT CENTER  
AIR RESEARCH AND DEVELOPMENT COMMAND  
UNITED STATES AIR FORCE  
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## FOREWORD

This publication has been prepared under contract No. AF 33(616)-3257 by Technical Publications, Government Service Department, RCA Service Company. The project was initiated by the Electronic Components Laboratory, Directorate of Laboratories, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio. The project monitor for the Laboratory originally was Mr. N. P. Kempton, and replacing him later in the project was Mr. R. W. Sevy. Mr. C. A. Golueke, as Section Chief for the Laboratory, was active in the conception and throughout the progress of the project.

RCA personnel participating in the project were: Mr. R. E. Barbieri, project leader and writer; Mr. W. Hall, engineering consultant and writer; and Messrs. P. M. Costanzi, G. P. Sakalosky, and R. F. Sweeton, technical writers.

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## ABSTRACT

Improvements in the mechanical design of airborne electronic equipment have not kept pace with advances in electrical design. The resulting major inadequacy has been the inability of some equipments to resist a shock and vibration environment. This book is a guide for the designer of airborne electronic equipment to aid him in designing resistance to shock and vibration into equipments.

Both the theoretical and practical aspects of shock and vibration are covered. The discussion of shock and vibration concepts and the philosophical approaches to shock and vibration problems serve as a foundation for understanding the complete publication. This makes it unnecessary for the electronics designer to have prior shock and vibration knowledge. Environmental data reveal to the designer the levels of shock and vibration excitation to which electronic equipments are subjected.

The practical applications of shock and vibration design practices are emphasized. The relative susceptibilities of component parts and the effects of the mounting orientation of component parts in the equipment are presented. Also covered are the design of racks and chassis for best resistance to shock and vibration and the use of auxiliary means, such as damping or isolation, for either resisting or changing the environment.

The balance of the design guide considers the test methods and facilities that can give reasonable assurance that the equipment will eventually be able to withstand its operational environment. A thorough discussion is given of shock and vibration simulators and instrumentation as well as the philosophy of performing laboratory tests.

## PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or conclusions contained herein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:



GEORGE F. WATKINS

Lt. Colonel, USAF

Chief, Electronic Components Laboratory  
Directorate of Laboratories

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## INTRODUCTION

Rapid improvements in the electrical design of electronic equipment have resulted in phenomenal technological advances. Unfortunately, the mechanical design of this same electronic equipment has not developed at a like rate. As equipments became more complex electrically, the more likely became the chances of equipment breakdown, especially when the equipments were subjected to environments where shock and vibration existed.

Recognizing the need for better mechanical design of electronic equipment, the Electronic Components Laboratory, Wright-Patterson Air Force Base, Ohio, established a project entitled "Vibration and Shock Design Criteria for Airborne Electronics Equipment." The project was divided into four phases: (1) the accumulation of service environmental data, (2) the correlation of laboratory tests with service conditions, (3) the evaluation of presently designed equipments, and (4) a design manual covering the mechanical design of equipment. The fourth and final phase, the preparation of a design manual, is the program under which this publication was written.

This design guide is intended for use by electronics designers rather than by specialists in the shock and vibration field. Therefore, the complexity of the manual will be limited to the more basic considerations, and no previous understanding of shock and vibration phenomena will be required of the reader. The scope of the design guide, however, is broad in order to give the reader a general background in all phases of shock and vibration applicable to airborne electronic equipment.

This manual could not possibly cover specifically all problems confronting the designer. The approach has been to present a general knowledge of shock and vibration so that the designer can intelligently solve his particular problem. Wherever possible, the advantages and disadvantages of techniques, materials, component parts, test procedures, etc., are given.

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## SECTION I

### THEORY AND PHILOSOPHY

#### 1-1. GENERAL.

1-2. Designers have made great strides in developing functional circuitry for electronic equipment. The most advanced electrical designs are worthless, however, if the equipment fails to perform reliably. Many factors enter into equipment reliability. One of these is the ability of the equipment to withstand vibration or shock in its operational environment.

1-3. Vibrations and shocks will impose forces on and will deform any flexible or elastic structure (such as electronic equipments and their individual parts). The severity of the deformation depends upon the nature and intensity of the imposed force and the geometrical configuration, total mass, internal mass distribution, stiffness distribution, and damping of the equipment or part structures. If the structure and all of its components are suitably designed, this deformation will not produce functional damage.

1-4. The mechanical requirements of airborne electronic equipment should be recognized early in the design stage of the equipment. The premise that a shock and vibration problem can be resolved by installing mounts under the equipment after it is built is invalid. This does not imply that mounts are no longer useful or have no place in design, but rather that they are only one method, and if used, should be considered not as an afterthought, but as part of a mounting system best suited for a particular equipment in an intended environment. Also, it must be emphasized that isolators are not a universal solution to all shock and vibration problems.

1-5. The importance of testing during the stages of designing the equipment cannot be overemphasized. Although many words can be written concerning the design of equipment to withstand a shock and vibration environment, all situations cannot be covered, and those cases which are covered are often so complex that the discussion is necessarily simplified. Furthermore, data on the resistance to shock and vibration of component parts will be far from complete and the inherent resistance will change with the mounting method used. Throughout the equipment design stages, testing serves an important function in evaluating component parts, determining natural frequencies, and uncovering poor or borderline designs.

#### 1-6. FUNCTION OF THE DESIGN GUIDE.

1-7. The primary functions of this design guide are to make the designer aware of the need for considering the effects of shock and vibration, and to provide the designer, whose specialty lies in another field, with the benefit of the experience of others, acquired through the design of good and bad equipment. The information provided in this manual will not ensure reliable electronic equipment; this the designer can achieve through an appreciation of the problem and by relating the

knowledge he gains from this manual to the other considerations of equipment design.

1-8. Many persons other than the designer have need for an understanding of shock and vibration. For example, the best efforts of a designer can sometimes be voided by the installation technique. A short lead of heavy cable from the equipment to a source of excitation can transmit vibration to an extent that isolation mounts will be bypassed. It is hoped that this publication will prove of value to the following personnel:

- a. Designers of airborne electronic equipment.
- b. Designers of electronic component parts.
- c. Government project engineers concerned with the development and procurement of reliable electronic equipment.
- d. Installation personnel.
- e. Reliability engineers concerned with evaluating and troubleshooting equipment, and determining the cause of equipment failures.

1-9. This publication is not expected to be of major interest to the engineer who specializes in shock and vibration, since, in most cases, the complexity is limited to considerations of primary value to electronics engineers or designers, installation engineers, and reliability engineers.

1-10. The reader should consider this publication as a guide and not a bible. The design methods are not set down as inflexible laws nor are they intended to supersede any specification requirements. The complexity of the shock and vibration problem is such that each design must be considered as somewhat of a special case, and the design must take into account the requirements for the equipment and the specific environment to which it will be subjected.

#### 1-11. VIBRATION AND SHOCK DEFINED.

1-12. Shock and vibration is a composite term used to describe an environment under which equipment must operate successfully. Although shock and vibration are treated as separate and distinct phenomena in the literature and are tested for by different procedures, actually, in some areas, the distinction between the two is not immediately evident. The difference between periodic vibration and a transient shock motion is obvious. Much less obvious, however, is the existence of any basic differences between random vibration (which is not periodic) and shock. For the sake of discussion, however, shock and vibration are treated differently in all respects. In those areas where they do overlap, vibration is considered as sustained excitation and shock as intermittent.

1-13. **VIBRATION.** Vibration is more easily defined than is shock, although the word vibration, used alone, is often thought to mean only a steady-state or periodic motion. The American Standards Association definition of vibration states that it is "the variation, usually with time, of the magnitude of a quantity with respect to a specified reference, when the magnitude is alternately greater and smaller than the reference."

1-14. In the study of kinetics, the principal form of vibration treated is motion that is steady-state or periodic in nature. The most elemental form of steady-

state vibration is simple harmonic motion. Simple harmonic motion\* can be identified by any two of the four parameters; frequency, amplitude of excursion, velocity, and acceleration (the other two can be calculated). Figure 1-1 shows the relationship between frequency, double amplitude\*\* and acceleration. If double amplitude remains constant, the acceleration increases as the square of the increase in frequency. Likewise, increasing the excursion while the frequency remains constant results in a proportionately higher acceleration. Any periodic motion may be considered to consist of motions at one or more frequencies, with the motion at each frequency being harmonic. A steady-state vibration may be completely defined by designating the frequency or frequencies, the maximum value of the harmonic variable at each frequency, and the phase relationships that exist between the component harmonic motions. The harmonic variable may be expressed in terms of displacement, velocity, or acceleration.

1-15. Two other types of vibration are termed random and white-noise vibration. They differ from one another, although the two terms are often used synonymously. Random vibration differs from steady-state vibration in that the amplitudes at the component frequencies vary randomly with respect to time. White-noise vibration has no defined component frequencies; both frequencies and amplitudes may vary randomly with time and power spectral density is constant over the bandwidth.

1-16. **SHOCK.** The meaning of the term shock is vague. Shock connotes a rapid change of load, or a rapid change of acceleration with a resultant change of load. A shock motion cannot be defined by giving numerical values to established parameters; it can be defined only by describing the history of a significant parameter, such as acceleration, velocity or displacement. The ASA definition for shock is "Mechanical shock occurs when the position of a system is significantly changed, in a relatively short time in a nonperiodic manner. It is characterized by suddenness and large displacements and develops significant inertial forces in the system."

1-17. In order that the designer be cognizant of the fundamental theories upon which he must base his design considerations, brief discussions of vibration and shock are presented. Electrical analogies are presented so that the designer can relate his knowledge of electrical theory to mechanical theory. A more detailed theoretical treatment of shock and vibration is given in the Appendix.

#### 1-18. BASIC VIBRATION THEORY.

1-19. A vibrating system must have two characteristics: inertia and a force which tends to restore equilibrium. Any mass restrained by an elastic support is able to vibrate. If the mass is forced from its static equilibrium position, potential energy is given to the elastic support. When the mass is released, the force due to elasticity will accelerate the mass toward the original (static equilibrium) position. As the mass reaches the static equilibrium position with a finite velocity, the mass' inertia will carry it beyond this point until the elastic forces decelerate the mass to zero velocity. At that time, the mass is again displaced from its

\* The motion varies sinusoidally.

\*\* Double amplitude is the excursion from one extreme of a harmonic motion to the other.

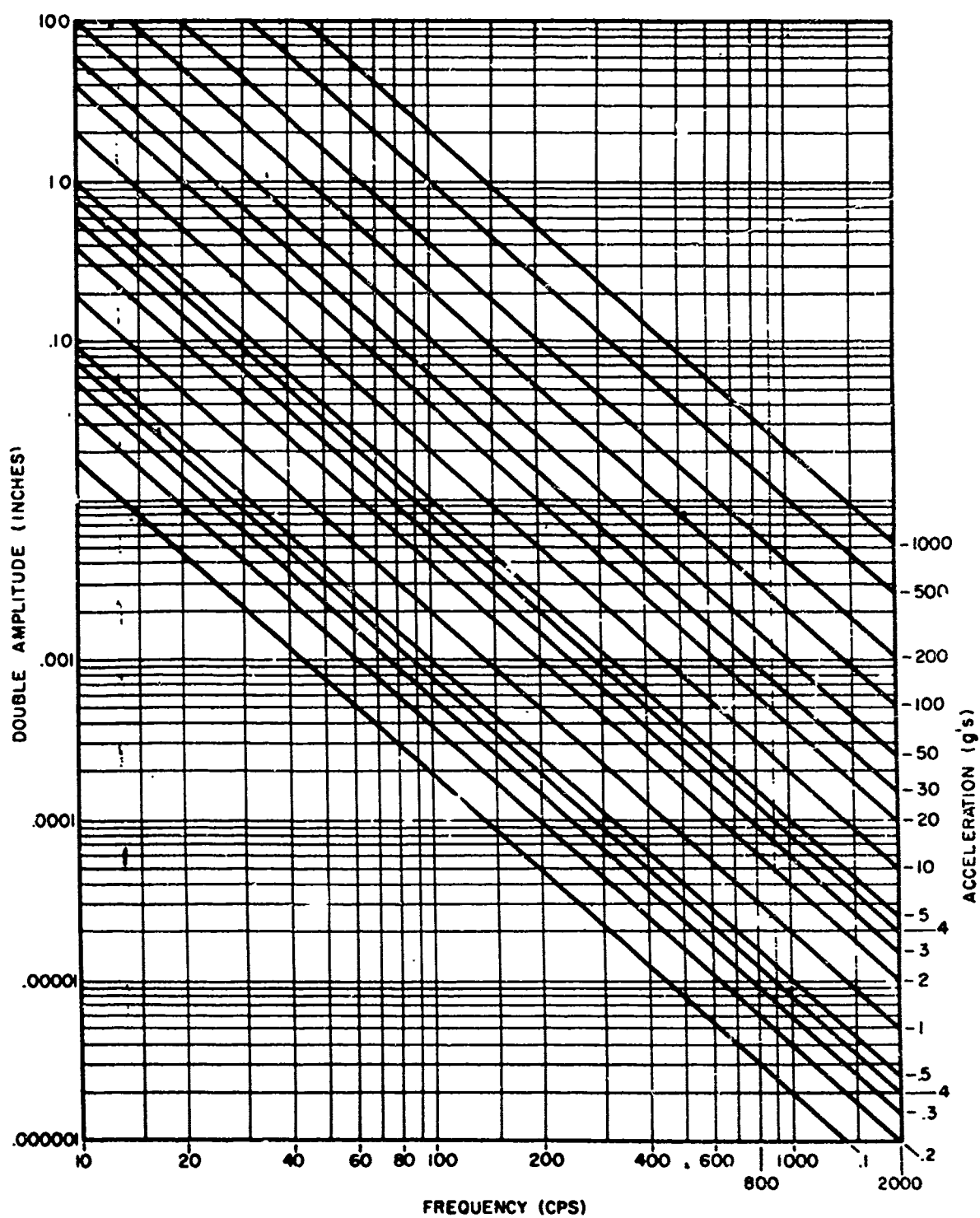


Figure 1-1. Relationship Between Frequency, Double Amplitude, and Acceleration

original position, and is again forced back by the elastic support. The mass again passes the original position and continues to move back and forth (vibrate) until the energy imparted to the system is dissipated through internal friction of the elastic support or by some other damping means. The frequency of oscillation about the equilibrium position depends on the elastic modulus of the support, the mass, and upon the amount of damping. (In most systems, the frequency of oscillation is not changed appreciably by damping.)

#### 1-20. SINGLE-DEGREE-OF-FREEDOM SYSTEM.

1-21. The number of degrees of freedom is the number of variables of the system which must be fixed to define the system completely. In a single-degree-of-freedom system, all the variables (e.g., time, displacement, spring force, etc.) are interdependent. By assigning a value to one parameter (variable), the values of all other parameters become fixed.

1-22. **FREE VIBRATION.** A single-degree-of-freedom system will exhibit the simplest type of vibration. This vibration can be either free or forced. Free vibration is the motion of the mass without external excitation. The following is a discussion of free vibration in a single-degree-of-freedom system.

1-23. The single-degree-of-freedom system is illustrated (figure 1-2) by a mass hanging from a spring, in which the only possible motion is along the vertical axis. Under static (equilibrium) conditions, the spring will be extended a given amount depending on the spring stiffness\* and the weight of the mass. Figure 1-2 shows this configuration and the forces acting on the weight when the weight is displaced (moved) from the static equilibrium position. If the weight is displaced from the static equilibrium position a distance  $y_0$  and is then released, it will oscillate. The equation of motion may be written as a sum of forces, taking the downward direction as positive.

$\Sigma F = Ma$ where $a = \ddot{y}$	$M = \text{mass}$
$-k(\delta + y) + W = M\ddot{y}$	$\ddot{y} = \text{acceleration}$
$-k\delta - ky + W = M\ddot{y}$	$W = \text{weight} = Mg$
But $W = k\delta$ and therefore:	$\delta = \text{static deflection} = W/k$
$-ky = M\ddot{y}$	$k = \text{spring constant}$
and $\ddot{y} = -(k/M)y$	$y_0 = \text{initial displacement}$
setting $\omega^2 = k/M$ .	$y = \text{location on Y axis}$
$\ddot{y} = -\omega^2 y$	$t = \text{time}$
and upon integrating.	
$y = y_0 \sin \omega t$	

This is the equation of a simple harmonic motion whose frequency ( $f_n$ ) in cycles per second is equal to  $\omega / 2\pi$  and whose maximum displacement equals  $y_0$ . If no

\* Spring stiffness is the ability of the spring to resist deformation. The measure of spring stiffness is the spring constant  $k$ , the force required to produce unit deflection; a spring deflected 2 inches by 4 pounds would have a spring constant  $k$  (4 lbs/2 inches) of 2 lbs/inch. Spring flexibility is the reciprocal of stiffness and is expressed in units of inches per pound.

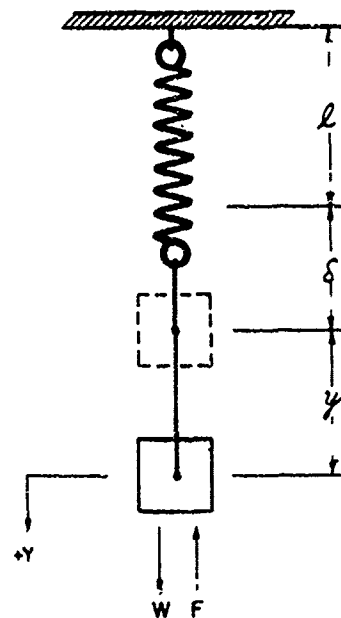
damping is assumed, the vibration induced by displacing the mass and then releasing it would continue indefinitely and the displacement ( $y$ ) of the mass plotted against time ( $t$ ) would be a sine curve as in figure 1-3a. If damping is included in the system, the potential energy put into the system by the initial displacement of the mass would be gradually dissipated. The excursion of the mass in this system for successive cycles would decrease as in figure 1-3b. It should be pointed out that  $f_n$  depends only upon mass and  $k$  and not upon weight. Weight determines only the static equilibrium position.

**1-24. FORCED VIBRATION.** Forced vibration is defined as the excitation of vibration of the mass through vibratory motion of the support. The vibration frequency of the mass is determined by the vibration frequency of the support. The amplitude of constant (steady-state) forced vibration depends on the ratio of the natural frequency of the mass-spring system to the frequency of the forcing vibration. Solution of the differential equation of motion for the forced vibration system gives the following expression for the amplitude of vibration of the mass:

$$A = \frac{X}{1 - \left(\frac{f_1}{f_n}\right)^2}$$

$A$  = amplitude of forced vibration  
 $X$  = amplitude of exciting force  
 $f_1$  = frequency of exciting force  
 $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$  = natural frequency of mass-spring system

The ratio  $\frac{1}{1 - (f_1/f_n)^2} = \frac{A}{X}$  is termed transmissibility and may be considered to be a magnification factor. If the ratio  $A:X$  is plotted against the exciting frequency expressed in terms of natural frequency, the curve of figure 1-4 re-



$l$  = SPRING LENGTH WITH NO WEIGHT.  
 $\delta$  = STATIC DEFLECTION =  $W/K$ .  
 $y$  = DISPLACEMENT FROM STATIC EQUILIBRIUM  
 $W$  = WEIGHT =  $Mg$ .  
 $F$  = TOTAL SPRING FORCE =  $K(\delta + y)$   
 $Y$  = AXIS ALONG WHICH MOTION OCCURS

Figure 1-2. Single-Degree-of-Freedom System

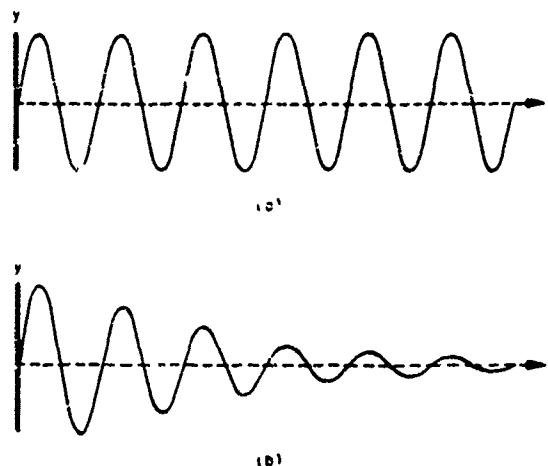


Figure 1-3. Plot of Motions of Undamped and Damped Vibrations

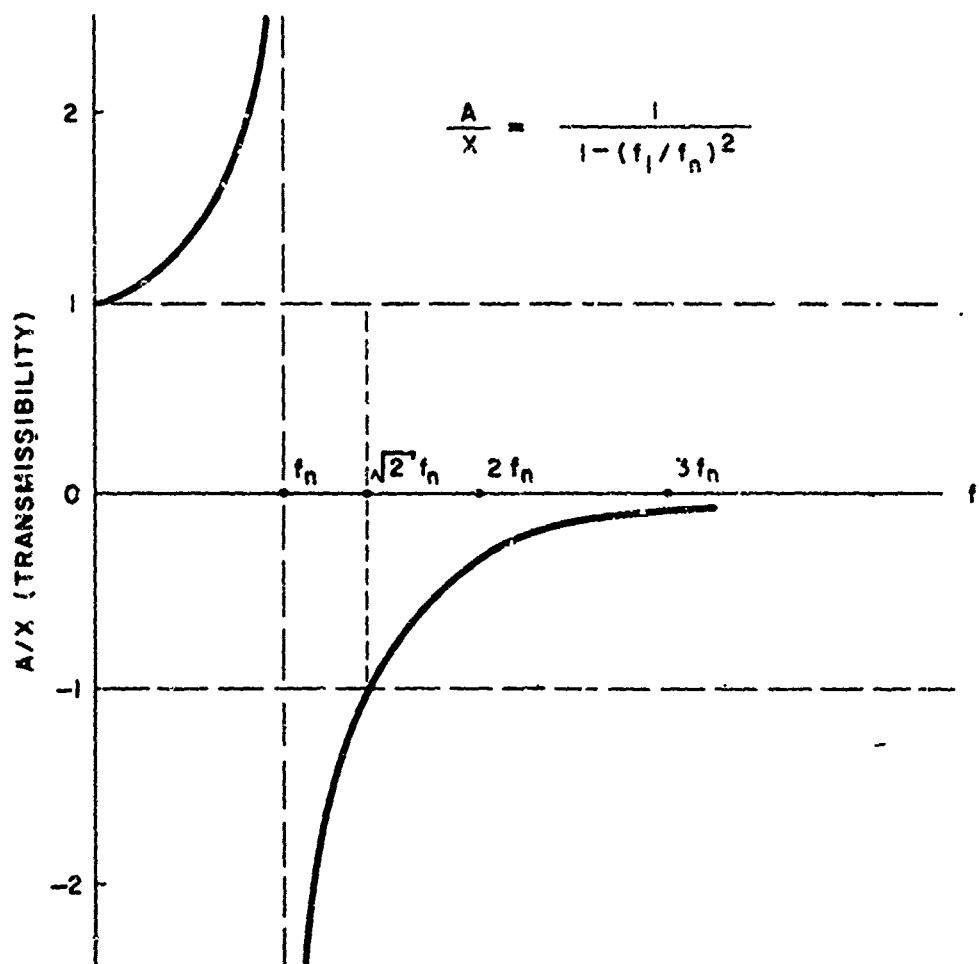


Figure 1-4. Plot of Transmissibility Curve

sults. The curve shows that whenever the exciting frequency is less than the natural frequency ( $f_1 < f_n$ ), the magnification is one or greater; and as the exciting frequency approaches the natural frequency (resonance) the magnification of the displacement increases rapidly to a theoretically infinite value. Above resonance ( $f_1 > f_n$ ), the magnification decreases as the exciting frequency ( $f_1$ ) increases, the magnification being 1 at a frequency ratio of 1.414, or more precisely  $\sqrt{2}$ . Figure 1-4 also shows that, above resonance, the curve is negative; this indicates that the exciting force is 180 degrees out of phase with the motion of the mass. This phasing is usually not important; the curve, therefore, is plotted as if all values were positive (see figure 1-5). Although magnification at resonance is theoretically infinite, all actual devices have inherent damping. The two dotted curves (a and b) of figure 1-5 show the effects of damping; curve a exhibits less damping than does curve b.

**1-25. VIBRATION ISOLATION.** As discussed, when the ratio of the forcing frequency to the natural frequency is above 1.414, the response amplitude is less than the exciting amplitude. This lessening of amplitudes is called isolation, and it is on this principle that vibration isolators depend. However, referring to figure 1-5, examination of the portions of the curve below a frequency ratio of 1.414

shows that in this region, the response of an isolator-mounted body is greater than the amplitude of the forcing frequency. A forcing frequency equal to the natural frequency of a body on its mounts will produce a particularly large response. It may be seen from the figure that the frequency ratio at which maximum response occurs depends upon the amount of damping. The greater the damping, the lower the resonant frequency. The shift in resonant frequency due to damping is usually very slight and for most cases the resonant frequency of a damped system may be taken to be  $\frac{1}{2\pi} \sqrt{k/m}$ . Section V discusses the effects of damping more fully.

1-26. The action described by the curve of figure 1-5 can be reproduced with a weight suspended by a rubber band. By holding the free end of the rubber band, the essential parts of a forced single-degree-of-freedom system are obtained. If the support (the hand) is moved up and down very slowly, the weight will follow closely, and its motion will be in phase with the motion of the hand. If the frequency of the hand's motion is increased, the motion of the weight will increase until the frequency of the hand's motion approximates the resonant frequency of the rubber band and weight system. At resonance, the motion of the weight will be

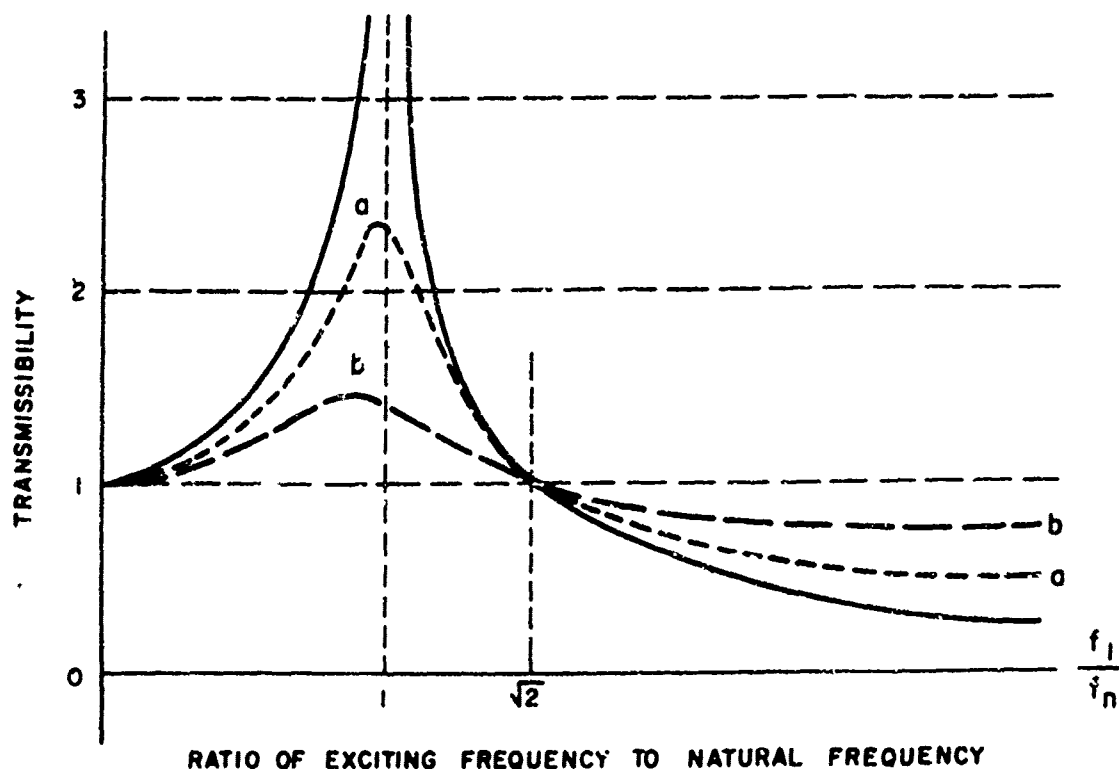


Figure 1-5. Transmissibility Curves for an Undamped and Two Damped Systems

much greater than the motion of the support (the hand). If the frequency of hand motion is increased above this point, the excursion of the weight will decrease and it will be out of phase with the hand motion, and at high frequencies of hand motion, the weight will appear to stand still.

#### 1-27. MULTI-DEGREE-OF-FREEDOM SYSTEM.

1-28. In a multi-degree-of-freedom system, more than one parameter (position of mass, force on spring, etc.) must be known to define the whole system. The single-degree-of-freedom system, which has only one natural frequency, is useful for explaining vibration, but it is seldom encountered in practice. In most applications, a multi-degree-of-freedom system exists. The many degrees of freedom may be coupled or uncoupled. In a multi-degree-of-freedom system, the system will have a natural frequency for each possible mode of vibration. If vibration in one mode does not affect vibration in the other modes, the vibration is said to be uncoupled. Conversely, if vibration in one mode affects vibration in another mode, the two vibrations are said to be coupled.

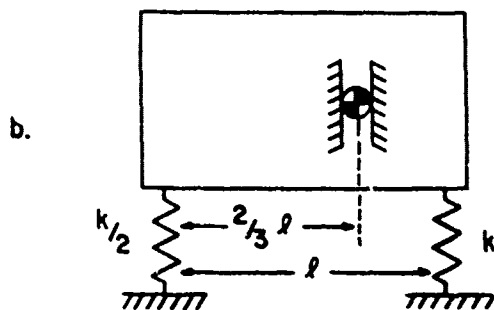
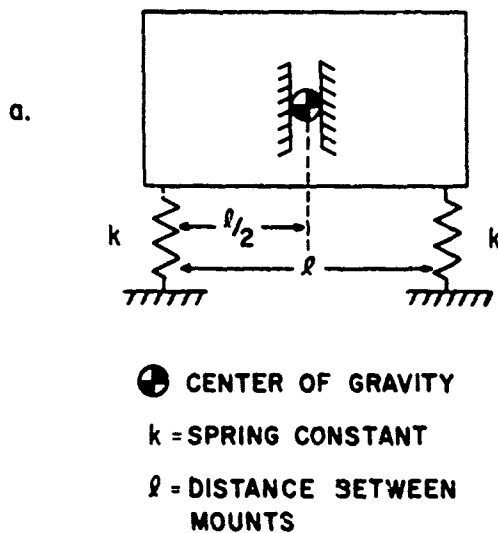


Figure 1-6. Systems with Uncoupled Natural Frequencies

ment around the center of gravity. Vibration of the center of gravity in one mode will be accompanied by vibration in the other mode. In each of the two depicted

1-29. UNCOUPLED DEGREES OF FREEDOM. A system having uncoupled modes of vibration is shown in figure 1-6a. If the mass is rotated about its center of gravity so that one spring is compressed while the other is stretched, upon release, the mass will vibrate in a rotational mode. If the mass is pressed downward so that both springs are compressed equal amounts, the mass will vibrate vertically in a translatory mode when the mass is released. In this simple case, since the mass is balanced on the springs, the motion of vibration in one mode does not affect the motion of vibration in the other mode; the two natural frequencies, one in rotation and the other in translation, are said to be uncoupled. In figure 1-6b, the center of gravity is displaced from the geometrical center of the mass, but the spring arrangement is such as to compensate for this. This configuration will also demonstrate uncoupled vibration.

1-30. COUPLED DEGREES OF FREEDOM. Systems with coupled natural frequencies are shown in figures 1-7a and 1-7b. In either of these systems, if the mass is pressed vertically downward and released, the reactions of the springs are such as to produce a mo-

configurations, there are two natural frequencies, one in which translation is the predominant motion, and one in which rotation is the predominant motion; and these two coupled frequencies will not be the same as the frequencies when uncoupled. In all cases, the highest coupled natural frequency will be higher than the highest uncoupled natural frequency and the lowest coupled natural frequency will be lower than the lowest uncoupled natural frequency. Although the illustrations used are two-degree-of-freedom systems, systems with higher degrees of freedom are probable.

### 1-31. ELECTRICAL ANALOGY FOR VIBRATION.

1-32. The behavior of a mass-spring system in mechanical vibration is similar to the behavior of an alternating-current LC circuit. In the mechanical system, a mass has inertia which is evidenced as a force that resists any change in the velocity of the mass; in an electrical system, an inductor has electromotive force which opposes any change in current. As a spring will store mechanical energy when subjected to a force and will return part of that energy if the holding force is lessened, a capacitor will store electrical charge when subjected to a voltage and will release a portion of the charge if the voltage is lessened. An LC circuit behaves like a mass-spring system in that the combination of the abilities to store kinetic energy and to store potential energy produces a system which will oscillate at a natural or resonant frequency during energy transfer between the kinetic storage component and the potential storage component. When a sinusoidal voltage is applied to a series LC circuit, the current flow will vary with the frequency of the applied voltage, reaching a peak (resonance) when the applied frequency is equal to the circuit natural frequency.

1-33. This analogy may be demonstrated mathematically. In a series RLC circuit, a summation of voltage drops around the circuit gives:

$$L\dot{i} + Ri + q/C = V_m \sin 2\pi ft$$

Substituting  $\dot{q}$  for  $i$ :

$$L\ddot{q} + R\dot{q} + (1/C)q = V_m \sin 2\pi ft \quad (1)$$

The differential equation of motion for a viscously damped single-degree-of-freedom mechanical system is

$$M\ddot{y} + c\dot{y} + ky = X_0 \sin 2\pi ft \quad (2)$$

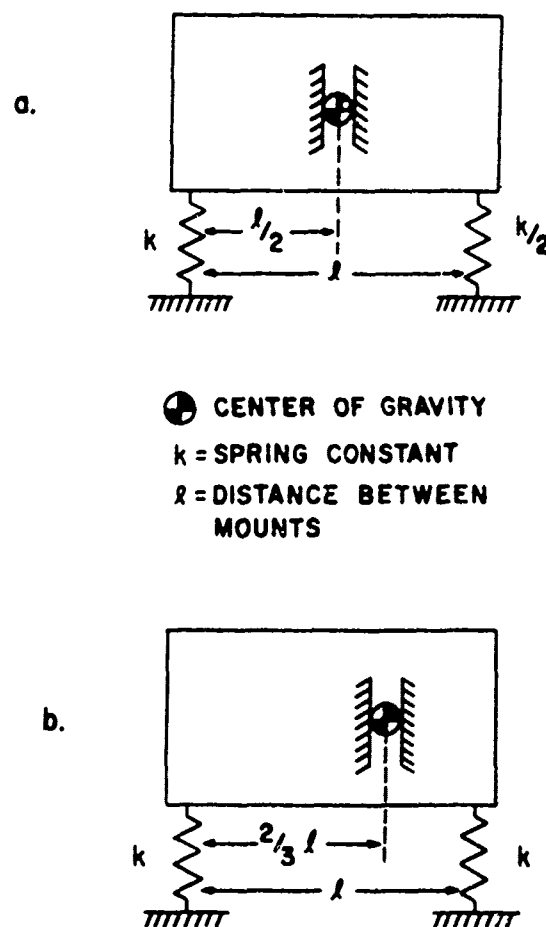


Figure 1-7. Systems with Coupled Natural Frequencies

The similarity between equations (1) and (2) is obvious with inductance analogous to mass, resistance analogous to the viscous damping coefficient, capacitance analogous to spring flexibility which is the reciprocal of stiffness, voltage drop analogous to force, and current analogous to velocity. In the same way in which a mechanical system has a natural frequency of vibration:

$$f_n = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{M}} = \frac{1}{2\pi \sqrt{M/k}}$$

so does the series circuit have a resonant frequency:

$$f_r = \frac{1}{2\pi \sqrt{LC}}$$

which is in keeping with the analogies stated above.

#### 1-34. BASIC SHOCK THEORY.

1-35. Shock has been defined in many ways, but all definitions include the concept of suddenness or transience. From a materials viewpoint, shock might be considered to be a rapid change in stress such as might result in the walls of a pipe subjected to water hammer. However, in the field of airborne electronics, the shocks received by an equipment are usually in the form of a motion of the air-frame or rack which holds the equipment. Therefore, shock is variously described as a sudden change in velocity, a transient displacement, or an acceleration experienced over a short time interval. The concept of suddenness as a requisite for shock is important. No matter which parameter, displacement, velocity, or acceleration, is used in defining shock, the implication of sudden change in acceleration is always present. Acceleration by itself does not constitute shock; a structure subjected to steady-state acceleration, although under a static acceleration load, cannot be considered to be undergoing shock. The time duration of a shock pulse is important since it helps in determining the way in which a structure will react to that pulse.

1-36. Dimensionally, any shock motion may be completely described by a time history of either displacement, velocity, or acceleration. Although theoretically an equation might be written for a shock motion, actual shock motions are usually complex and the customary method of describing the time function of the motion is graphical. For example, figure 1-8 might represent a typical acceleration versus time curve. There are an infinite number of possible shock motions since the motion may vary in pulse shape, time duration, and peak acceleration.

1-37. INSTANTANEOUS VELOCITY CHANGE OF SUPPORT. To illustrate the concept of shock, consider the simple case of the mechanical system shown in figure 1-9. The system consists of a small mass B connected to the support A by a flexible member (spring). Figure 1-10 shows the velocity of the support and the mass when the support is given an instantaneous velocity upward. As the support moves with its acquired velocity, the spring compresses due to the inertial force of the mass B. Although the mass begins to accelerate due to the spring force, the spring continues to compress until, at time  $t_1$ , the mass has been accelerated to the velocity of the support. At this point, although the velocity of the mass is equal to the velocity of the support, the spring is compressed and will continue to accelerate the mass toward the static equilibrium position. The mass will vibrate with

free vibrations relative to the support about the static equilibrium position until the energy which the spring obtained in accelerating the mass to the support velocity is dissipated through damping.

1-38. Figure 1-11 shows the displacement curves for the support and mass when the support instantaneously acquires an upward velocity. The absolute motion of B is the sum of the motion of the support A and the motion of B relative to A. The spring will be damaged if the relative motion of B to A becomes excessive. The amount of this relative motion (spring deflection) is proportional to the force on the mass B. This force, neglecting the weight of B, is an inertial force. Therefore, the maximum spring deflection will occur concurrently with the maximum acceleration of B.

1-39. The stress in the spring is proportional to the deflection of the spring and, therefore, the maximum stress is proportional to the maximum relative motion between A and B. This might appear to lead logically to the conclusion that the stiffer the spring, the less chance of breakage but this is not necessarily true. This may be seen by looking at the motion in another way. At the time the support acquires its velocity, B is not moving and hence must acquire kinetic energy which must be transferred from the spring which stores it in the form of potential energy. The energy stored in a spring is equal to the area under the force-deflection curve. Figure 1-12 shows force-deflection curves for two springs with spring  $S_1$  stiffer than spring  $S_2$ . Spring  $S_2$  can store more energy than can spring  $S_1$  for the same compressive load although a larger deflection is required to do it. For the same deflection, the stiffer spring can store the most energy.

1-40. SEMISINUSOIDAL ACCELERATION OF SUPPORT. In the above dis-

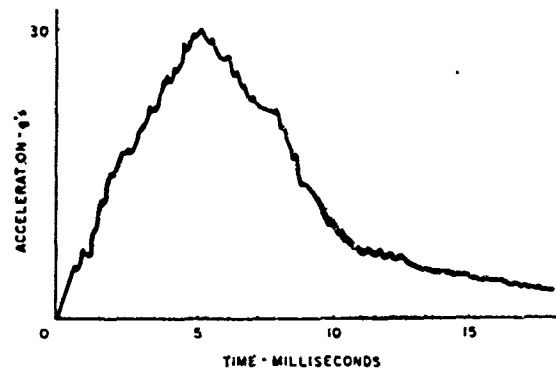


Figure 1-8. Plot of Shock Motion

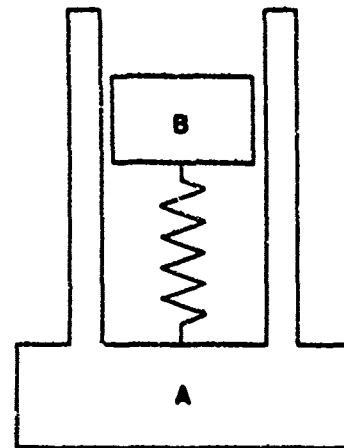


Figure 1-9. Simple Mechanical System

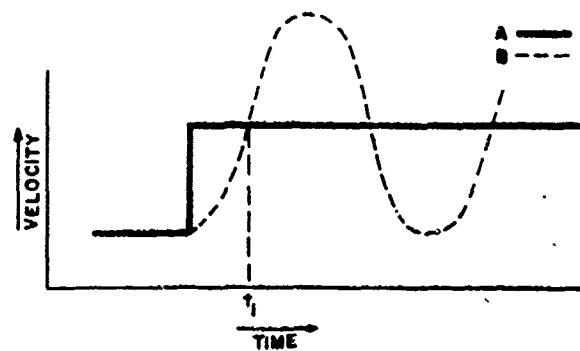


Figure 1-10. Instantaneous Velocity Change of Support and Resulting Motion of Mass

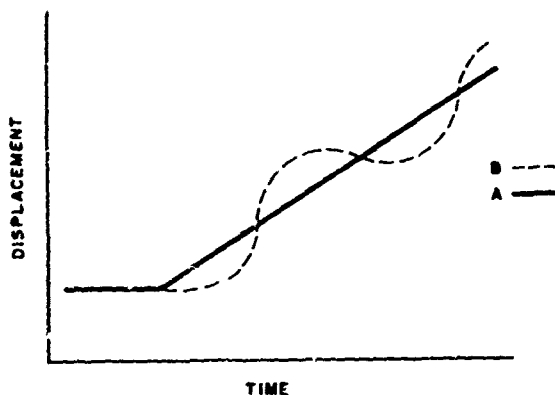


Figure 1-11. Displacement Curves for Motions Shown in Figure 1-10

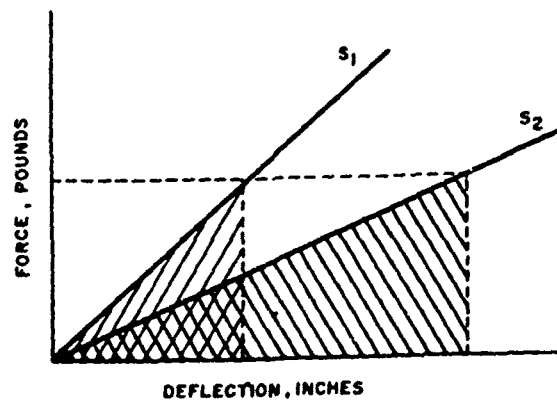


Figure 1-12. Effect of Spring Constant on Energy Storing Capacity

cussion it was assumed that the velocity change of the support A was instantaneous, but this assumption is not a very practical one. The effect of time duration of the shock pulse can be illustrated most simply by considering the effect of an acceleration pulse which is semisinusoidal in nature upon a single-degree-of-freedom system similar to that in figure 1-9. The shock pulse, shown in figure 1-13, may be defined mathematically in terms of acceleration by the following expressions:

$$\ddot{y} = a \sin \Omega t \quad \text{for } 0 \leq t \leq \frac{\pi}{\Omega}$$

$$\ddot{y} = 0 \quad \text{for } \frac{\pi}{\Omega} \leq t$$

where  $\ddot{y}$  = acceleration of the support at any time  $t$ .  
 $a$  = maximum acceleration of support  
 $\Omega$  = circular frequency, radians per second

The equations for the velocity and displacement of the supports can be found by successive integrations of the acceleration equations.

1-41. The maximum acceleration of the mass B when the support A is subjected to the semisinusoidal acceleration pulse depends upon the ratio of the time duration of the pulse to one-half the natural period of the spring-mass system. When the pulse duration is less than half of the half natural period of the system, the maximum acceleration of the mass is less than the maximum acceleration of the support. When the pulse duration is greater than half but less than five times the half natural period of the system, the maximum acceleration of the mass is greater than the maximum acceleration of the support, with the highest acceleration of the mass being approximately 1.75 times the maximum acceleration of the support. When the pulse duration is greater than five times the half natural period of the system, the maximum acceleration of the mass is almost equal to the maximum acceleration of the support.

1-42. VELOCITY CHANGE AS SUPPORT COMES TO REST. At the completion of the semisinusoidal acceleration pulse discussed above, the velocity of the support had changed from what it was originally. If the support originally had a velocity which was reduced to zero by the acceleration pulse, no further shock motion will occur. In many cases, however, the support is initially at rest and the

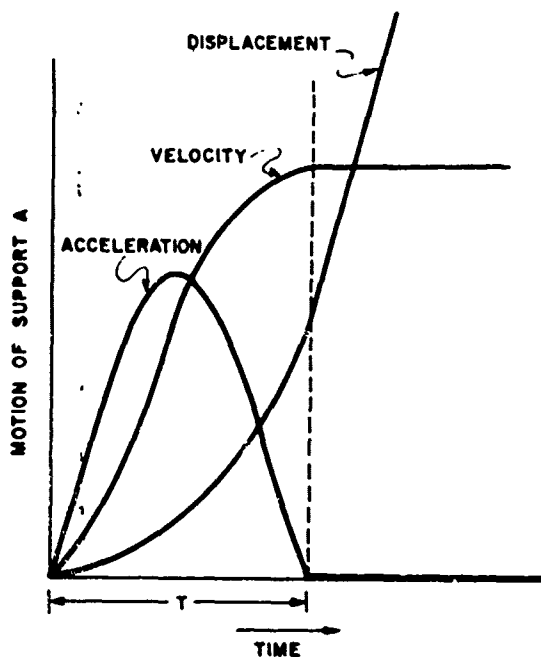


Figure 1-13. Shock Pulse

relative displacement and zero relative velocity. Displacement-time curves for the support and mass are indicated respectively by  $A_1$  and  $B_1$  in figure 1-14. Assume now that the velocity of the support is reduced to zero at  $t = 3\tau^*$ . The velocity of the mass is then twice as great as the initial velocity of the support. The displacement of the mass relative to the support is zero. Free vibration will continue, therefore, with twice the initial amplitude. Displacement-time curves for the support and mass are indicated respectively by  $A_2$  and  $B_2$  in figure 1-14.

#### 1-43. ELECTRICAL ANALOGY FOR SHOCK.

1-44. Assuming that a constant current is flowing in the theoretical circuit of figure 1-15a, the pure inductance  $L$  will have a steady-state magnetic field surrounding it. There will be no current flow in the left branch of the parallel circuit, hence there will be no voltage drop across the resistance  $R$ . Furthermore, since the inductance is pure and has no voltage drop across it, there will be no charge on the capacitor. If the switch  $S$  is opened, the flux field around the inductor collapses, and in so doing creates an emf which causes current to flow and to charge the capacitor. When the magnetic field has disappeared, the capacitor will discharge through the inductor in the opposite direction, cre-

\* An odd number of half cycles.

first acceleration pulse is closely followed by a second motion which brings the support to rest. The time at which the second pulse occurs determines the effect of this pulse on the motion of the mass relative to the support. Assume that the support has acquired a velocity as in figure 1-11 and that the support velocity is later reduced to zero as in figure 1-14. If the half natural period of the spring-mass system is  $\tau$  and if the velocity of the support is arrested when  $t = 2\tau$ , there is, at that instant, no displacement of the mass relative to the support and the absolute velocity of the mass is zero. Since the velocity of the support is reduced to zero at the moment that the velocity of the mass becomes zero, there is no relative velocity. A new time interval may thus be considered to start at time  $t = 2\tau$  for which the initial conditions are zero

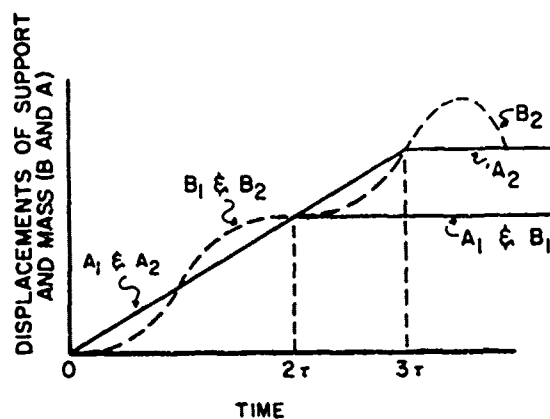


Figure 1-14. Effect of Subsequent Velocity Change

ating a new magnetic field which is reversed in polarity from the original. The electrical oscillations will continue at a frequency of

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

until the energy left in the inductor when the switch was opened is dissipated by the resistor.

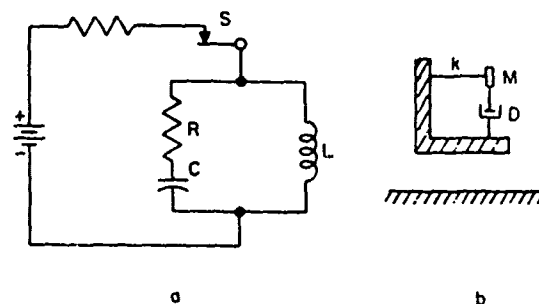


Figure 1-15. Electrical Circuit and Mechanical System for Shock Analogy

1-45. This excitation of the resonant circuit by a sudden change in current is analogous to the excitation of the mechanical system of figure 1-15b by a sudden velocity change. Assuming that the mechanical system is approaching the floor at the end of a free fall, the mass M has inertia just as the indicator of figure 1-15a had a magnetic field; the cantilever beam k is unloaded just as the capacitor of figure 1-15a was uncharged; and the dashpot is exerting no force just as the resistor caused no voltage drop. When the mechanical system strikes the floor, the support velocity abruptly drops to zero just as the "supporting" current dropped to zero when the switch was opened. The energy of the mass is expended in bending the cantilever beam, and the mass vibrates at the natural frequency until the energy is dissipated in the dashpot. The same analogies of mass-inductance, flexibility\*-capacitance, velocity-current, etc, hold for this case just as they did for viscously damped free vibrations.

#### 1-46. RANDOM VIBRATION.

1-47. Random vibration consists of a continuous spectrum of frequencies rather than the distinct harmonics of any single fundamental frequency. The amplitude varies in a random manner, and, consequently, the amplitude at any time cannot be predicted. This type of randomness is illustrated for a single frequency in figure 1-16a. Although the peak acceleration at any time cannot be predicted, if the variation is truly Gaussian, the probability that the acceleration will be a certain value can be expressed mathematically and therefore it can be plotted as a curve, as shown in figure 1-16b. This curve, known as a Rayleigh distribution, is a plot of the probability of occurrence of a given value of peak acceleration versus that value.\*\* The ordinate of figure 1-16b is peak acceleration expressed in multiples of the root-mean-square (rms) value of the acceleration. The probability that the peak acceleration will not exceed a given value is equal to the area under the curve between zero and that value. A probability of one is certainty, and the area under the Rayleigh distribution curve is one when integrated between zero and infinity. This means that in a Rayleigh distribution, while all of the values must fall within the curve, there is a small probability of occurrence of very high (theoretically infinite) peak accelerations.

\* of the beam

\*\* This statement is a simplification and is not precisely correct. The Rayleigh distribution is covered more completely in the Appendix.

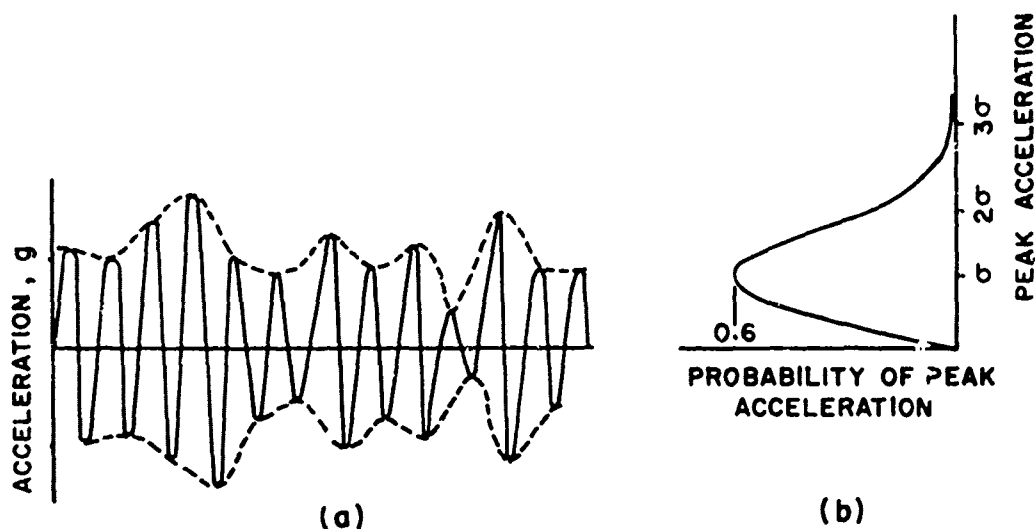


Figure 1-16. Random Vibration at a Single Frequency

1-48. Ordinarily, random vibration is more complex than the single frequency shown in figure 1-16a; this random-amplitude, single-frequency vibration is characteristic of the response of a single-degree-of-freedom structure to the complex random vibration, with the frequency of the response being the resonant frequency of the structure. It is also similar to the response of a bandpass filter with the frequency of the response being the center frequency of the filter. The bandpass filter is analogous to the high  $Q$  single-degree-of-freedom structure. The Rayleigh distribution depends upon the rms value of the acceleration and is independent of the frequency.

#### 1-49. ACOUSTICAL ENERGY.

1-50. The noise generated by jet and rocket engines is another form of excitation that can result in the fatigue failure of airborne electronic equipment. The high levels of sound impinge on the aircraft skin, and the sound energy is converted to mechanical energy that can eventually reach the equipment in the form of vibration. The sound, generally attenuated when it reaches the equipment compartment, can also impinge directly on the equipment. The following discussion of the nature of sound is intended to serve as background information for the frequent references to sound energy throughout the book.

1-51. A sound wave is a fluctuation in pressure transmitted through an elastic medium. A pressure change in an elastic medium is always accompanied by a change in particle position, particle velocity, and density of the medium, and a change in any of these is always accompanied by a change in pressure. A sound wave, therefore, could also be defined as a propagated change in any of these parameters. Progressive wave sound consists of successive sound waves (pressure fluctuations) moving through a medium. For progressive wave sound of a constant frequency, the pressure gradient along the line of propagation at any time will be as shown in figure 1-17. In figure 1-17a, the sound is assumed to be made up of plane progressive waves, and there is only a slight decrease in

peak pressure with distance from the source. In figure 1-17b the wave fronts are spherical and the peak pressure drops much more rapidly with distance from the source.

1-52. Plane progressive wave propagation, then, is a wave phenomenon in which the variable is pressure. As sound moves through the atmosphere, each portion of the air is alternately compressed and expanded. Sound in air can originate with a vibrating body which imparts energy to the adjacent air as in a loudspeaker, stringed instrument, or squeaking door hinge. It can also result from a combustion process in which, due to uneven burning, rapid pressure fluctuations exist in the gaseous combustion products as in a blow-torch or rocket motor, or from mechanically caused pressure variations as in an air stream which is rapidly chopped.

1-53. A localized high pressure area in an elastic medium is contained only by the forces exerted on the area by the surrounding material. A pressure wave moves through a material by a transfer of energy from the high pressure area to the adjacent material and, as in mechanical vibrations, the speed of energy transfer depends upon the inertial properties of the material receiving the energy and the flexibility of the transfer medium. The speed with which a sound moves through a medium is dependent upon the density (inertia) and elasticity (spring constant) of the medium. In the case of a gaseous medium, the compressibility of the gas is the elastic property. Due to the speed with which sound travels, there is insufficient time to allow heat transfer to occur within a gaseous medium and the expansions and compressions are adiabatic. For this reason, the equation for the velocity of sound in a gas contains the adiabatic constant. \*\* The following formulas for velocity of sound express the dependence upon density and elasticity (or compressibility).

Gaseous medium:

$c =$

$$\sqrt{\frac{Kpg}{\rho}}$$

Longitudinal wave in a bar

$c =$

$$\sqrt{\frac{Eg}{\rho}}$$

\* Noise can be defined as unwanted sound.

\*\* Ratio of specific heat for a constant pressure process to the specific heat for a constant volume process. The constant is 1.4 for air.

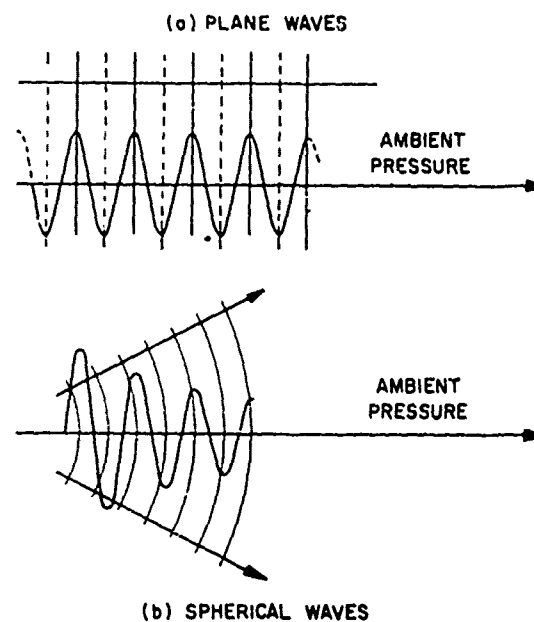


Figure 1-17. Pressure Gradient for Single-Frequency Sustained Sound

$$\text{Torsional wave in a bar} \quad c = \sqrt{\frac{Eg}{2\rho(\sigma + 1)}}$$

$c$  = velocity of sound                      in/sec                      cm/sec

$K$  = adiabatic constant =  $\frac{cp}{cv}$

$p$  = static pressure                      lbs/in<sup>2</sup>                      dynes/cm<sup>2</sup>

$g$  = acceleration due to gravity                      386 in/sec<sup>2</sup>                      1 cm/sec<sup>2</sup>

$\rho$  = density of medium                      lbs/in<sup>3</sup>                      grams/cm<sup>3</sup>

$E$  = Young's modulus of elasticity                      lbs/in<sup>2</sup>                      dynes/cm<sup>2</sup>

$\sigma$  = Poisson's ratio

1-54. Three terms which are commonly used in sound measurement are sound (or acoustic) power, sound intensity, and sound pressure. Sound power is the total power expressed in watts (one watt equals  $10^7$  ergs/sec or one joule/sec). Sound intensity is the power per unit area expressed in watts per square centimeter. Sound pressure (or effective sound pressure) is the root mean square of the instantaneous values of the pressure fluctuations and is expressed in dynes per square centimeter. The range of values encountered for any of these three characteristics is so great that the numbers become cumbersome. For this reason the logarithms of the numbers were adopted for convenience, and the use of logarithms led to the adoption of the decibel which was originally used for electrical power losses. The decibel is, of course, one-tenth of a bel, and a bel is the logarithm of the ratio of two powers.

$$db = 10 \log_{10} \frac{W_1}{W_2}$$

Although the decibel is correctly based on a ratio of powers, it is used for other ratios such as intensity and pressure. Since intensity is proportional to the square of the pressure, the decibel formula for sound pressure contains a factor of 2 ( $\log x^2 = 2 \log x$ ).

$$\text{Acoustic power level: } db = 10 \log_{10} \frac{W_1}{W_0}$$

$$\text{Sound intensity level: } db = 10 \log_{10} \frac{I_1}{I_0}$$

$$\text{Sound pressure level: } db = 20 \log_{10} \frac{P_1}{P_0}$$

A tenfold increase in intensity corresponds to a 10-db increase, and doubling the intensity adds approximately 3 db.

1-55. Since the decibel is a function of a ratio, if decibel units are to be used to express a single absolute quantity, a reference level must be chosen in order that the ratio of the quantity to the reference may be established. In all cases where decibel units are used, the reference level should be stated. For sound in air, the generally accepted reference levels are as follows:

- A. Sound pressure reference.  $2 \times 10^{-4}$  dynes/cm<sup>2</sup>. This represents the average threshold of hearing for young ears for a 1000-cps note.
- B. Sound intensity reference.  $10^{-12}$  watts/cm<sup>2</sup>. In air, this corresponds to a sound pressure of  $2 \times 10^{-4}$  dynes/cm<sup>2</sup>.
- C. Acoustic power reference.  $10^{-13}$  watts. Approximately equal to the power over one square foot at an intensity of  $10^{-16}$  watts/cm<sup>2</sup>.

1-56. When sound power, sound intensity, or sound pressure is expressed in terms of decibels, the word "level" should properly be added and they should be referred to as sound power level, sound intensity level, or sound pressure level.

1-57. Other systems of units have been devised for sound and, although they are not generally seen in engineering, it is well to be able to recognize the units. A phon is a unit of loudness used by psychological workers, the number of phons being numerically equal to the db level of a 1000-cps sound of equal loudness. The response of the human ear is not flat with respect to frequency, and the shape of the response curve depends upon loudness; therefore, for a given loudness level in phons, the db level is dependent upon frequency and loudness. Another scale used by psychologists is the sone and is based upon the fact that people can judge one sound to be "so many times as loud" as another sound. A 40-db, 1000-cps sound has a defined loudness of one sone. A sound which is  $n$  times as loud as a one-sone sound has a loudness of  $n$  sones.

#### 1-58. EQUIPMENT RELIABILITY IN A MECHANICAL ENVIRONMENT.

1-59. The term reliability, as applied to electronic equipment, signifies that the equipment will operate satisfactorily for a desired period of time after exposure to or in all anticipated environments. Reliability implies that critical moving parts (e.g., relay contacts) will not change their positions and cause malfunction, or that spacing between parts will not change and alter electrical characteristics (e.g., capacitor plates, grid and cathode of tubes). Progressive deterioration can also destroy reliability, since improperly designed structures and components subjected to recurrent shocks and vibrations can fail eventually because of fatigue. The problem of designing reliable electronic equipment becomes more and more difficult as the complexity of the equipment increases. In a small electronic assembly containing 50 component parts, each of which is 99.9% reliable in the particular application, the probability of failure is 3.8%. If the number of parts were 500, the probability of failure would be 39%; and if the number of parts were 2000, the probability of failure would be 82.7%.

#### 1-60. EQUIPMENT FAILURE IN A MECHANICAL ENVIRONMENT.

1-61. The theory of vibration and shock previously described concerned single masses. It was stated that, according to the system of mounting, each mass would have one or more natural frequencies or natural periods. In an electronic equipment made up of hundreds of individual masses (resistors, capacitors, tube filaments and plates, etc), each of these masses may have one or more natural frequencies or natural periods. If the natural frequencies lie within the range of frequencies imposed upon the equipment, various components will come into resonance at various points in the range of excitation frequencies. Depending on the

nature of the shock pulse, different components will react in accordance with their individual natural periods.

1-62. **COMPLETE FAILURE.** Material or parts failures result from mechanical stresses imposed within the material. They can occur either through fatigue, excessive single stress, or excessive deflection. Although fatigue failure usually implies a large number of stress cycles, the time required for these stresses to accumulate is short when a component is vibrating at hundreds of cycles per second. Excessive single stress may cause brackets or other supporting structures to yield or fracture. Excessive deflections of parts may result in their hitting one another with resultantly high impacts.

1-63. **MALFUNCTION.** Malfunction arising from changes in the electrical characteristics of the equipment may be exemplified by the opening or closing of relays and the displacement of tube elements under heavy single stresses. With vibration, resonant frequencies may cause relays to chatter or the elements of tubes to create microphonics.

#### 1-64. **DESIGN APPROACHES.**

1-65. When designing equipment to resist malfunction or rupture of component parts, several approaches may be used. It is possible (1) to reduce or remove the excitation at the source, (2) to attenuate the excitation between the source and the equipment, or (3) to design the equipment to withstand the excitation.

1-66. **SOURCE SUPPRESSION.** Reducing or removing sources of vibration and shock excitation is mainly the concern of the airframe manufacturer. Engines are mounted on isolators, rotating parts are balanced, and damping is introduced through the use of paints or other such skin coverings and the use of sound suppressors (mufflers). The environment existing at the point of equipment attachment is thus determined outside the realm of the equipment designer, and it is often a problem for him simply to get information on what constitutes the true environment.

1-67. **EXCITATION ATTENUATION (ISOLATION).** The attenuation of excitation between the airframe and the equipment is accomplished by the use of isolators. An isolation system effectively alters the environment by changing the level (amplitude) of vibration which the equipment experiences at each frequency. Consequently, the amplitude that the mounted equipment will experience is dependent upon the natural frequencies of the equipment on its mounts and the exciting frequencies in the support. However, isolation mounting may also introduce a rotational natural frequency which may not exist for a rigidly mounted equipment.

1-68. As shown in the transmissibility curve of an undamped single-degree-of-freedom system (figure 1-5), isolation takes place when the ratio of exciting frequency to natural frequency is above 1.414; when the exciting frequency is equal to the natural frequency (resonance), the transmissibility is very high. (Damping is used in commercial isolation mounts to lessen the transmissibility at resonance.) Where the exciting frequency is below 1.414 times the natural frequency, the equipment will be subjected to more severe vibration than if a rigid mounting were used. In general then, an equipment mounted on isolators depends, for vibration attenuation, upon having the exciting frequency of the environment above

the natural frequency of the mount system. For exciting frequencies well above the natural frequencies, the transmissibility is low and the equipment will experience only a small percentage of the vibration of the support.

1-69. The ability of the electronic equipment to withstand shock is dependent on its inherent shock resistance and the ability of the shock or vibration mount to attenuate the magnitude of the shock. Resistance to shock is aided by using materials which can absorb energy through elastic straining. When the shock results in a permanent change in velocity, the shock mounts attenuate by increasing the time over which the change in velocity occurs. This reduces the peak acceleration of the equipment, and, therefore, its g loading.

1-70. The use of an isolation system requires that space be allotted not only for the placement of mounts but also for the excursion of the equipment. Equipments on mounts are subject to motions in several directions and sufficient space must be left around the equipment so that the equipment will not strike adjacent structures or other equipments.

1-71. In environments composed of both shock and vibration, the choice of isolators is particularly difficult. A soft, "vibration" isolator does not have the capacity for storing large amounts of energy since the deflection necessary for so doing would be impractically large. A high-impact shock which produces considerable velocity change may bottom a vibration isolator resulting in a shock to the equipment of higher acceleration than the original shock. A harder "shock" isolator, while it does have a higher energy-storing capacity, also has a higher natural frequency which may not be low enough to provide ideal vibration isolation.

1-72. **EXCITATION RESISTANCE.** If an equipment is used without isolators, it will be subjected to the shock loadings and the amplitudes and frequencies of the vibration of the supporting structure. Rigidizing, damping, and miniaturizing are three methods for designing equipment to resist excitation. Rigidizing is intended to make the natural frequencies of the equipment and components fall above the excitation frequency range. During rigidizing, the ratio of natural frequency of the equipment and its components to the exciting frequency should be made as high as possible to approach a transmissibility of one (which is the best that can be accomplished by rigidization).

1-73. In many cases, particularly in missiles, the excitation frequency range is so high that it is impractical to rigidize so that all internal natural frequencies will be above the excitation frequencies. In these cases, if possible, attempts are made to choose frequencies in a range in which the excitation level is low. In any event, when the internal natural frequencies coincide with the excitation frequencies, inherent damping must be depended upon to limit the magnification due to resonance. Additional damping is sometimes added to or designed into an equipment to decrease the response to frequencies near the resonant frequency.

1-74. Miniaturization lends itself to designing for resistance to mechanical environments. The small dimensions of miniaturized equipment result in more rigid construction with higher natural frequencies, and, in addition, the small masses have low inertia so that the forces imposed under a given acceleration level are lower.

#### 1-75. ISOLATION VERSUS RESISTANCE.

1-76. While a large segment of the airborne electronics design industry uses isolation mounts as standard procedure for all equipments, a second group, mainly those designing equipment for installation in missiles, believes that isolation mounts are either unnecessary or disadvantageous. Mounts with resilient elements that will resist bottoming when exposed to the severe shocks of the missile environment will have a natural frequency so high as to fall in the vibration excitation range. If isolators are used with a natural frequency below the excitation range, the mounts will bottom magnifying the shock forces. There are various considerations that can be helpful in deciding whether to design an equipment to be mounted rigidly or on shock mounts.

1-77. VERSATILITY OF ISOLATORS. During the life of an equipment, its location and the type of vehicle using it may change. An equipment designed for one set of frequencies and amplitudes may not be workable in a changed environment. Isolators tend to attenuate most frequencies. Also, the effects of random vibration are more easily predicted for an isolator installation.

1-78. COST AND WEIGHT CONSIDERATIONS. The cost and weight of designing and manufacturing components to better withstand shock and vibration must be considered. Also, the additional weight, cost and space required for isolators must be weighed in any decision.

1-79. ISOLATOR BOTTOMING. For a shock condition, the shock forces which must be tolerated (and the component still remain functional) may cause bottoming of any isolators and subsequently subject the equipment to a higher loading than it would receive if mounted solidly.

1-80. SHOCK MAGNITUDES MORE EASILY PREDICTED FOR RIGID MOUNTINGS. If the equipment is mounted solidly, the shock it will receive is that imposed upon it by the support. This magnitude is known within limits and all parts can be designed to this value. However, the shock loading due to striking surfaces or stops is not well defined and designing for this may be difficult.

## SECTION II

### VIBRATION AND SHOCK ENVIRONMENTS

#### 2-1. VIBRATION AND SHOCK DATA.

2-2. **VIBRATION DATA.** There are two schools of thought on how to interpret flight vibration data, depending upon whether the purpose of a program is to investigate structural failures or installed equipment failures. In both cases, the flight data represents local responses of the airframe to various flight-condition inputs. Structural failure investigators usually consider only the maximum value of the airframe response as significant, since this is directly related to maximum stress in the structure. (The number of repetitions of this stress will indicate whether fatigue failure is probable.) To determine the possibility of equipment failure, it is necessary to evaluate thoroughly the airframe responses in the vicinity of the equipment because of the many natural frequencies present in an equipment. Since most equipment failures are caused either by structural rupture due to fatigue or by malfunctioning of components at resonance, it is of prime importance to obtain accurate frequency and amplitude information. Hence, in contrast to the structural type of data analysis, it is necessary to analyze harmonically all recorded data so as to ascertain the true contribution of the various frequencies and amplitudes present in a complex waveform.

2-3. **SHOCK DATA.** The shocks to which airborne electronic equipments are subjected are the end results of shock loads which have been applied to the aircraft and transmitted through the airframe to the point of equipment attachment. Because of the elasticity of the airframe, applied shocks are received by the equipment as transient vibrations. These transient vibrations often have considerable amplitude and occur at various frequencies, depending upon (1) the natural vibrational frequencies of the airframe and (2) the frequency components of the applied shock load. However, the transient vibration transmitted to the equipment is well damped by the airframe; and, when the equipment receives it, it is, in effect, a shock pulse. This new shock pulse may differ considerably from the shock applied to the aircraft, making it necessary to measure the shock at the equipment locations.

#### 2-4. DATA PRESENTATION.

2-5. **STEADY-STATE VIBRATION.** In contrast to the problems associated with transient-environment data, mathematical relations exist for computing the response motions of systems subjected to steady-state vibration environments. It is most convenient, in this instance, to define the environment in frequency and amplitude parameters descriptive of the disturbance itself. (Also, a vibration environment expressed in frequency and amplitude is readily simulated in vibration test machinery.)

2-6. **RANDOM VIBRATION.** Practically all of the vibration data recorded in missiles to date is nonperiodic and almost completely random. Assuming that

missile vibration is truly random and that theoretically infinite peak acceleration values are possible, the conventional method of describing vibration by plotting peak acceleration against frequency is not realistic. Based on this reasoning, investigators have proposed the use of a method based on the statistical properties of the random vibration. If the Rayleigh distribution of a random vibration does not change with time, then it can be used to describe the severity of the vibration by describing the response to the vibration at a given frequency. Since the Rayleigh distribution is a function only of the rms value of the response, either the rms value or the square of this value (mean squared acceleration) can be used to describe the vibration. Since the rms value obtained by playing a recording of the random vibration through a bandpass filter is dependent upon the bandwidth of the filter, the units chosen for plotting were  $g^2/\text{cps}$ , which are obtained by dividing the mean squared acceleration by the bandwidth of the filter. When  $g^2/\text{cps}$ , which is referred to as the "mean squared acceleration density," is plotted against the center frequency of the bandpass filter, the resulting plot is termed a "mean squared acceleration density spectrum" or a "power density spectrum."

2-7. For testing purposes, reproduction of a vibration environment which is defined in terms of a mean square acceleration density spectrum may be accomplished with an electrodynamic exciter using an input signal which originates in a random signal generator and is shaped by filters and amplifiers. It is also possible, of course, to generalize random vibration data by using a flat spectrum (white noise excitation) for testing purposes. The need for random test excitations is not universally accepted and methods for determining "equivalent" single-frequency testing have been devised.

2-8. The use of a mean square acceleration density spectrum is based on the assumption that the vibration is continuous in frequency over the range of interest and that the instantaneous accelerations have a normal probability. It is not certain that the service vibrations are continuous in frequency over the range of interest, and other forms of data presentation are encountered which attempt to examine the basic nature of the vibration.

2-9. SHOCK. Although a shock motion may be defined by describing the time history of displacement, velocity, or acceleration, visual examination of a time-history curve does not lead to a numerical value which can be used for structural design. Data defining a given shock is conveniently summarized by maximum response acceleration as a function of natural frequency. Data in this form is referred to as "shock spectra." If several single-degree-of-freedom components with various natural frequencies are subjected to a shock motion and the maximum response of each component is measured, the results may be summarized by a plot of maximum response versus natural frequency. This plot may be used to predict the response of any single-degree-of-freedom system to the shock motion when the natural frequency of that system is known. However, the response of a system is dependent upon the damping in the system and a shock spectrum is meaningless unless the amount of damping used in recording it is known. Although it is theoretically possible to determine a shock spectrum by mechanical means, it is more convenient to introduce the input to an electrical analog of the system to determine the response. \* The problem involved in using such an analysis is the

\* The pulse is fed into a resonant circuit with a controlled amount of damping and the response at each natural frequency is measured.

inability to duplicate the damping from the actual model to the test article

#### 2-10. APPLICABILITY OF FIELD DATA. \*

2-11. The field data presented in the following portions of this section has been culled from a large quantity of material. Attempts have been made to use the latest and/or the most reliable data available. While much field data exists, in some cases the data is questionable and in other cases it is not available for a particular condition.

2-12. Field data is presented to show the types of environment that have been encountered to date. Assuming no radical design changes, these environments are indicative of those that may be expected in future designs. Although generalized data is sometimes used in preliminary design, it should not be used in place of data for the specific application, if available, since generalized data includes several types of aircraft under many conditions.

#### 2-13. TURBOJET AIRCRAFT.

2-14. The shocks and vibrations encountered in turbojet aircraft arise from many sources. The engine produces random excitation and vibrations at multiples and fractions of the engine speed. Minor vibrations result from combustion, unbalance in the engine, and accessory unbalance. The high rotational speeds of jet-engine turbines result in high-frequency, engine-induced vibrations. The amount of this vibration felt at a particular location depends upon its proximity to the engine. The major excitation source in turbojet aircraft is high-intensity noise, which can impinge on the airframe or equipment; much of the vibration experienced by turbojet-engine aircraft is induced by this noise. The wings, empennage, and fuselage of the aircraft are subjected to periodic aero-elastic vibrations and to shock and transient-vibration loads due to wind gusts and buffeting. During taxiing and landing, the aircraft receives shock loads and transient vibrations; the transient vibrations due to taxiing and landing are at different frequencies than the vibrations experienced in the air, since the natural frequencies of the aircraft structure when on the ground are different than when it is airborne. The igniters of JATO units sometimes cause transient forces of considerable magnitude, and shock may result from rupture of the nozzle closure, if any. During flight maneuvers, particularly in fighter aircraft, high g loads of relatively long duration are imposed upon the aircraft concurrent with the usual flight vibrations. Crash landings impose high shock loads on aircraft, and, although the equipment may not be required to operate afterward, the mount system must prevent the equipment from breaking loose entirely.

2-15. VIBRATION. Vibration in aircraft varies, depending upon the type of craft. A small plane has higher structural natural frequencies than does a large plane and will, therefore, transmit a different vibration spectrum to the equipment locations. The environment varies, depending upon the location of the equipment within the aircraft. A large bomber, for instance, may have low-frequency, high-amplitude vibration, particularly at the tail or wingtips, where the effects of flexibility are exaggerated. Also, the basic differences in performance requirements will result

\* The data presented is not intended to supersede either detailed or general specifications.

in varied input excitations; maneuvers by military fighter aircraft will subject equipment to a large, steady-state acceleration. For example, during dive pull-out, steady-state accelerations up to  $+7g$  concurrent with the usual flight vibrations can occur. One aircraft manufacturer has chosen  $4g$ , steady-state acceleration with superimposed vibration up to 500 cps as a realistic requirement for fighter craft.

2-16. Figure 2-1 is a plot of double amplitude versus frequency for turbojet-engine fighter aircraft. The figure shows vibration at frequencies up to 2000 cps at low  $g$  levels. Only two data points fall above  $4g$ , and for the F-102A aircraft, which is reported as being a very quiet aircraft (vibrationally), all data points fall below  $0.5g$ . Figure 2-2 is a plot of acceleration level versus frequency for two turbojet-engine bomber aircraft. The acceleration levels for these bombers are higher in the high-frequency range than those shown in figure 2-1. This appears contrary to what could be expected, considering the lower airframe natural frequencies present in bombers. It must be remembered, however, that in most fighter aircraft the engine exhaust is behind the plane, whereas in bomber aircraft, the engines are mounted in such a position that portions of the aircraft are subject to impingement of high-level acoustic excitation from the exhaust.

2-17. SHOCK. The shock an equipment receives is also dependent upon the performance and structural flexibility of the plane. A ranking of planes in order of decreasing landing shock would place carrier-based fighters first, land-based fighters second, and large land-based planes third. Admittedly, the landing shocks encountered can vary greatly with any particular craft, and it is possible within limits to make soft and hard landings with any plane. However, it is more difficult to land a "hot" plane softly, particularly on the deck of a carrier. For large, land-based craft, the shocks transmitted to the equipment are less severe. Because of the performance level and the structural flexibility of the airframe, which responds to the shock by transiently vibrating or by acting as an isolator, the landing shock is often no worse than a wind gust and may be considered to fall within the envelope of flight vibrations.

2-18. Figures 2-3 and 2-4 present landing shock spectra for land-based aircraft and carrier-based aircraft respectively. The spectra of figure 2-3 are envelopes of several landing records for each aircraft. In some of these landing tests, the pilots were instructed to "land hard" and, consequently, the spectra may be considered to be close to the maximum expected service conditions. The spectra of figure 2-4, for carrier-based aircraft, are much more severe than are those of figure 2-3.

#### 2-19. MISSILES.

2-20. Missiles, in general, present rather severe shock and vibration environments. Missiles are subject to shock and transient vibrations prior to launching due to ground handling, and during takeoff and flight due to igniters, boosters, motor cutoff, and booster separations. During the initial stages of flight and after motor cutoff, there are acceleration forces along the missile axis. Acceleration forces result from flight maneuvers, and during flight the missiles are subjected to aerodynamic buffeting and gust loading. The highest levels of missile vibration generally occur during the boost or lift-off phase and only exist for a short time.

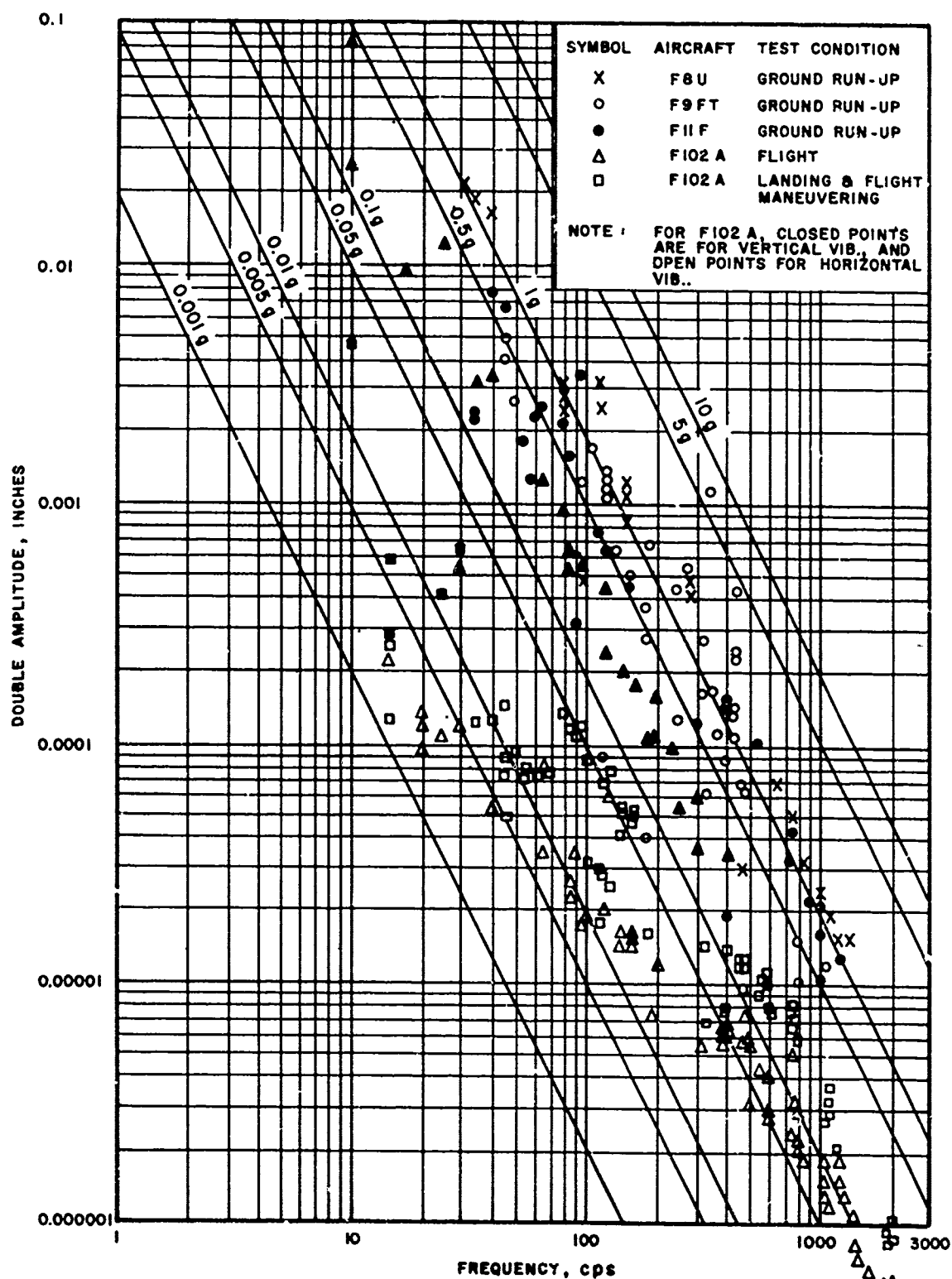


Figure 2-1. Vibration Data for Jet-Engine Fighter Aircraft

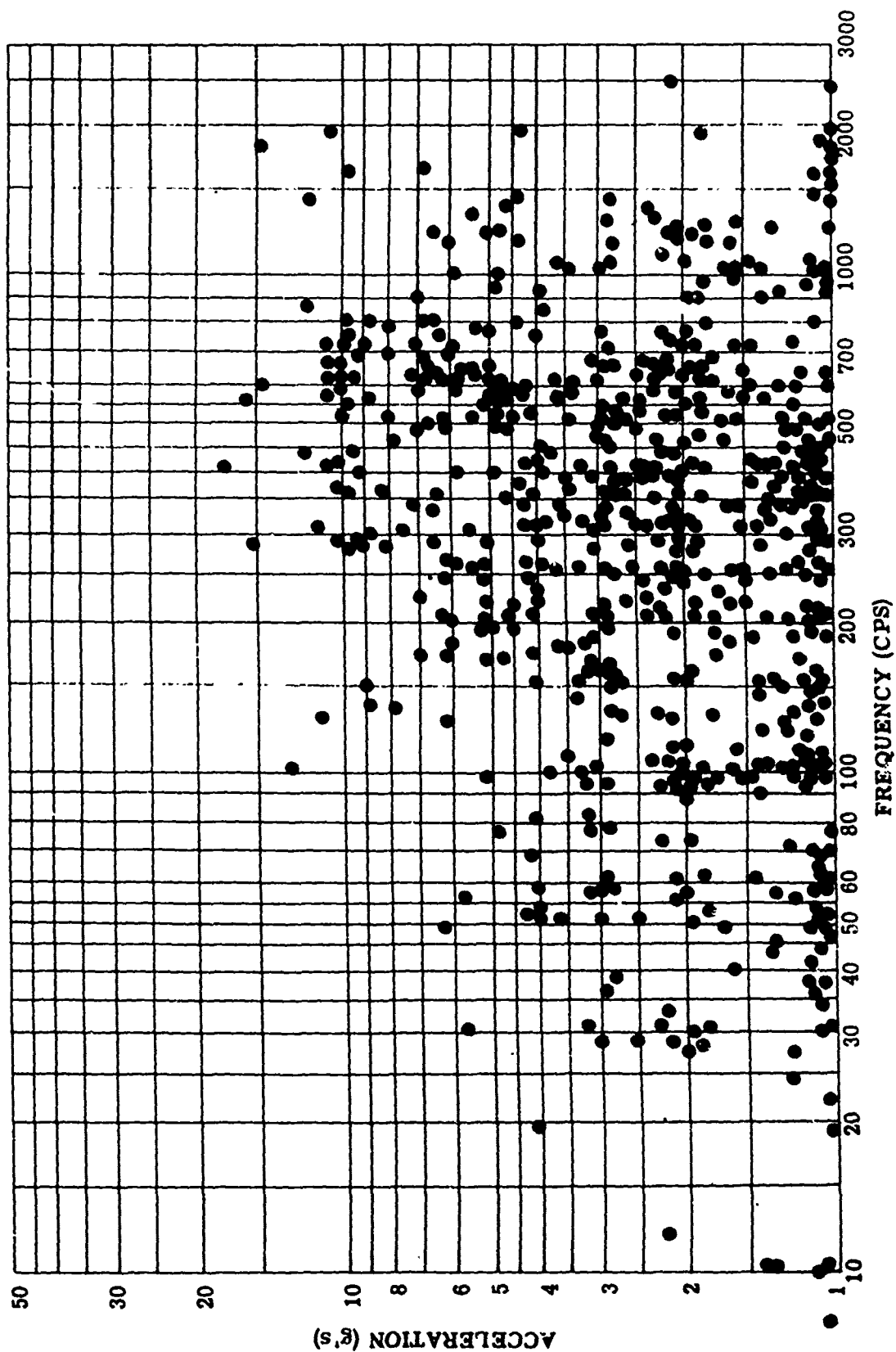


Figure 2-2. Vibration Data for the B-47 and B-52 Aircraft

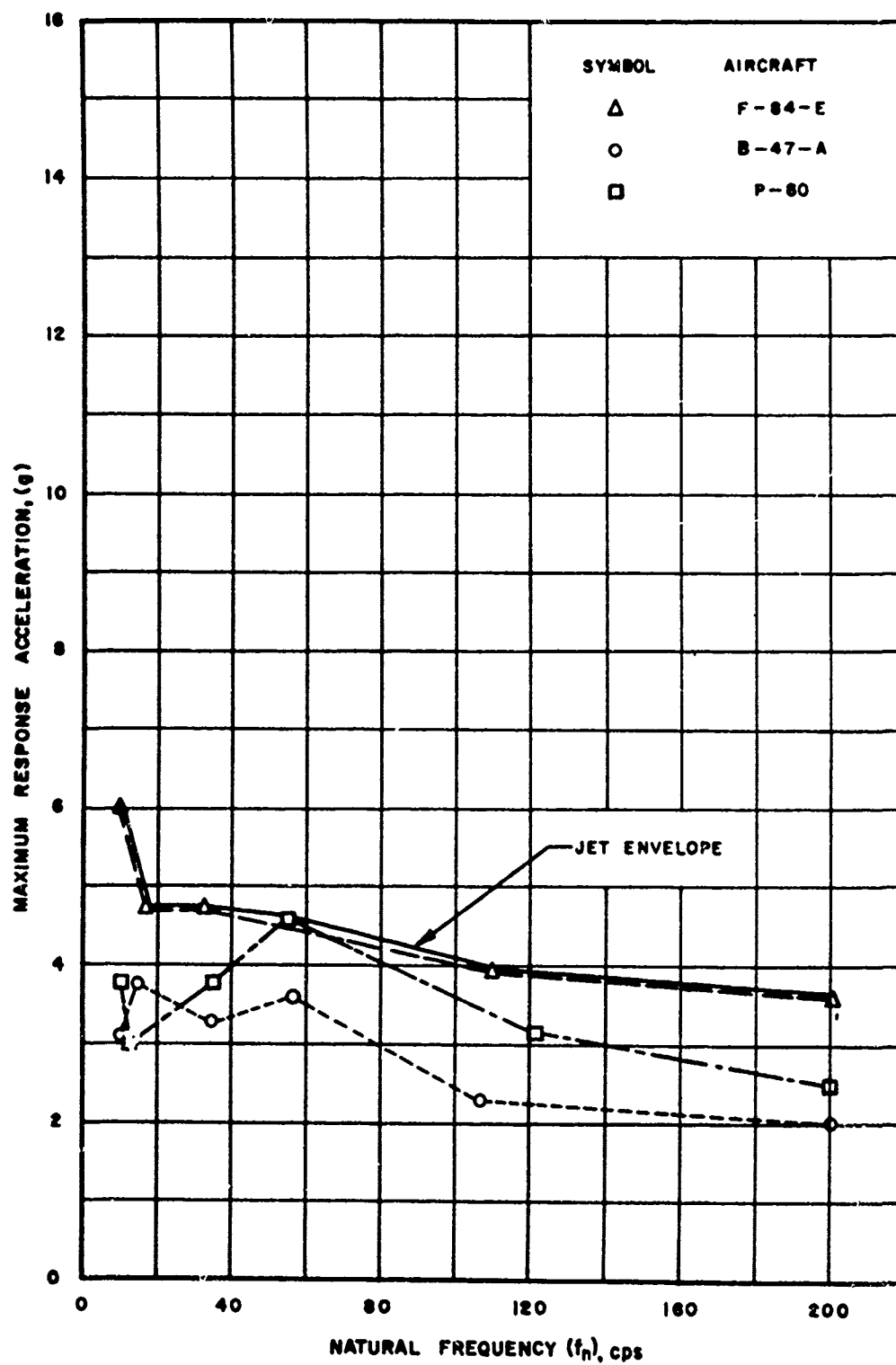


Figure 2-3. Shock Spectra for Land-Based Jet Aircraft

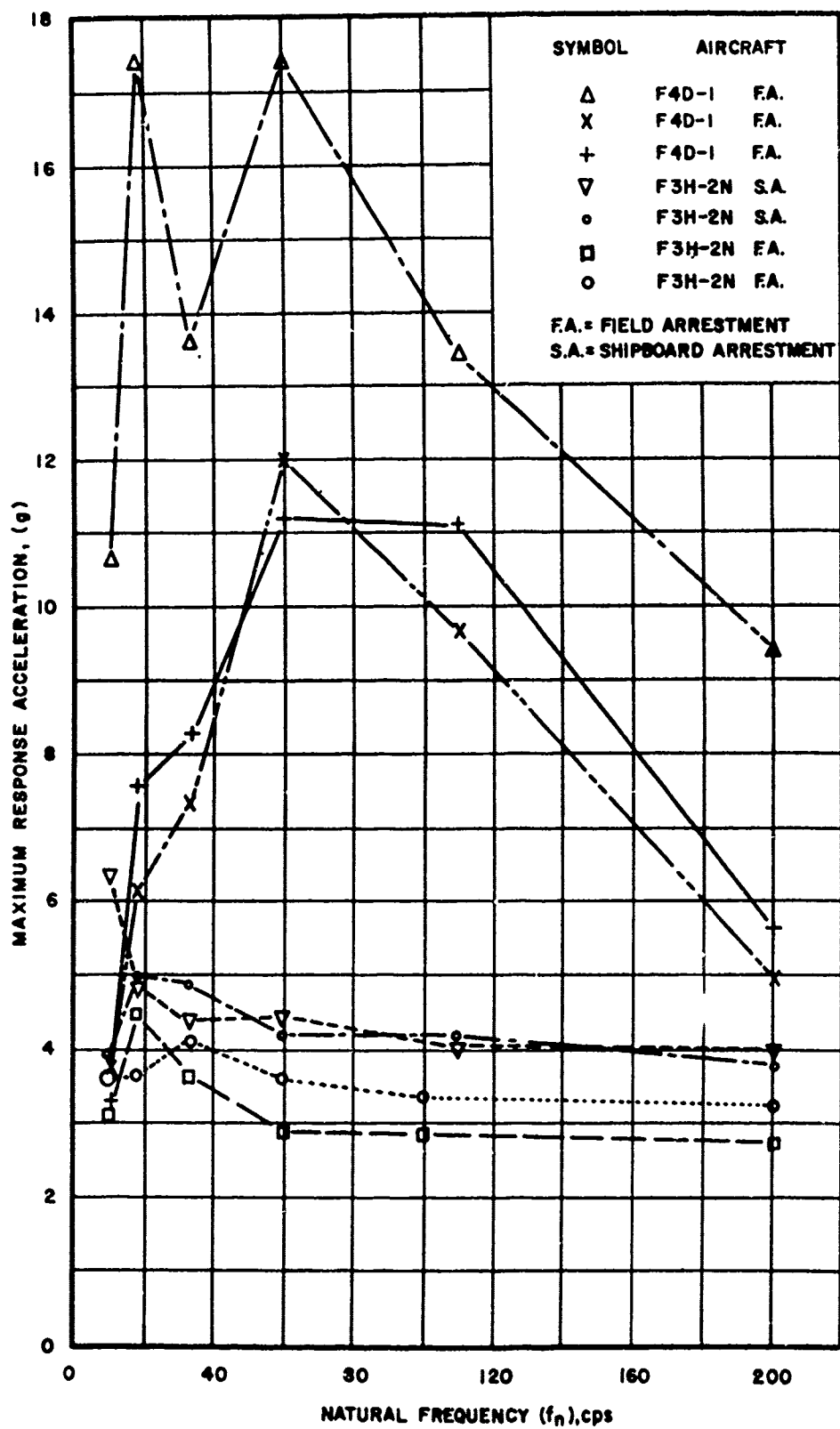


Figure 2-4. Shock Spectra for Carrier-Based Jet Aircraft

2-21. Reliable missile data is scarce in comparison with data for aircraft. The vibrations recorded have been complex and random\* in amplitude; for this reason, statistical methods have been proposed as the best means for expressing missile vibration environments. The lack of uniform means of data reduction, however, tends to complicate the presentation of data. The reduced data often does not contain enough information on the nature of the environment to allow an accurate comparison between data groups. The expense of test-firing a missile and the limited number of telemetering channels available per missile are additional reasons for the scarcity of reliable information.

2-22. Figures 2-5 to 2-10 give vibration data for four different missiles. The figures are summaries of data from either several locations within an individual missile or from several flights of a particular missile type. The telemetered accelerometer signals were recorded on magnetic tape and each signal was filtered through band-pass filters. The distribution of peaks in each filtered signal were then examined.

2-23. The data for missile A (figure 2-5) is based on measurements at six stations within the missile during a single flight. The acceleration peaks of the filtered signals were found to be distributed in accordance with a Rayleigh distribution and for each filtered signal the rms acceleration and the maximum acceleration which occurred were recorded. Figure 2-5 shows the average and greatest value of the rms and maximum accelerations at each filter center frequency. The data in figures 2-6 to 2-10 was obtained in the same general way as that of figure 2-5; however, in some cases, it was not as complete as that for missile A, and some assumptions were necessary for showing the data in this form. Also, the source of the data in figures 2-5 to 2-10 does not indicate the bandwidth of the filters used in reducing the data although the filter output would depend upon the bandwidths, unless the vibration existed only at discrete frequencies and the filter was tuned to these frequencies.

2-24. Figure 2-11 shows two envelopes of maximum values of mean square acceleration density. For missile E, the curve is the envelope of data for three directions at each of four stations within the missile. The individual mean squared acceleration density spectra exhibited sharp peaks showing that the energy distribution over the frequency range was far from uniform. The envelope for missile F is for data from three accelerometers on a single flight. The accelerometers were attached to an electronic chassis with their axes mutually perpendicular. The portions of the telemetered signals selected for analysis were chosen intentionally as representative of the most severe vibration.

## 2-25. HELICOPTERS.

2-26. The problem of shock and vibration in helicopters is unique because of the use of a moving airfoil as the sole means of obtaining lift. Vibrations may be due to the engine, the reduction gears, the universal joints, and the rotors. The basic

\* The term "random" is used here somewhat loosely to suggest the impossibility of accurately predicting or defining the exact amplitude of the vibration at any instant.

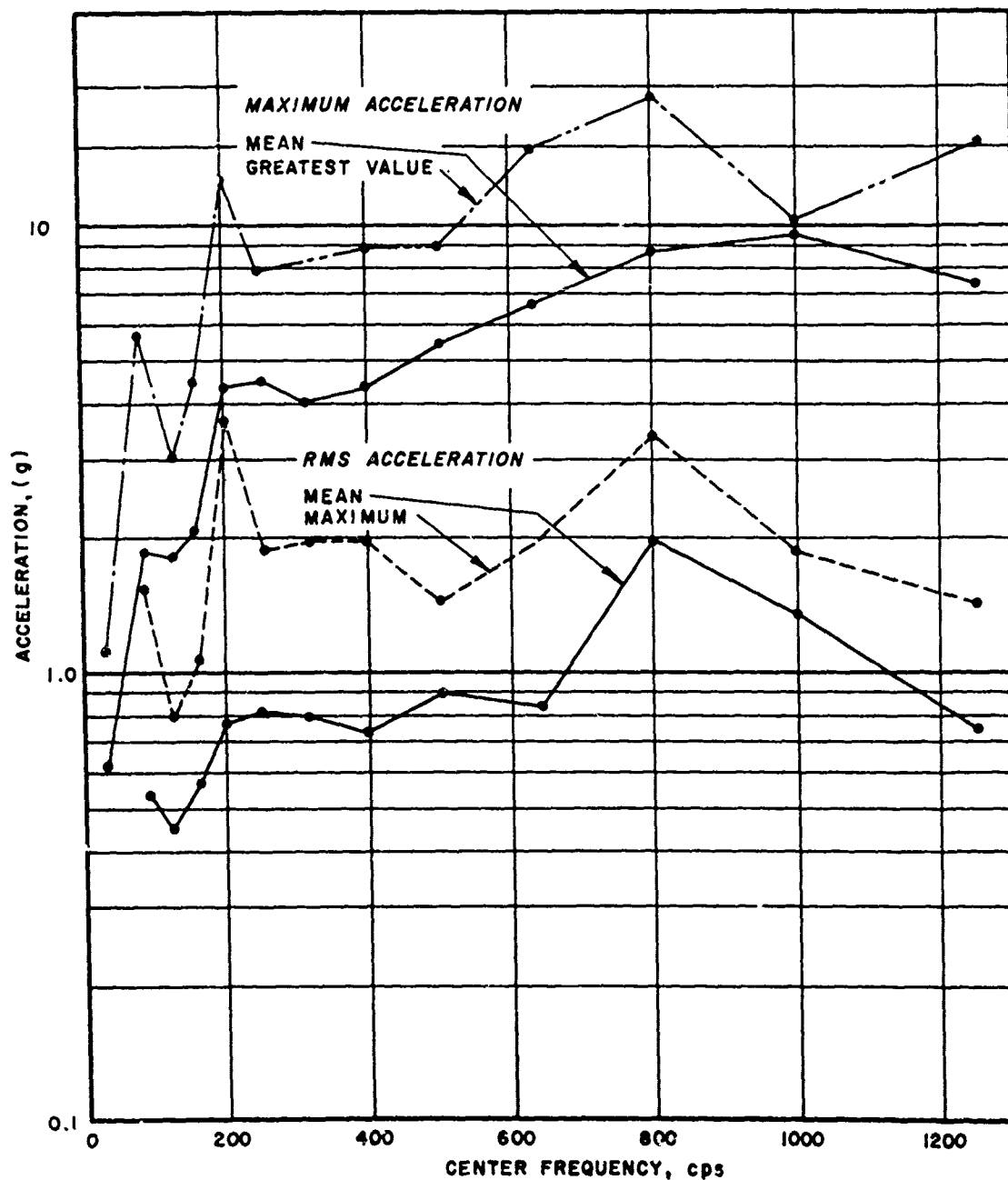


Figure 2-5. Maximum and RMS Accelerations for Missile A

engine vibrations in helicopters are similar to those encountered in winged aircraft with like power plants. Vibrations arising from gear reduction are more likely in helicopters than in fixed-wing aircraft because of the greater difference between engine speed and blade speed; however, transmission parts lend themselves to accurate balancing, and transmission vibration due to unbalance is usually of low magnitude. Tooth loads on the transmission reduction gears produce high-frequency forces which may be important in some helicopters. Universal

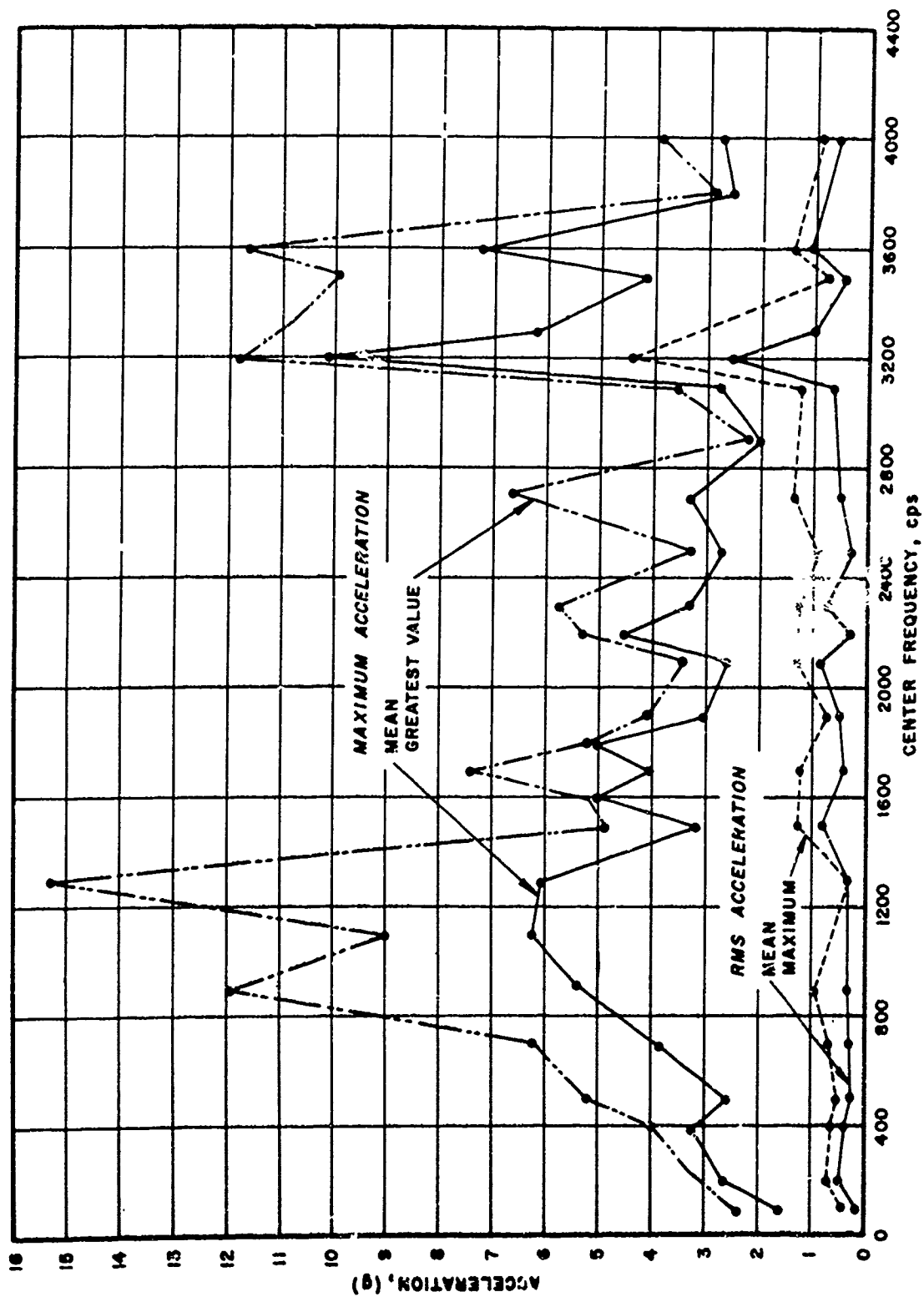


Figure 2-6. Maximum and RMS Accelerations for Missile B. Boost Phase

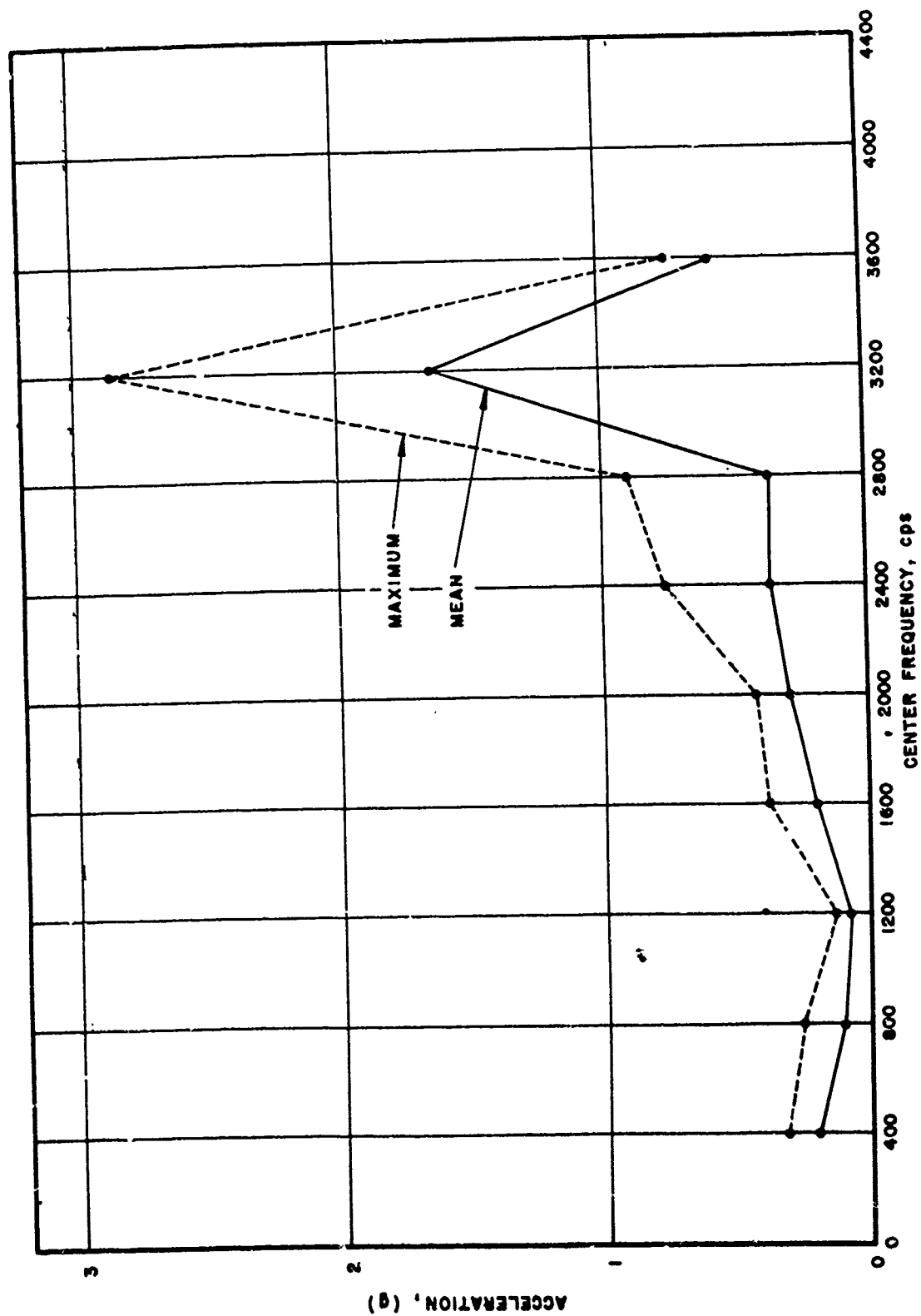


Figure 2-7. RMS Accelerations for Missile B, Sustained Flight

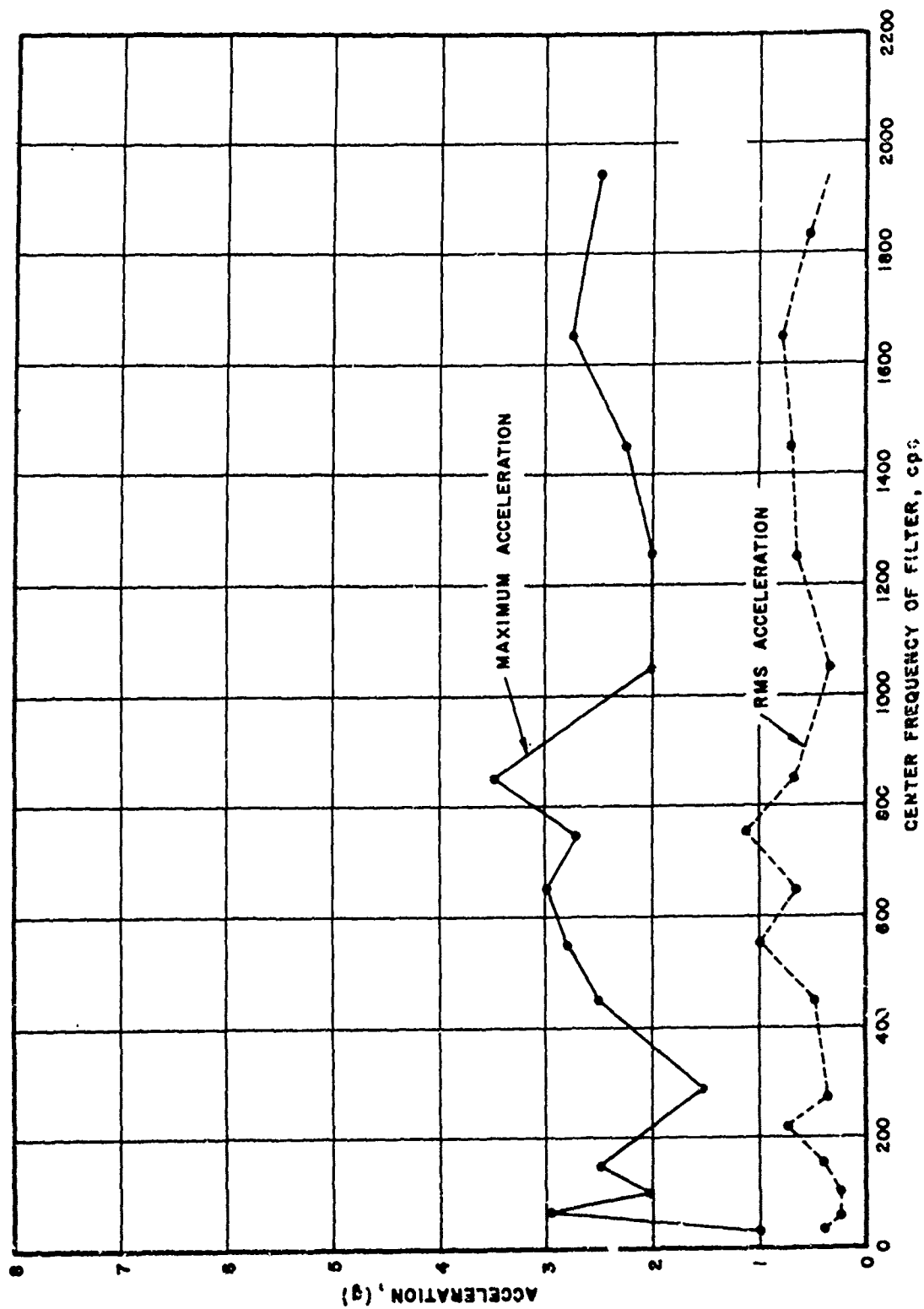


Figure 2-8. Maximum and RMS Accelerations for Missile C, Lift-Off

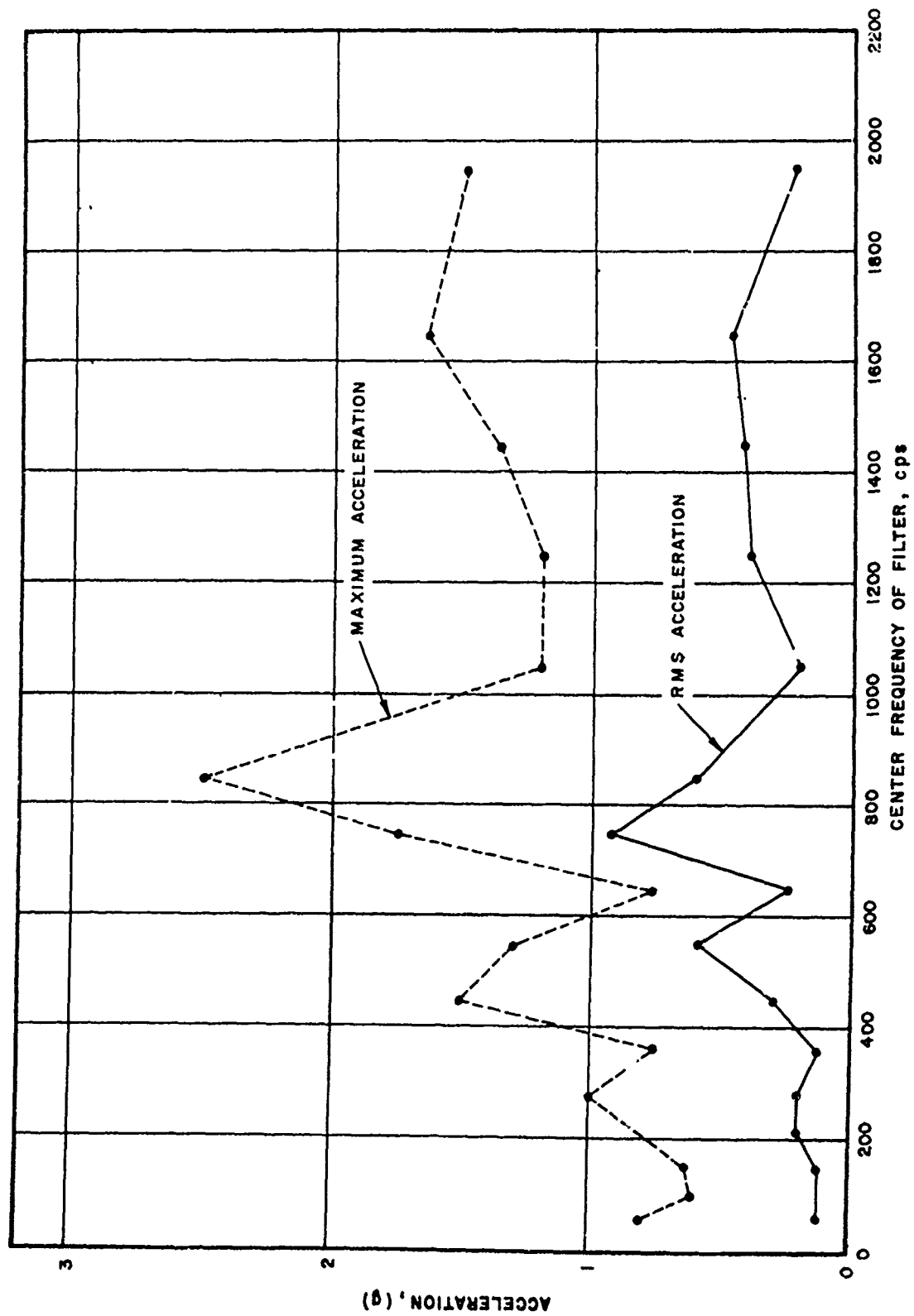


Figure 2-9. Maximum and RMS Accelerations for Missile C, Sustained Flight

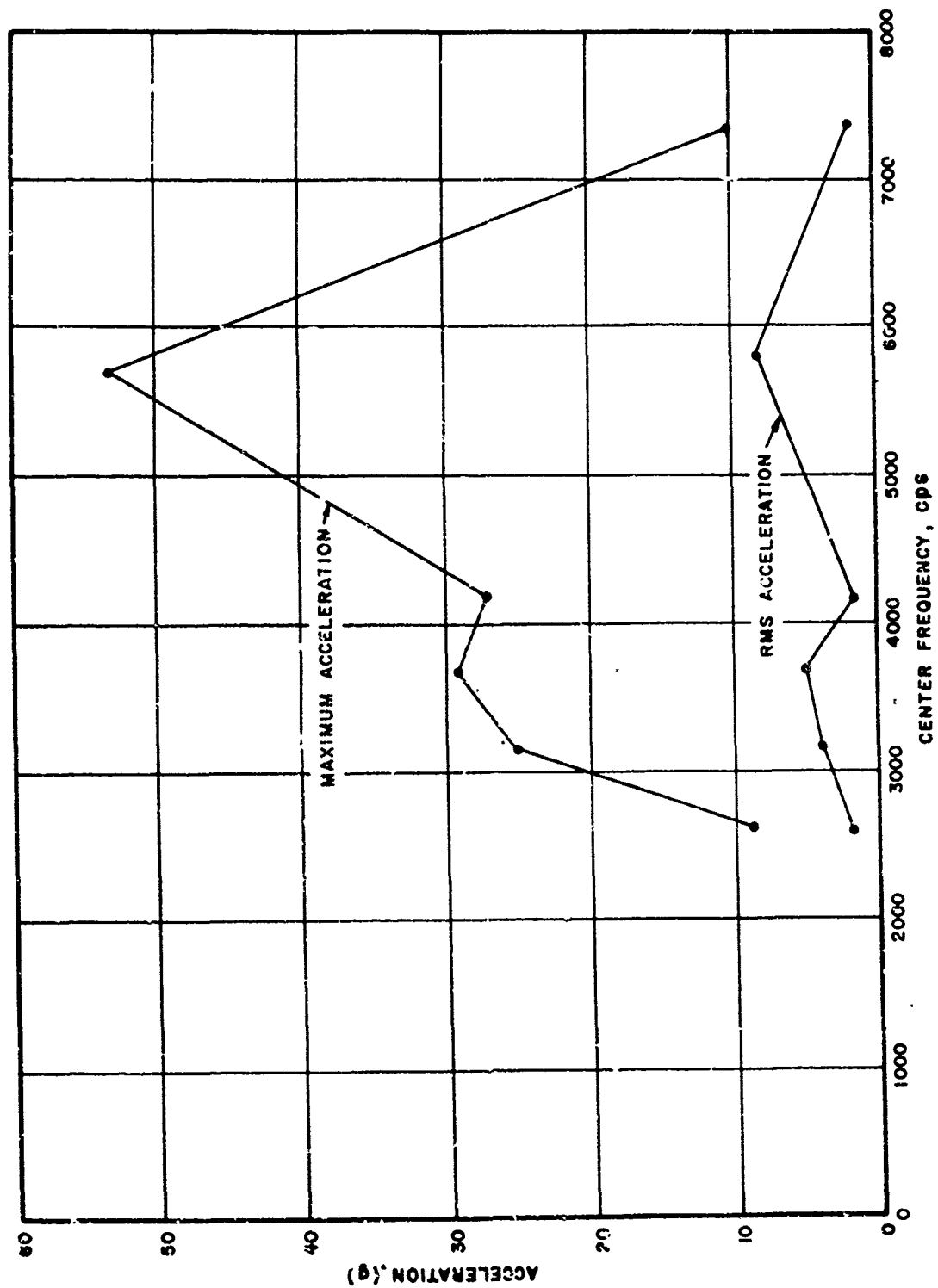


Figure 2-10. Maximum and RMS Accelerations for Missile D

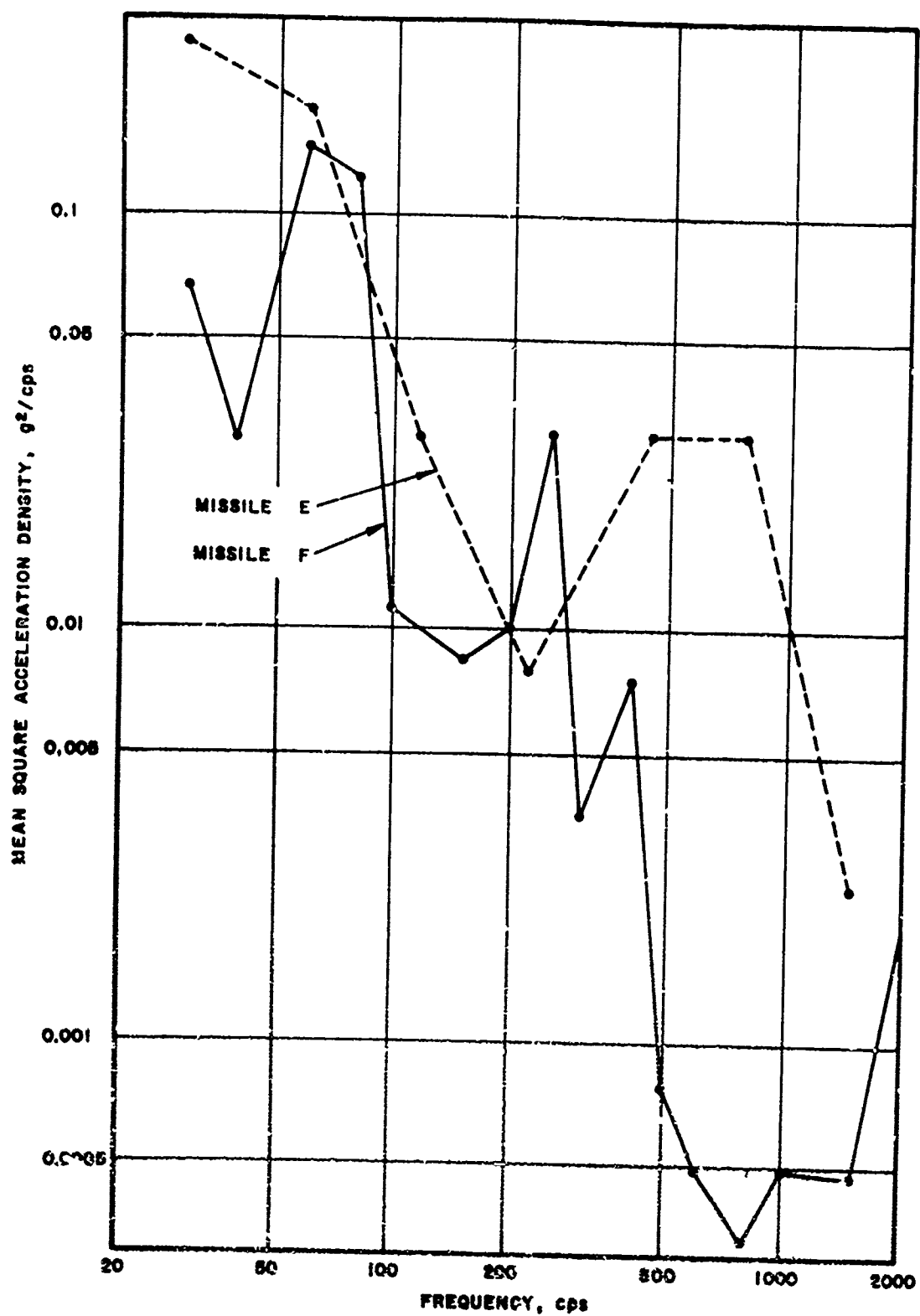


Figure 2-11. Envelopes of Mean Square Acceleration Density for Two Missiles

joints, especially those that are not of the constant-velocity type, can produce forces of considerable magnitude. The main rotor of the helicopter produces vibrations at frequencies corresponding to the number of blades times multiples of the rotor speed. In the case of helicopters which have an antitorque tail rotor, vibrations due to this rotor will be present in the environment, although they are often negligible because of attenuation in passing through the tail boom. The response of the helicopter fuselage is different when the craft is on the ground than when it is airborne, since the natural frequencies of the structure are different. Therefore, ground as well as flight operation must be considered in determining the excitations of equipment supports.

2-27. Figure 2-12 is a plot of double amplitude versus frequency for helicopters in flight. The data is from tests on four helicopters and shows a large amount of low-frequency vibration. Although no definite data on landing shock is available, one helicopter manufacturer considers 2g a reasonable value. Acceleration loadings due to maneuvering (which is limited to about 5% of operation time) are about 2g maximum.

#### 2-28. TRANSPORTATION AND HANDLING.

2-29. The shocks and vibrations to be expected during transportation vary, depending upon the type of carrier. Figure 2-13 shows the frequency ranges encountered in various modes of transportation. In view of the latest data for aircraft vibration, the frequency range for air transport could be extended up to 500 cps, particularly in the case of turbojet transports. The environment encountered by equipment is further dependent upon the protection afforded by any packaging and upon the stowage technique.

2-30. TRUCK TRANSPORT. Vibration frequencies in motor trucks are dependent upon the natural frequency of the unsprung mass on the tires, the natural frequency of the spring system, and the natural frequencies of the body structure. The vibration amplitudes are dependent upon road condition and speed of travel. Inter-  
mittent road shocks of high magnitude can occur with resultant body displacements of 12 to 24 inches. These large displacements may result in unlash cargo being bounced from the truck floor with large impacts upon recontact. Vibrations due to the engine and transmission system are relatively insignificant in the cargo area. Table 2-1 shows the ranges of three predominant natural frequencies for common truck types. Figure 2-14 gives measured data for truck transport; this chart also shows a high density of readings within the three critical ranges. \*

2-31. RAIL TRANSPORT. Vibrations in moving freight cars arise from track and wheel irregularities; these vibrations occur principally in the lateral and vertical directions. Shock and transient vibrations during coupling and during starting and stopping are generally considered to be the most damaging phases of rail shipment. Figure 2-15 gives vertical shocks and vibrations measured in railroad cars in motion. The enclosed areas, which are at the natural frequencies of the car on its springs, represent continuous and peak vibrations. Two types of springs are represented. The section of curve noted by a dashed line gives approximate data for the frequency range of 7.5 to 35 cps. The data for frequencies above 35 cps is for eleven train speeds and two freight cars and indicates average ampli-

\* The data in this figure shows the trend to be one of constant acceleration.

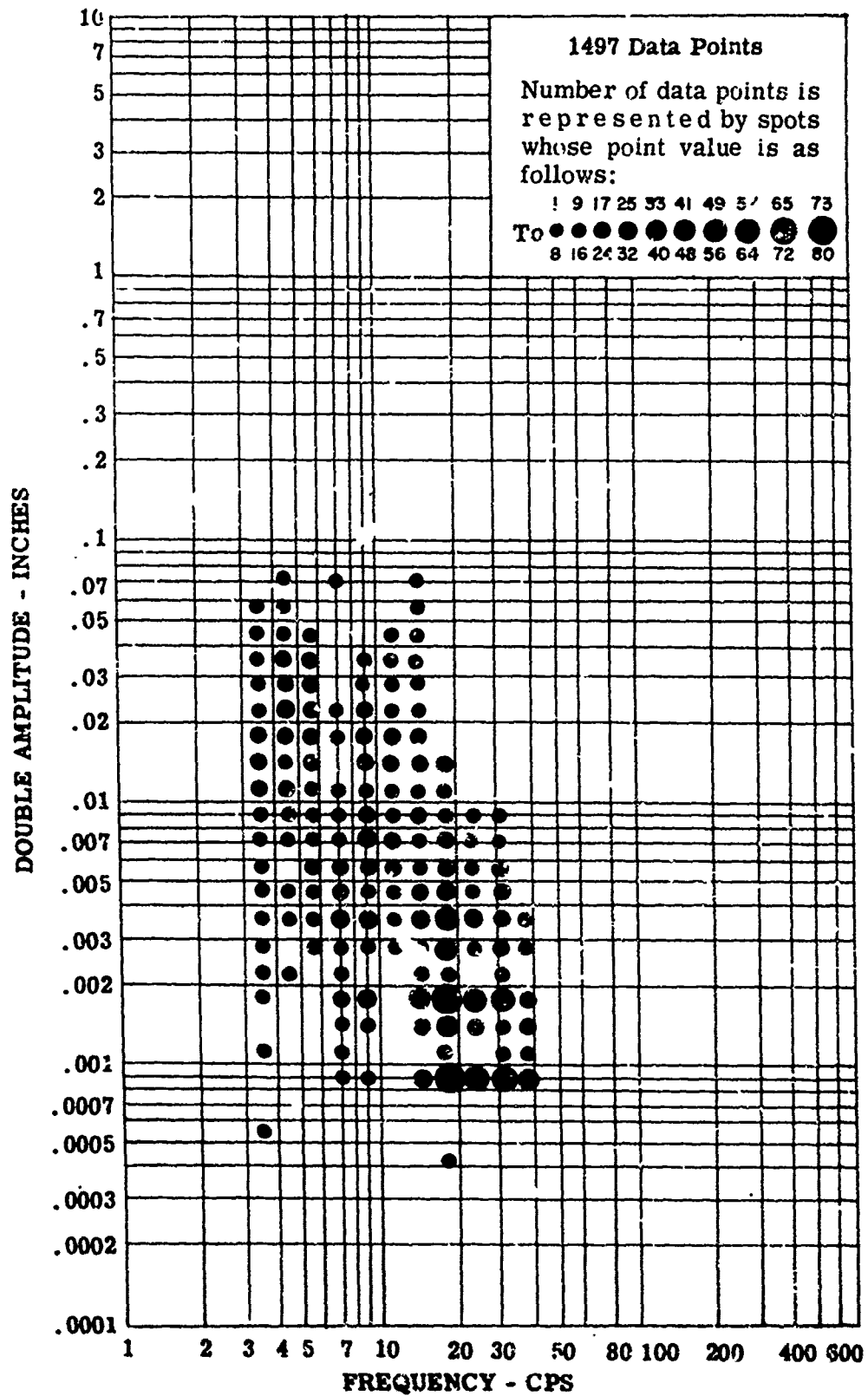


Figure 2-12. Vibration Measured During a Helicopter Flight

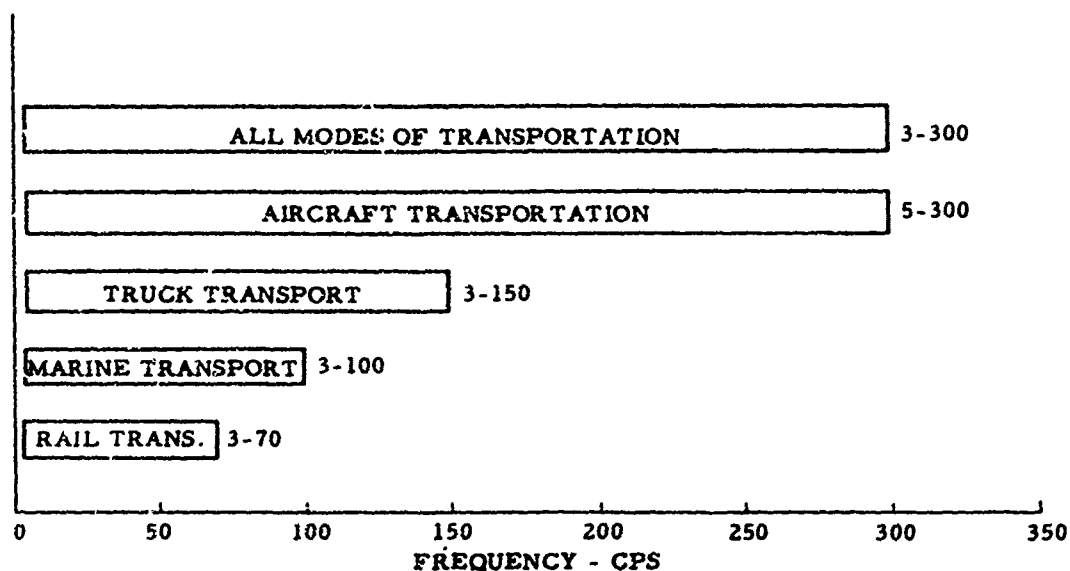


Figure 2-13. Range of Vibration Frequencies Likely to Occur in Transportation

TABLE 2-1. TRUCK VIBRATION

Type of Vehicle	Direction of Acceleration	Predominant Frequencies (CPS)		
		Springs	Tires	Body
Truck (2-1/2 ton)	Vertical	2 to 4	8 to 13	70 to 180
	Longitudinal	-	10 to 20	70 to 100
	Lateral	2	10 to 20	100 to 200
Truck (3/4 ton)	Vertical	2 to 3	5 to 10	60 to 110
	Longitudinal	-	-	70 to 100
	Lateral	-	-	60 to 70
Trailer (1 ton)	Vertical	3 to 5	8 to 10	50 to 100
	Longitudinal	-	-	50 to 100
	Lateral	2	-	50 to 120
M-14 Trailer	Vertical	1 to 4	7 to 10	50 to 70
	Longitudinal	3 to 4	8 to 10	200 to
	Lateral	2 to 4	-	-
M1, 2T Trailer	Vertical	2.5 to 5	7.75 to 10.5	100 to 150

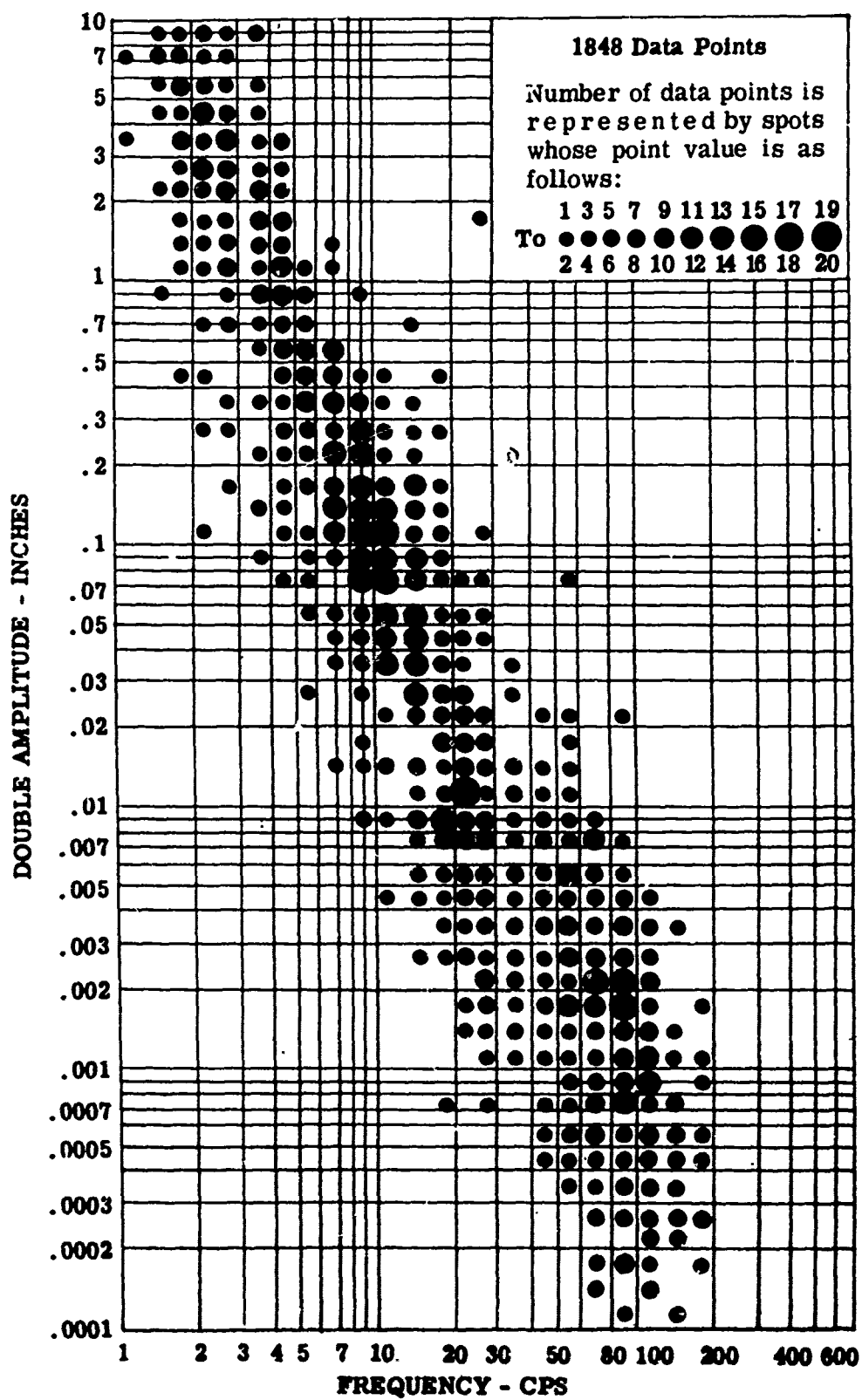


Figure 2-14. Vibration Measured During Truck Transportation

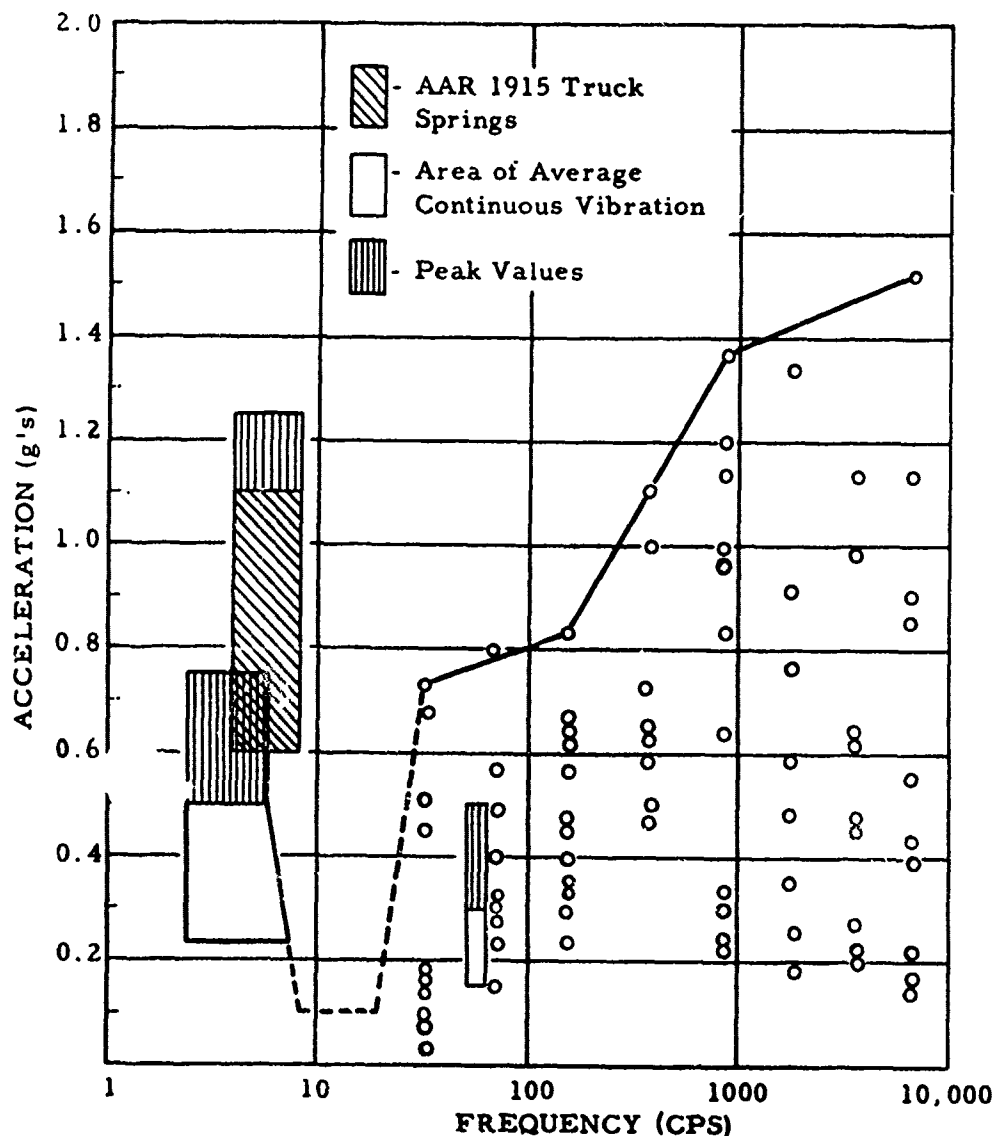


Figure 2-15. Summary of Vibration Data Recorded in Railroad Freight Cars

tudes as a function of time. Figures 2-16, 2-17, and 2-18 give data on the number of occurrences of shocks recorded on a 700-mile trip. The data is divided into shock ranges and direction. For each direction and within each shock range, the shocks are plotted against the speed range during which they occurred. Table 2-2 gives the percentage of travel time in each speed range. The time durations of the shock impulses, while not measured accurately, were estimated to be between 10 and 50 milliseconds.

2-32. As mentioned above, the shocks and vibrations that occur during switching may be the most damaging phases of rail transport. Figure 2-19 is a curve of horizontal acceleration experienced in a freight car versus the impact speed in a two-car impact. Only one draft gear\* was used in these two-car impact tests;

\* The coupling and associated equipment for absorbing shock.

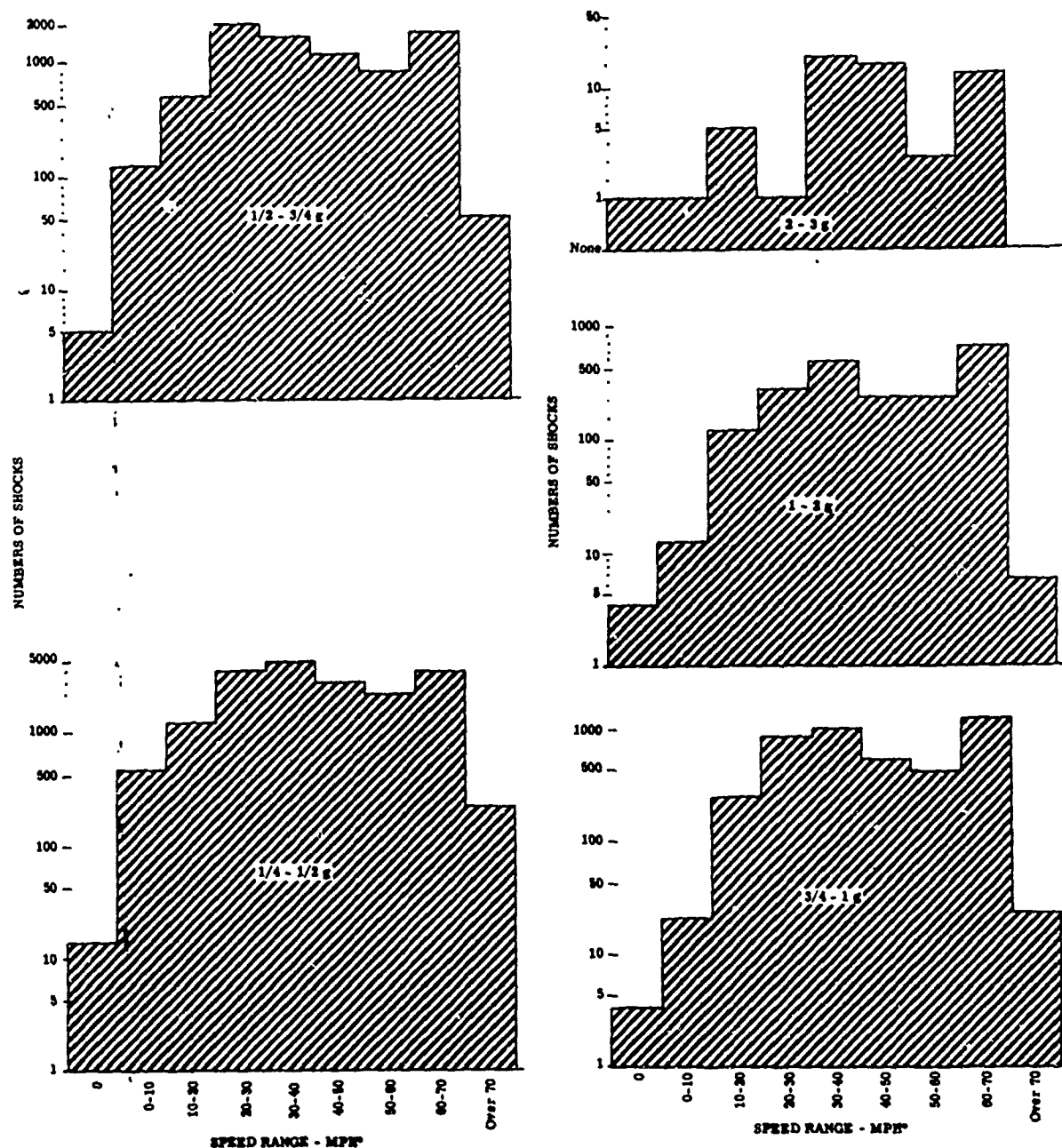


Figure 2-16. Vertical Shocks Measured on Freight Car Floor

but, particularly at the higher speeds, the draft gear can store only a small part of the total energy, and an additional draft gear would not change the curve much. Figure 2-20 shows the distribution of the relative velocities of freight cars being coupled as observed in a number of yards. It is obvious that a high percentage of the impacts occurred above five mph.

\* Table 2-2 lists the travel and recording time in each speed range.

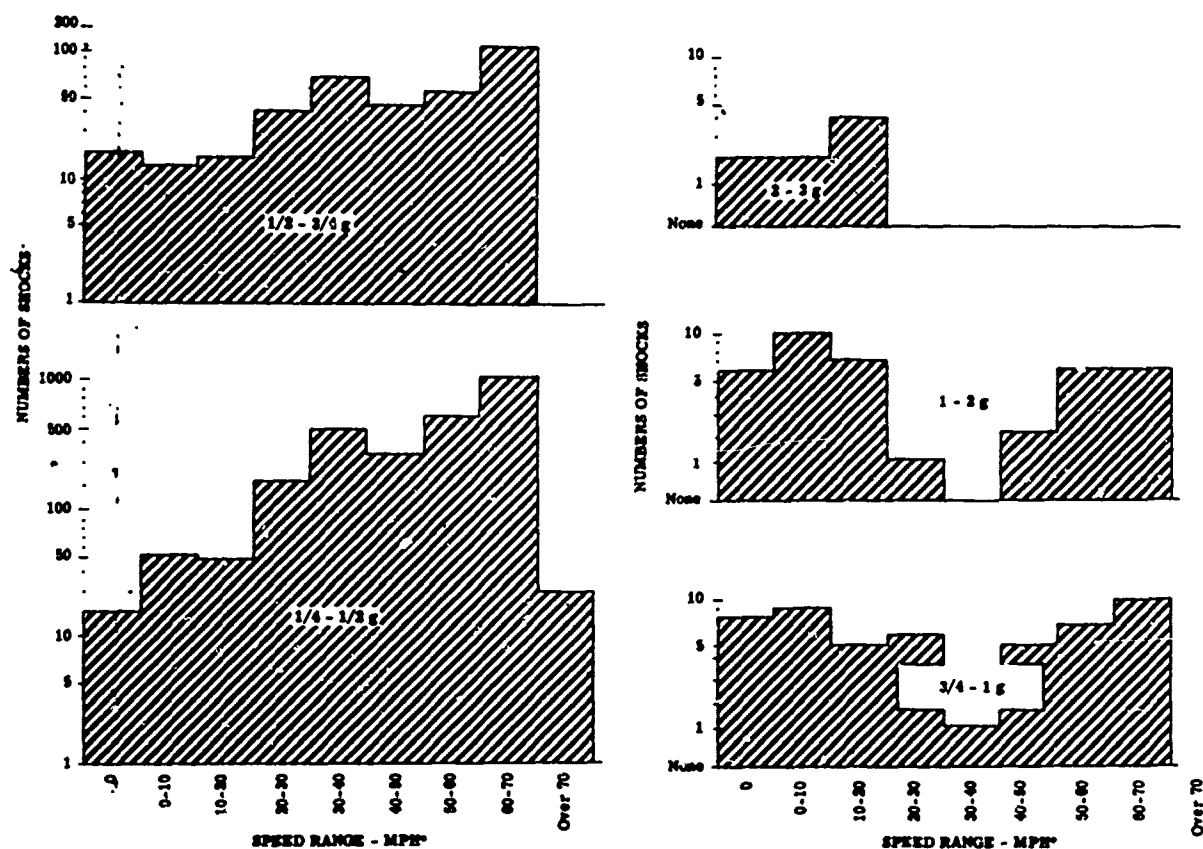


Figure 2-17. Longitudinal Shocks Measured on Freight Car Floor

2-33. **AIR TRANSPORT.** The shock and vibration environments of air transport are much the same as previously discussed under aircraft except for differences due to location within the plane. In air transport, as in all other modes of transportation, the shocks encountered in handling, as in loading and unloading, must be considered in the overall picture. Figure 2-21 shows the maximum shocks recorded in a test shipment by a large commercial airline. Two impact recorders were placed in a wooden box (having 73 pounds total weight) so as to record shocks both vertically and longitudinally. It is evident that the most severe shocks recorded arose from handling.

2-34. **HANDLING.** Handling during transportation and maintenance produces shocks and vibrations which are difficult to predict although generalizations can be made. Equipment in transit may be dropped, thrown, tumbled, sat upon, stacked, and wedged. However, equipment in transit does have the protection of a package. Where transportation environments are more severe than service environments, it is an economic decision whether to strengthen the equipment to withstand transportation shock and vibration or to protect the equipment through packaging. Barring accidents, maintenance handling can be assumed to be less severe than transportation handling since the maintenance handler usually has a better concept of the fragility of the equipment and in many cases he must fix what he breaks.

\* Table 2-2 lists the travel and recording time in each speed range.

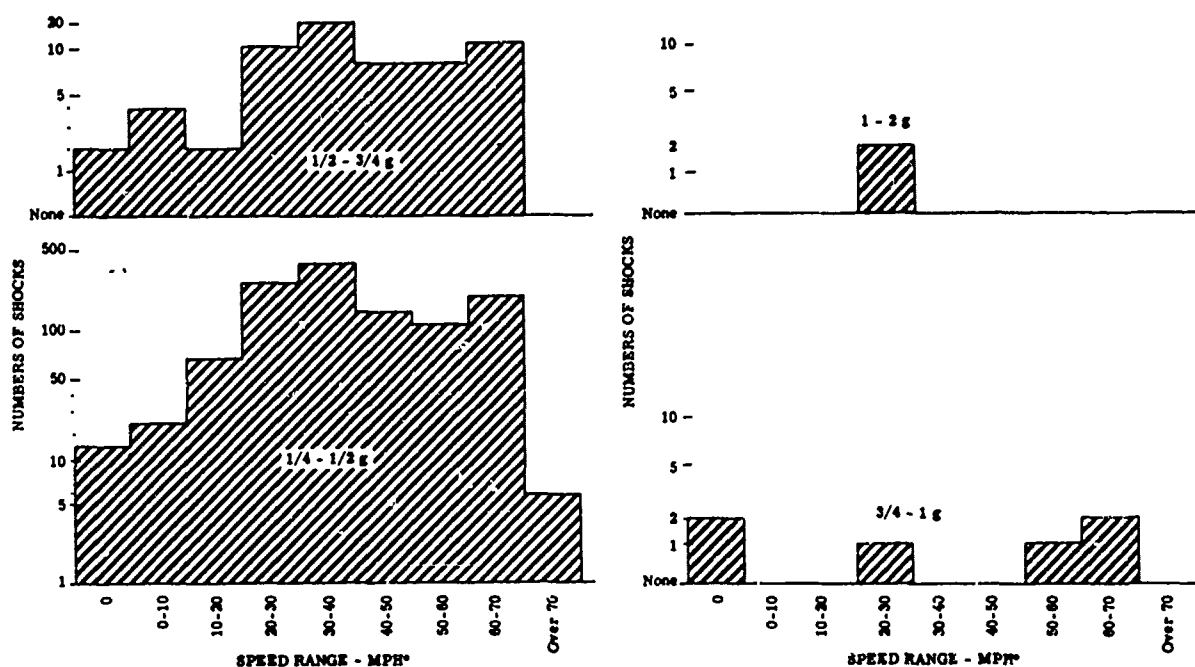


Figure 2-18. Lateral Shocks Recorded on Freight Car Floor

TABLE 2-2. TRAVEL AND RECORDING TIME VERSUS SPEED RANGE

Speed Range (mph)	Travel and Recording Time (%)
0	9.3
0 - 10	14.3
10 - 20	9.4
20 - 30	20.3
30 - 40	21.2
40 - 50	10.8
50 - 60	6.1
60 - 70	8.3
Over 70	0.3

#### 2-35. ACOUSTICAL EXCITATION.

2-36. The aircraft industry is devoting increased attention to the existence, and effects, of high levels of acoustical energy resulting from the use of turbojet- and rocket-propulsion systems. Noise as it affects human beings has long been a problem, but these new propulsion systems have focused attention on the effects of noise upon mechanical systems, including electronic equipment. Typical jet-engine aircraft data is shown in the two curves of figure 2-22. Curve A represents measurements made 10 feet from the tailpipe; curve B is derived from measurements made at the inboard equipment bay. The noise level in this region reached 140 db.

\* Table 2-2 lists the travel and recording time in each speed range.

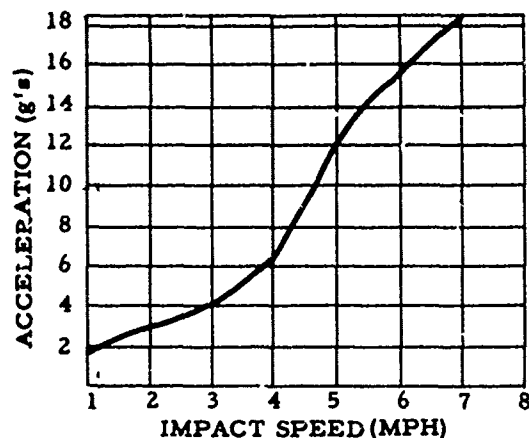


Figure 2-19. Horizontal Accelerations of Car Body Versus Impact Speed

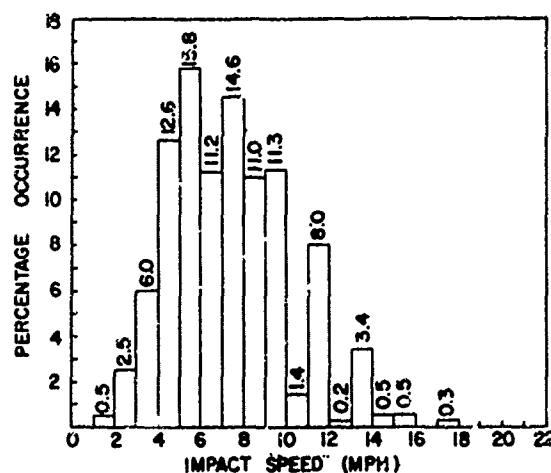


Figure 2-20. The Relative Velocities of Freight Cars being Coupled (555 Impacts)

Figure 2-23 shows the variation in overall sound level with engine thrust. Curves are given for both internal and external noise and show the attenuation afforded by an untreated compartment. These curves indicate only the levels which exist at positions aft of the nozzle.

2-37. A large portion of the structural vibration in the B-52 aircraft is believed to originate as acoustical excitation which excites the airframe in the vicinity of the engines. Measurements made at the wing behind the outboard engine pod of a B-52 with the two engines of that pod operating under full power showed an overall noise level of 158 db, with levels of from 149 to 133 db in frequency bands ranging from 25 to 19,200 cps. As vibrations are transmitted to the equipment compartments, the airframe will act as a mechanical filter and only relatively low frequencies will be carried any distance. High frequencies in the equipment compartment can exist from impingement of the acoustical excitation directly on the skin of the compartment. Measurements made in equipment compartments have shown 130 db at 2000 to 2500 cps.

#### 2-38. TEMPERATURE EFFECTS.

2-39. Vinyl chloride, neoprene, and rubber become more brittle at low temperatures, as do almost all materials. Conversely, at higher temperatures, the stiffness of these materials decreases (especially true of thermoplastic materials). Shock or vibration imposed upon electronic equipment exposed to temperature extremes necessitates considerations of the changes in mechanical properties of the materials within the electronic equipment. For example, the change in stiffness of hook-up wire insulation will result in a change in resonant frequency, or a change in stiffness of a potting material may either cause it to flow at high temperatures or to lose its toughness at low temperatures.

2-40. Shock and vibration excitation to equipment at low temperature can occur when starting an aircraft which has been standing at arctic temperatures, or when equipment is in an unheated portion of a low-speed plane at high altitudes. High

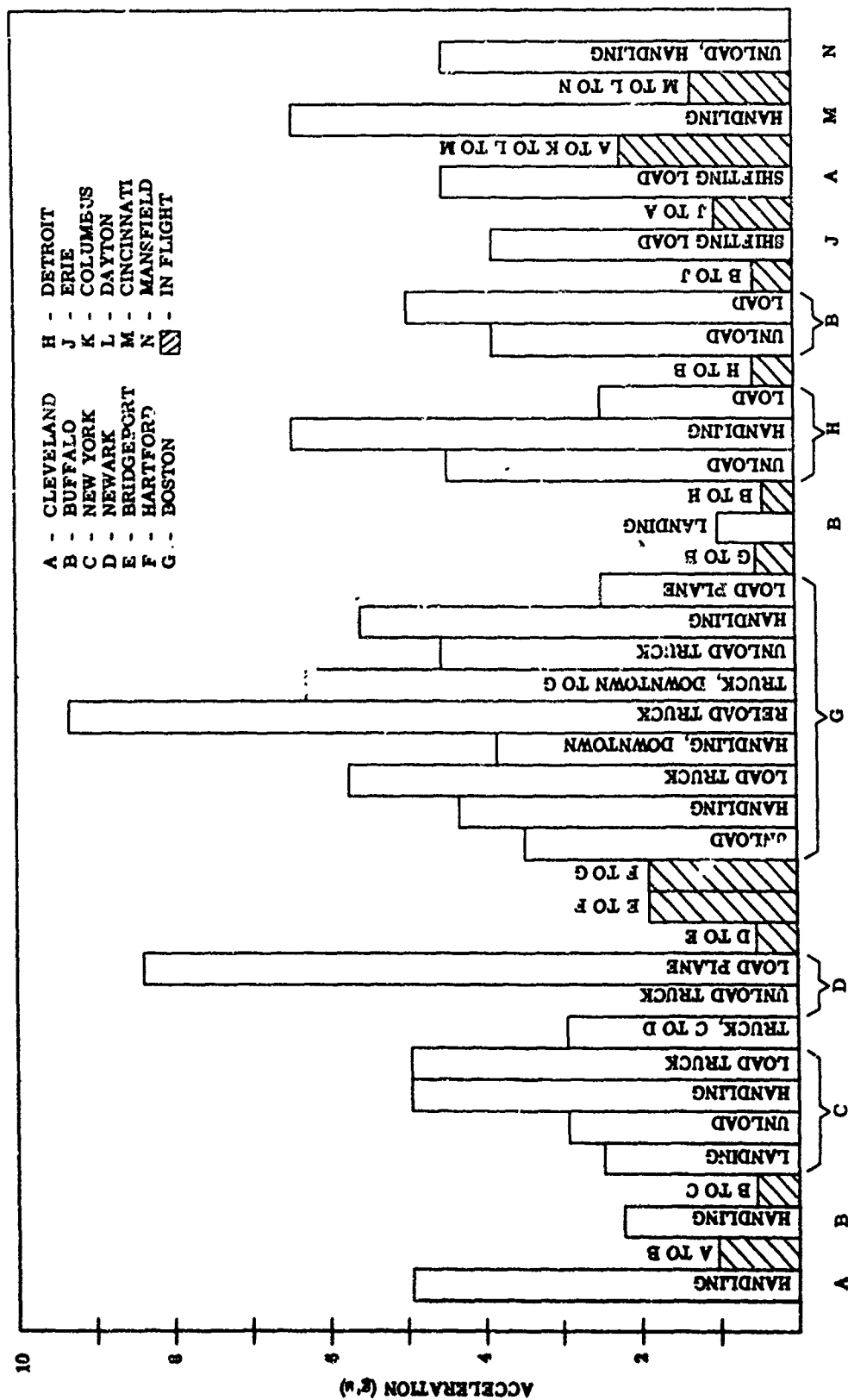


Figure 2-21. Shocks Recorded During a Test Shipment

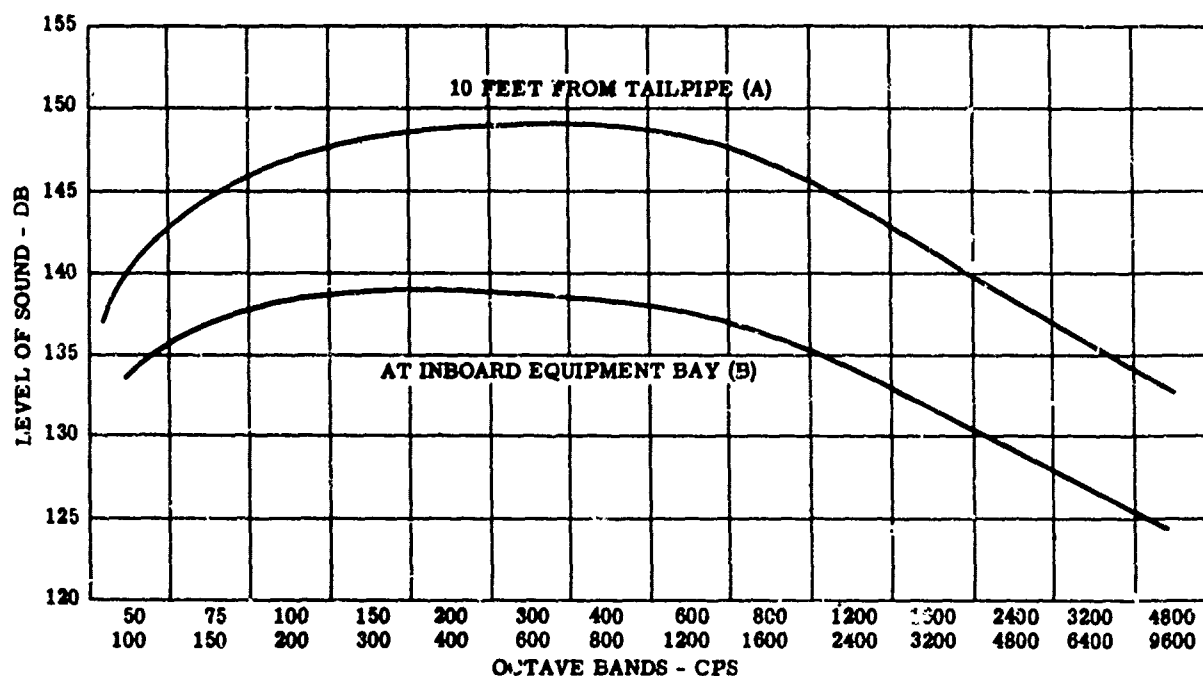


Figure 2-22. Noise Measurements Made on Jet Aircraft at Full Thrust

temperatures in equipment may result from the inability of blowers or other devices to dissipate heat generated within the equipment. This is particularly true at high altitudes where the low air density results in low heat capacity for a given air volume. High speeds result in high boundary-layer temperatures which raise the aircraft skin temperature. Where high speeds occur for only short time intervals, the lag in temperature rise of equipment due to its own heat capacity may prevent the equipment from reaching excessive temperatures. For aircraft with sustained high speeds, however, the thermal lag of the equipment cannot be relied upon. This would be the case in an aircraft flying at mach 2 at 70,000 feet with an operating skin temperature of approximately 120°C (248°F).

2-41. Changes in temperature can create problems of dimensional tolerance because of the difference in the coefficient of expansion of two materials. Parts of dissimilar materials which mate at assembly room temperatures will not fit the same at other temperatures. Plastics, in general, have a high coefficient of thermal expansion, and the effect of dimensional change due to temperature change should always be considered when plastics are mated with metals.

#### 2-42. RELIABILITY OF DATA.

2-42. The data, although not completely applicable to future design, does portray some of the levels and frequencies to be encountered. The available data tends to be rough and spotty; in addition, the environment changes as aircraft designs change. In the past 10 years considerable effort has been expended by members of government and industry in collecting data and in developing methods of presenting and using data. The increased awareness of electronics designers of the need for such data, coupled with the efforts of the instrumentation, flight test, and laboratory simulation engineers, should result in further improvement in future years.

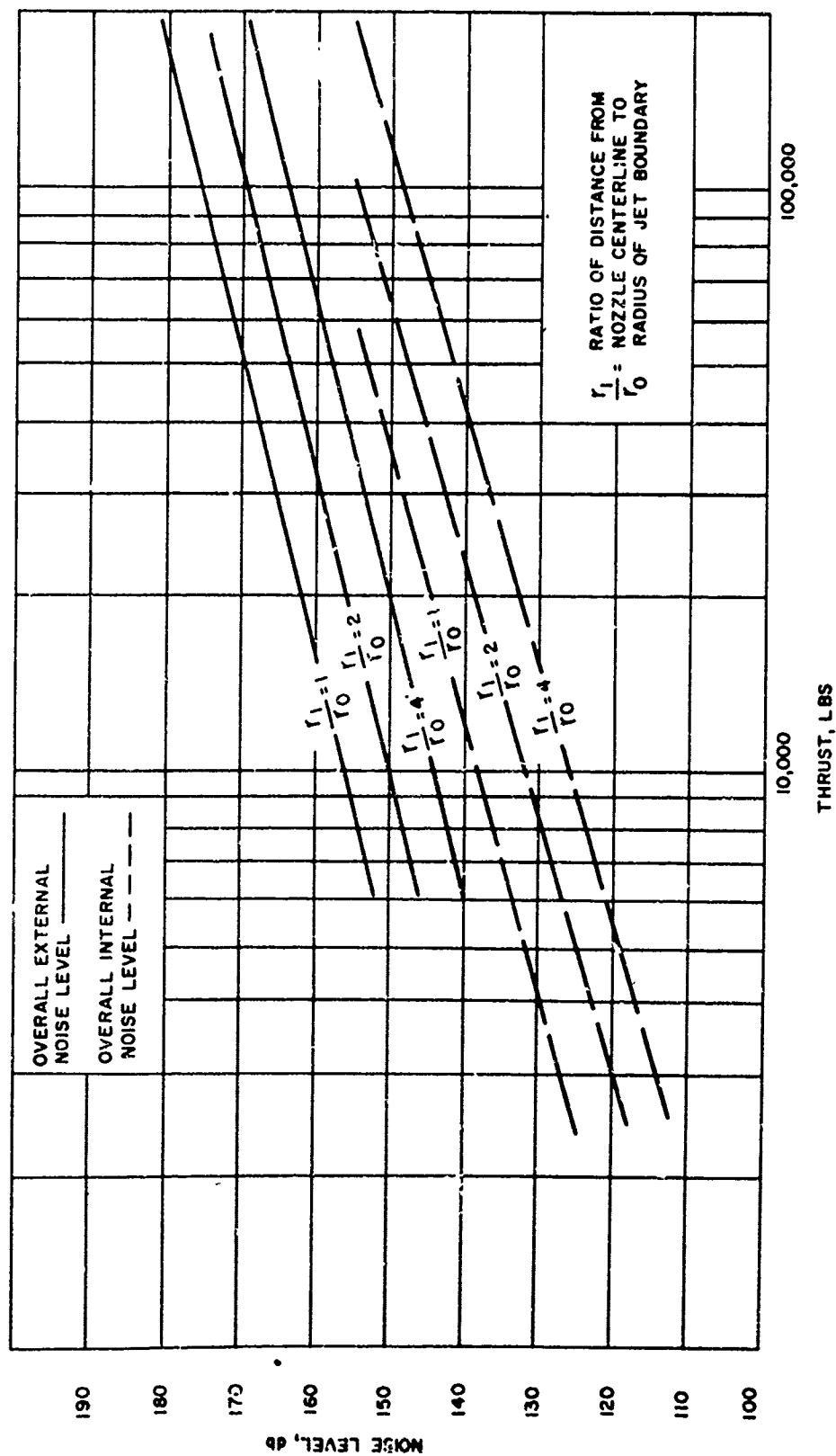


Figure 2-23. Variation of Overall Noise Level With Thrust

### SECTION III

#### ELECTRONIC COMPONENT PARTS AND THEIR CHARACTERISTICS RELATIVE TO SHOCK AND VIBRATION

##### 3-1. GENERAL.

3-2. The ability of a part to withstand a shock and vibration environment depends upon the response of the part to the encountered excitation and upon its ability to withstand that response without failure or malfunction. The size and shape of a part, the way in which it is mounted, and such material properties as density and elastic modulus affect its natural frequencies. The internal damping, a result of the material composition of the part, affects the response of the part to vibration. The toughness or impact resistance of the material plays a large role in determining the part's ability to withstand shock forces without rupturing. The effect of temperature upon material properties should also be considered if the part must function in a combination of environments.

3-3. The resonant frequencies and behavior of any electronic component part, after considering its particular configuration, mass distribution, and elasticity, are determined largely by the mounting method used. The mounting arrangement is the one aspect of design over which the electronics designer has the greatest control. Although the designer must work with a limited variety of component parts, he has the prerogative of locating these parts on the chassis and of choosing the means by which the parts will be held to the chassis. Of course, in order to locate and mount the parts in the best way, the designer must have a thorough understanding of the mechanical susceptibility of each part he chooses as a component.

3-4. Circuit design philosophies should be influenced by a consideration of the effect of mechanical environment upon component part performance. In some cases, it might prolong equipment life to de-rate component parts or to design a less critical circuit in such a way that any mechanically induced changes in part performance which occur early in the equipment's life would not render the equipment useless. This particular section of the guide should provide the designer with various, general, common-sense rules regarding component characteristics upon which he may base his decisions when selecting and applying various component parts.

##### 3-5. SUSCEPTIBILITY OF COMPONENTS TO FAILURE.

3-5. There are certain groups of component parts, that, by the nature of the function they must perform, are more susceptible to failure. A study was made in which several airborne electronic equipments were subjected to shock and vibration and the component part failures were recorded. Table 3-1 summarizes the findings of this study.

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3-7. Over half the total number of failures resulted from lead and connector failures, with resistor and capacitor leads accounting for the majority of these. Nearly all the lead failures were caused by the repeated stretchings and flexures resulting from the relative displacements of the lead terminations during chassis resonances. (While the flexible chassis themselves did not fail, they were responsible, indirectly, for more vibration failures than any other factor.)

3-8. More tubes failed (a total of 54) than any other component part. Most developed short circuits or excessive microphonics caused by vibration, and a few envelopes were broken during shock excitation. Eleven tube failures resulted when the tubes came out of the sockets.

3-9. Numerically, the relay proved to be the second most vulnerable component part. Fewer relays than tubes were tested, however, so they must be ranked at least as susceptible to failure, if not more so, than tubes.

3-10. Capacitors and resistors (disregarding lead failures), also contributed a significant number of failures. Considering the large numbers contained in the equipments tested, however, it is difficult to attribute particular shock and vibration susceptibility to these component parts. The miscellaneous mechanical failures category, which includes broken lock washers, loosened screws and nuts, and component mounting-stud failures, contributed approximately 11 percent of the total number of failures.

3-11. Shock excitation produced few electrical failures. Most equipments withstood several 5-foot drops (approximately 75g), the maximum obtainable with a 150-lb, drop-type shock machine, before failure. Tubes and relays accounted for half the total number of failures; lead and connection failures were few compared to those occurring during vibration.

### 3-12. WIRES, CABLES, AND CONNECTORS.

3-13. Wires, cables, and connectors are not in themselves considered dangerously susceptible to shock and vibration. They can cause considerable difficulty, however, if improper methods are used in their application, as evidenced by table 3-1.

3-14. WIRE AND CABLE FLEXIBILITY. Wires and cables have a high degree of flexibility as compared with other electronic component parts. This flexibility results from (1) the type of construction, (2) the kind of material, and (3) the high ratio of length to cross-sectional area. Because of this flexibility, the natural frequency of a span of wire or cable is low and may easily fall within the frequency range of the environmental excitation.

3-15. Wire, particularly hook-up wire, is quite soft and will permanently deform under light loading. If a relaxed span of wire is subjected to acceleration forces high enough to cause permanent deformation, the response might best be described as a flopping action rather than vibration. Both the flopping action and the vibration of a wire or cable produce varying stresses throughout the component's length; these stresses can ultimately lead to fatigue failure. The areas of maximum varying stress, and therefore the areas which usually are first to fail, are the ends of

the vibrating span. i.e., the wire or cable terminations or supports. Other areas and modes of failure are found where the component's insulation rubs against an adjacent part, causing the insulation to wear away, or where, in a multiconductor cable, one conductor rubs against another within the cable body.

3-16. In instances where several wires extend in the same direction, their stiffness may be increased by harnessing the wires with lacing cord or some other material. Harnessing also provides damping due to friction between the wires. The harnessed length, however, should be adequately supported in order that its weight will not endanger the wire termination points.

3-17. Coaxial cables, when subjected to flexing, may become a troublesome source of noise. Cables such as RG-8/U can generate sizable noise voltages in high-impedance circuits. In transducer signal circuits, for example, considerable noise difficulties can arise from movements of the coaxial cable. These noise voltages are generated by a phenomenon involving the mechanical configuration of the cable and the materials used in the cable construction. In the design of airborne electronic equipment packages, it is advisable to avoid, wherever possible, the use of high-impedance, low-signal-level circuits running external to the electronic packages. If this cannot be avoided, special low-noise cables must be used.

3-18. **MATERIALS AND CONSTRUCTION.** Although all wire and cable is flexible when compared to other components, the degree of flexibility depends in part upon the material composition. The flexibility may vary widely with temperature, since many organic insulating materials stiffen considerably with decreased temperatures and soften with increased temperatures. Fluorinated hydrocarbons and silicone elastomers maintain flexibility at low temperatures better than most others.

3-19. Stranded wire and cable have a relatively high degree of damping. In addition to the internal damping offered by the insulation, internal friction between multiconductor cable components or between individual strands of a conductor provides damping which reduces vibration amplitude at resonance.

3-20. **LEAD LENGTH.** The bending action of a wire or a cable may be reduced at its termination or support, under conditions of shock and vibration, by shortening the length of the lead between its supports. This applies only in cases where the supports do not move relative to one another. The shortening of the span of wire or cable accomplishes two things; it raises the natural frequency of the span and it reduces the weight and inertial loads at each support. Both of these accomplishments are extremely beneficial for the shock and vibration resistance of a cable or wire.

3-21. In cases where supports or terminals do move relative to one another, lead failures can be reduced considerably by allowing some slack in the lead. Electrical considerations dictate against making leads excessively long, but, as shown in figure 3-1, lengthening a lead slightly (by about 20 percent of its stretched length), reduces the failure hazard considerably. The additional length, although lowering the natural frequency of the lead, prevents repeated straining of the lead, and the use of stranded, insulated wire provides damping to limit responses during lead resonances.

3-22. **METHODS OF ATTACHMENT.** Wire or cable failures due to shock and vibration usually take place where the lead is interrupted, either at a termination point or at a support along the lead. Failures occur here chiefly because these particular locations are subjected to the greatest amount of bending stress. A termination is much more susceptible to shock and vibration, however, than a support. At a termination, it is necessary to strip the wire of its protecting insulation; this decreases the ability of the wire to withstand the encountered bending stresses.

3-23. When wire is terminated by soldering, the area most susceptible to shock and vibration lies between the end of the insulation and the rigidly "frozen" section at the terminal post. As shown in figure 3-2, this particular area is subjected to a great amount of flexing and undergoes fatigue very quickly. Figure 3-2 also indicates the effect that soldering has on a wire termination point. The solder flows up the strands of the conductor by capillary action and concentrates all bending action into a very small area of wire. Very little can be done to improve this condition, but failures in the soldered area can be minimized by reducing the wire's motion, as shown in figure 3-3. Properly clamping the wire to the framework can restrict the motion.

3-24. When wire is terminated by an end support lug, the lug should preferably grip both the insulation and the conductor. This prevents the flexing motion from being concentrated entirely on the extremely flexible conductor by transferring the motion to the more uniformly rigid insulated area.

3-25. When cable is terminated by a connector, the connector is usually attached to the cable so that it grips the cable jacket. This restricts flexing of individual wires within the cable body. Attaching a sleeve (as shown in figure 3-4), a spring, or some other restricting device to the connector prevents severe cable bending at a termination point. Bending of individual wires can also be restricted by potting the cable end permanently into the connector. In this procedure, the connections are made in a normal fashion, then the connector body is filled with a self-curing plastic material. This completely encapsulates the end of the cable. Other purposes are served by the potting process, such as the elimination of moisture condensation and the prevention of dirt accumulation between terminals and connector body. There is a disadvantage to potting a cable, however, as potted components are difficult to repair.

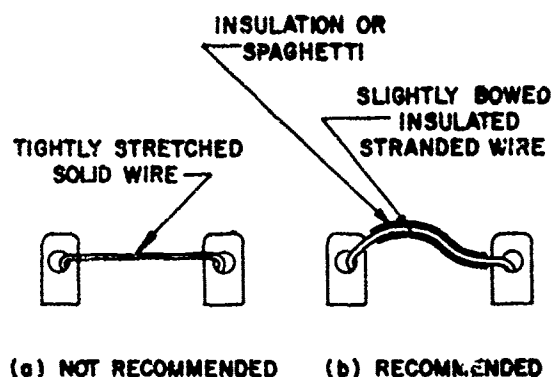


Figure 3-1. Recommended Method of Applying Jumper Leads

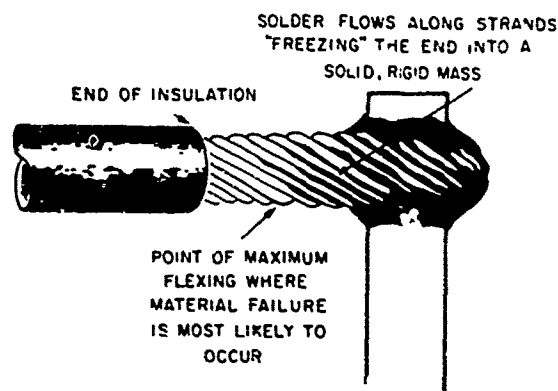


Figure 3-2. Wire Terminal Shock and Vibration Danger Area

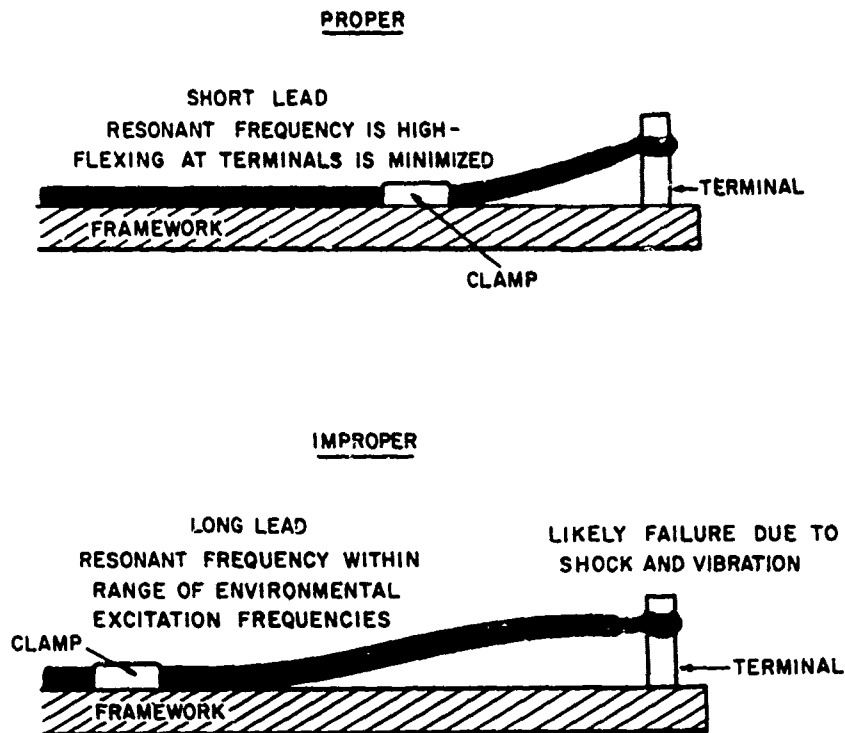


Figure 3-3. Proper Clamping of Cable End

### 3-26. RELAYS.

3-27. The relay ranks with the vacuum tube as a source of malfunction in airborne electronic equipment. Unlike the simple capacitor or resistor, the relay is inherently prone to failure because of its more intricate electrical and mechanical design. A plot of relay failures is shown in figure 3-5. Each point indicates the lowest g level failure for continuous sweeps between 10 and 2000 cps. This data indicates that more than 70 percent of the failures occur above 500 cps.

3-28. The operation of a relay in a circuit under conditions of vibration and shock generally is unpredictable. Typical shock and vibration failures are: contact chattering, armature binding, armature jarring from position, and mechanical warping and breakage. One of the lesser effects of vibration is change of contact resistance. While this may not cause permanent damage to the relay, it can result in interference which cannot be tolerated in most circuits. In the past, little was accomplished in designing relays to resist shock and vibration environments; however, some progress is being made in this field. One approach to increasing relay reliability is to use two relays (where one would normally be used) with different natural frequencies, each performing the same function. The relays, because of their different natural frequencies, are not likely to malfunction simultaneously.

3-29. **CONFIGURATION.** Relays designed with particularly long body dimensions often amplify the mechanical excitation appearing at their mounting base when cantilever mounted. The cantilever structure of the long-body relay shown in figure 3-6, in combination with its position on the flexible chassis, is wholly undesirable.

From the 10g motion of the base, successive amplifications occur until a 30g level is reached at the armature.

3-30. COMPARISON OF RELAY TYPES. Test results on three popular types of relays show various resistivities to vibration and shock. Also, when tested along different axes, certain relays were capable of withstanding as much as 5 to 10 times the acceleration along one axis without malfunction as they could along other axes.

3-31. The clapper-type relay shown in figure 3-7 failed as a result of vibration and shock forces along axis 1. At low

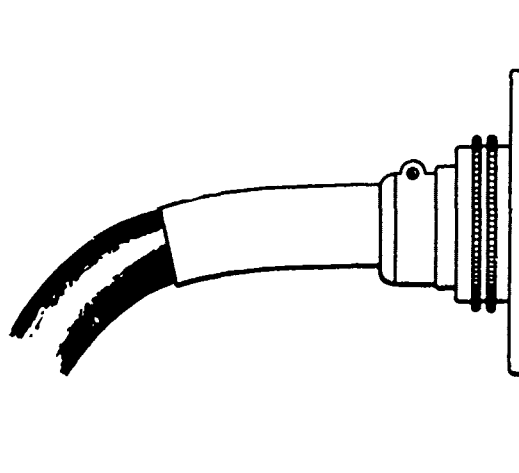


Figure 3-4. Restricting Sleeve

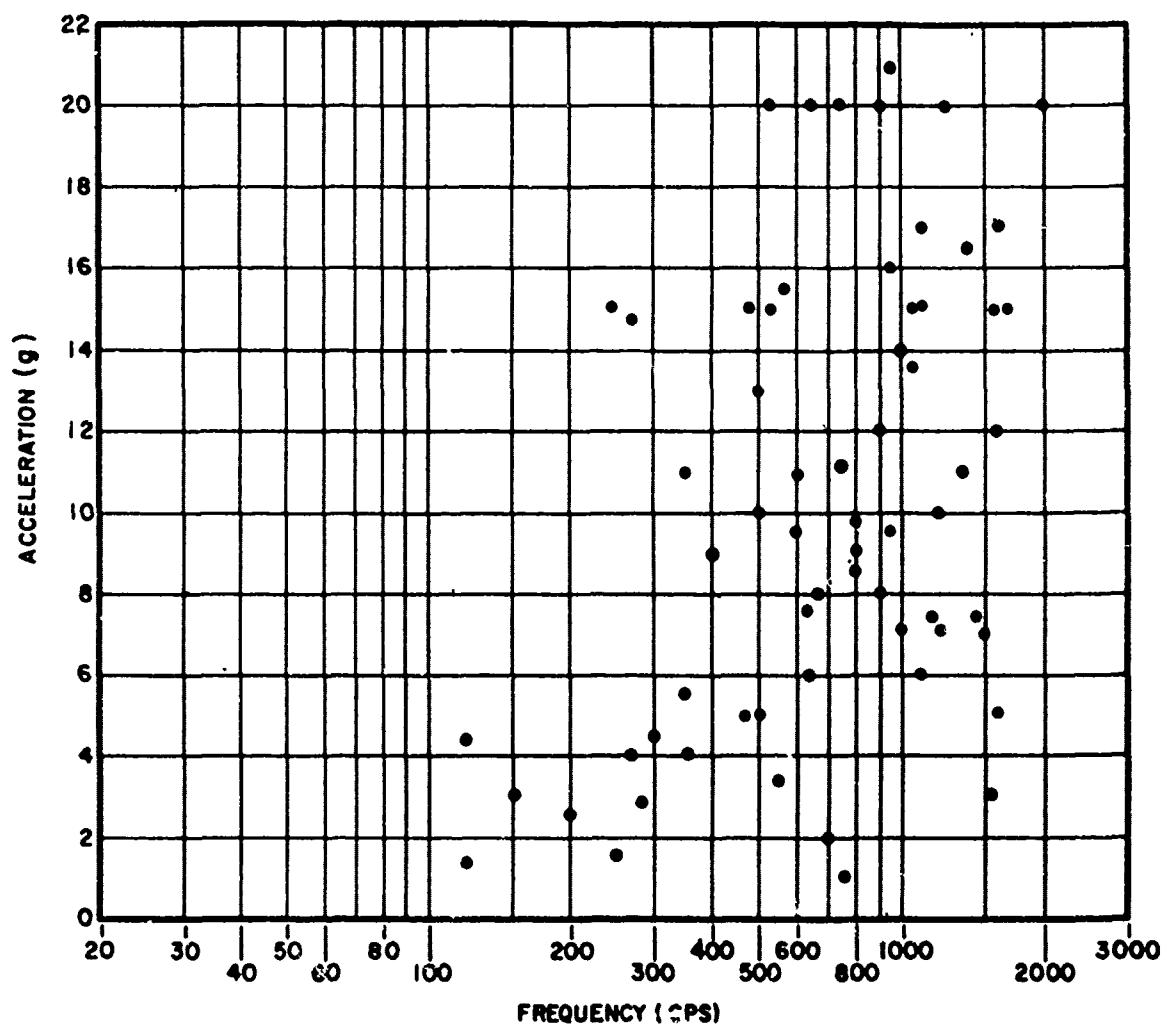


Figure 3-5. "G" Level Relay Failure Spectrum

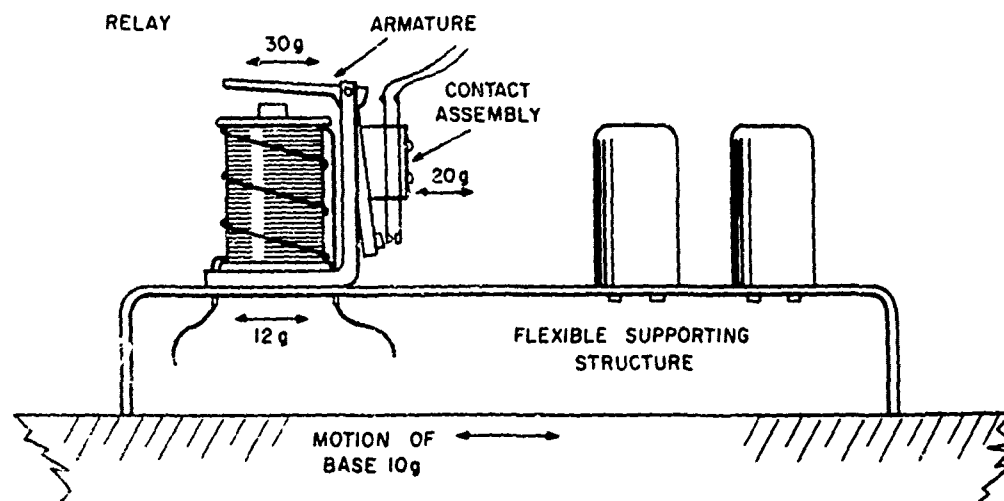


Figure 3-6. Shock Amplification Throughout a Long-Body Relay Mounted on Flexible Chassis

vibration frequencies along this axis, some contact chatter occurred, but the degree of chatter varied depending upon the contact-tip pressure for the particular relay design. Severe chatter occurred at or close to the resonant frequencies of the relay's contact support members. Continued vibration at resonant frequencies resulted in fatigue failure of the support members. Contact chatter also occurred as a result of shock forces. Axis 2 of the clapper-type relay appeared least sensitive to shock and vibration. Unless specialized clapper types are used, designers can usually be assured of optimum relay performance if the relay is oriented so that acceleration forces are directed parallel to this axis. Along axis 3, the average clapper-type relay is not considered susceptible to effects of vibration accelerations. Shock accelerations of 30 to 50g, however, caused considerable contact chatter.

3-32. The rotary-type relay shown in figure 3-8 malfunctioned as a result of vibration and shock forces along axes 1 and 2. Contact chattering occurred at low frequencies but was not serious; however, when the exciting frequency reached 500 cps, resonance of the contact support members resulted in serious contact chatter. Shock and vibration accelerations did not cause difficulty along axis 3. With rotary-type construction, rotational-mode accelerations also must be considered if the rotational axis is parallel to axis 3.

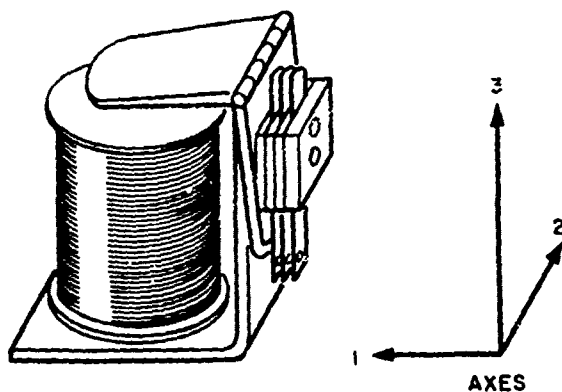


Figure 3-7. Clapper-Type Relay

3-33. Most plunger-type relays (figure 3-9) are immune along all three axes to vibration and shock accelerations encountered during airborne operation. Because of its construction, this relay is not susceptible to shock and vibration

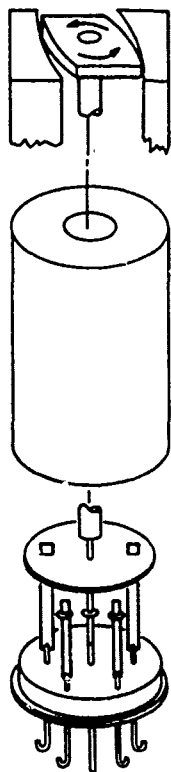


Figure 3-8. Rotary-Type Relay

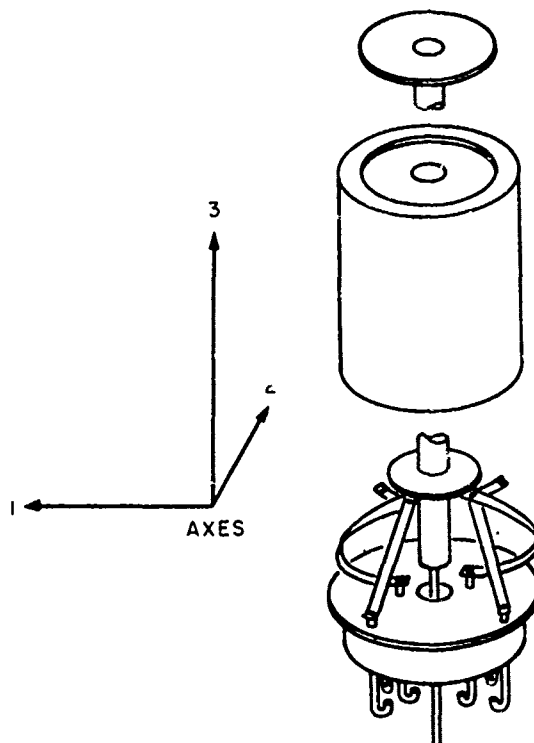


Figure 3-9. Plunger-Type Relay

along axes 1 and 2. For axis 3, the magnetic force of the armature counteracts forces due to shock and vibration. Another favorable feature is that plunger-type relays have short contact support members with natural resonant frequencies above 500 cps. However, certain models of plunger-type relays chattered when subjected to shock along axis 3 over the 30g level. The position of the plunger as the coil is being energized or deenergized is a factor as to whether or not the relay will chatter. An attempt should be made to orient plunger relays in operating positions so that the most severe acceleration forces are along axes 1 or 2.

**3-34. MOUNTING METHODS.** Most relays for airborne equipment are designed to be small and compact, as shown in figure 3-10. When rigidly mounted on firm decks or panels, these relays may withstand an operational shock of up to 50g, or a vibration between 10 and 2000 cps at 10g plus. When such nonflexible mounting locations are unavailable, sufficient rigidity must be designed into the mounting in order to increase the natural frequency of the assembly (if possible) beyond the environmental exciting frequencies. There must be sufficient clearance between the relay and other component parts, frame members, or panels to prevent bumping.

**3-35.** Experience shows that mounting orientations usually are dictated by space factors or convenience of attachment. Certain optimum orientations of relays, however, exist for various types of environments. The dynamic forces in fixed-wing aircraft and helicopters are primarily along the vertical axis, but in helicopters, forces nearly as great appear along the transverse axis. Missiles launched from the ground often experience large dynamic forces along the longitudinal axis. The vulnerability of the relay to these forces depends, in a great part, upon the

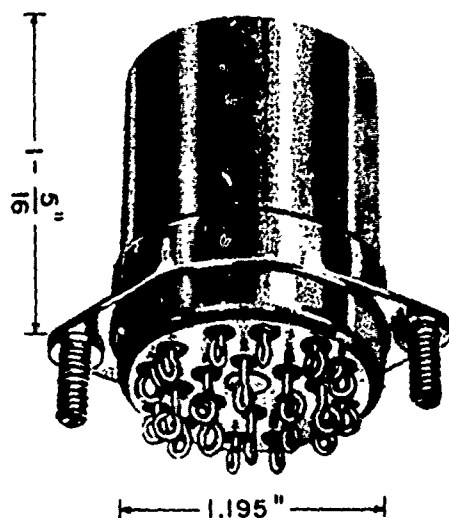


Figure 3-10. Small and Compact Relay Design

particular relay used and its particular mounting orientation in relation to the dynamic force encountered. Figure 3-11 illustrates mounting orientations for the three popular types of relays. These mounting configurations assume the support is rigid, and motion occurs along or about one axis only.

### 3-35. TUBES.

3-37. The highest percentage of all failures in components comprising airborne electronic equipment is attributed to the electron tube. Under conditions of shock, vibration, or acoustic noise, an electron tube may fail completely or may malfunction by acting as a transducer and creating noise or signals, or reacting to shock in such a way as to momentarily paralyze a circuit. Complete failure may result through breakage of the glass envelope, breakage of

the filament, or distortion of the elements. Figure 3-12 shows the vibration failure rates of three tube types for one level of vibration.

3-38. Tube failures may be reduced in three ways: improvement of the conventional electron tube to endure mechanical excitation, development of an entirely new design which would be inherently more rugged, and wiser application of the available tubes. Each of these three approaches is being used with varying degrees of success.

3-39. CONSTRUCTION. The conventional electron tube consists of several long, slender elements held close to one another by spacing discs, and the entire assembly is enclosed in a tubular envelope. Electrical connections are made at one end of the elements and this same end is used to mount the entire tube as a cantilever by pushing the exterior connecting pins into a mating socket. The relative positions of the elements are critical; changes in spacing will change the electrical characteristics of the tube. The internal structure of a tube has little damping and, therefore, a high mechanical  $Q$ . Typical tube failures which result from shock and vibration are: broken filament, broken mica spacers, enlargement of holes in mica spacers, pin breakages, loose metal particles, and cracked envelope or base seal. These primary failures result in tube failures evidenced as open circuits, short circuits, microphonics,  $G_m$  changes, and gassiness.

3-40. FAILURE DUE TO SHOCK. Envelope breakage due to shock is relatively uncommon. When it does occur, it usually results from stress concentrations caused by inadequate envelope support. Shock failure of the filament is generally associated with the stresses imposed by short-duration accelerations (the filament, although able to withstand high accelerations of long duration, will sometimes fail under relatively low accelerations of short duration). Shock failure of

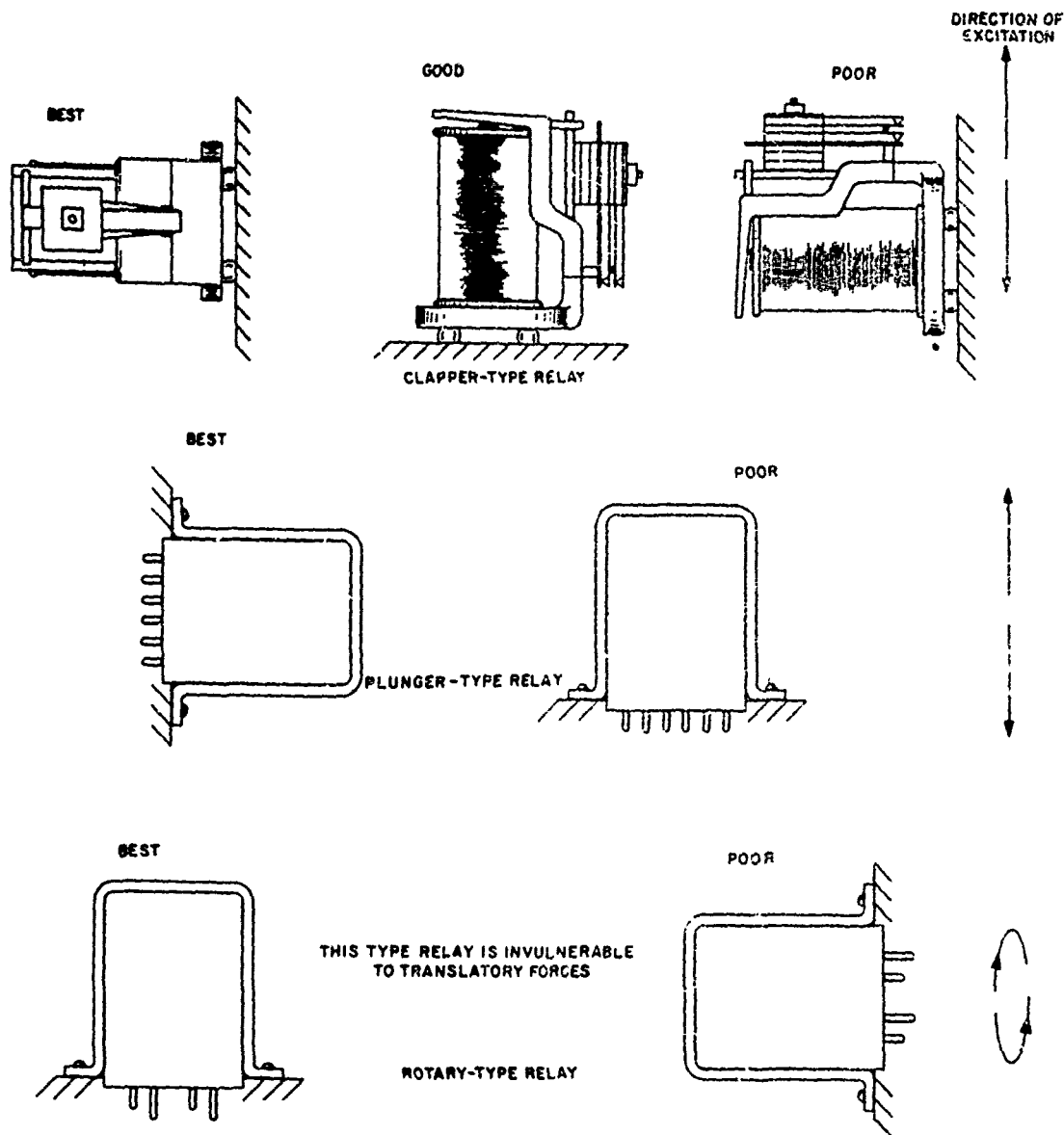


Figure 3-11. Mounting Orientations for the Three Popular Type Relays

other parts of the internal structure usually is associated with shock forces of long duration.

**3-41. FAILURE DUE TO VIBRATION.** In general, low-frequency vibration has little effect on the average size electron tube since the internal frequencies are relatively high. Even the larger size tubes, which are used under certain conditions in airborne equipment, seldom have natural frequencies below 180 cps. If the equipment is rigidly mounted, the environmental exciting frequencies can coincide with tube natural frequencies. Although the excitation amplitudes are low at the tube natural frequencies (usually 400 to 600 cps), the high mechanical  $Q$  of the tube structure can lead to destructive response amplitudes. Individual tube ele-

ments may resonate in the 500 to 8000 cps range and as a result may generate electrical noise.

3-42. Ruggedized vacuum tubes provide more rugged element supports and better fitting parts, and materials and processing are more closely controlled in their manufacture. These tubes are premium priced but are generally better with respect to shock and vibration resistance. Nevertheless, in some applications, tubes which were specially engineered and sold at premium prices had no effect whatsoever in changing tube failure rates.

3-43. Completely new tube designs (such as those shown in figure 3-13) are evolving, primarily out of the attempt to develop an automatic tube assembly. The stacked or ceramic type is more resistant to shock and vibration than conventional tubes because of its compact design and the rigidity of the materials used. The new, small box-shaped tube with leads all in one plane is certainly convenient for mounting in a circuit, and its size and shape indicate desirable shock and vibration resistant characteristics. It may also be noted that tubes are being replaced, wherever practicable, by transistors which are inherently resistant to shock and vibration.

3-44. MOUNTING METHODS. Except for the special case of individually isolated tubes, the best that can be accomplished in tube mountings is to subject the entire tube to the vibration environment existing in the chassis. The tube mounting must not only restrict the tube's motion under shock, but should also attenuate the shock, if possible. As with any cantilever type of structure, the electron tube has a least sensitive axis in respect to shock and vibration. If possible, the tube should be mounted so that its longitudinal axis is in the same direction as the most severe environment.

3-45. The simplest way to mount an electron tube is to push its pins into a socket which has been fastened to the chassis. While this method of mounting is completely satisfactory for mild service environments, it does not provide a good support for the tube. Tubes which are pin mounted may, under shock or vibration, pop out of their sockets, strike other components (unless abnormally large clearances prevail), or amplify the base motion and subject the interior of the tube to high accelerations. It is not uncommon, upon unpacking home electronic appliances, to discover that such damage has occurred during shipping.

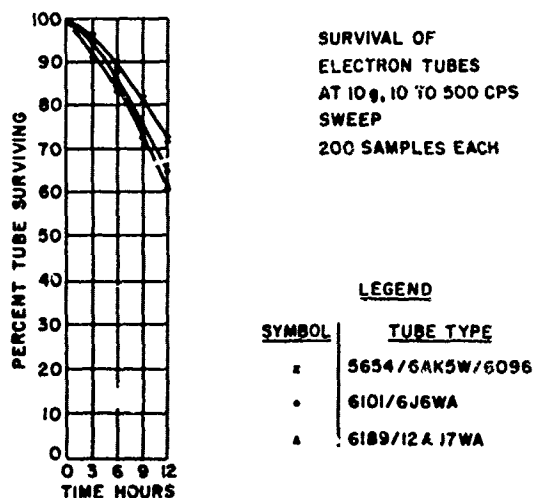


Figure 3-12. Typical Failure Rates of Electron Tubes Due to Vibration

3-46. One method used for securing tubes with bases has been to clamp the tube by its base, with the clamp secured to the chassis. Base-gripping clamps are usually more rigid than

other types and tend to transmit full shock forces directly to the tube. Although the base-gripping clamp holds the base of the tube securely, the envelope containing the tube elements is still a cantilever and under severe environment, the cement which holds the envelope in the base often fails. Once this happens, the wiring to the envelope will soon fail. Also, the use of a base-gripping clamp on a phenolic base tube may cause distortion and cracking of the base.

3-47. Clamps which grip the envelope are preferable to the base-gripping type. There are two general types of envelope-gripping tube clamps, the "hat and post" type and the shield type. The "hat and post" type, shown in figure 3-14, is available in many varieties. Hats of various sizes and shapes are available to fit all tubes or plug-in units. The "hat and post" type of retainer is available with either one or two posts.

3-48. Tube shields (in addition to providing electrical shielding) give the tube mechanical protection and help to hold the tube in its socket. Many varieties of tube shields are available, with most of the variations arising from thermal problems. One type of shield is tight-fitting and makes good contact with the tube envelope and chassis, thereby conducting the heat to the chassis (figure 3-15). Some electronic airborne equipment is constructed with a forced air cooling system, in which air is controlled and directed to the most temperature-sensitive parts of the equipment. Special tube sockets have been developed to direct the cooling air over the tube (figure 3-16). Air is forced into channels which carry it to the inside of the tube shield and over the tube envelope (figure 3-17).

3-49. Tube shields can be used more efficiently with subminiature tubes. The absence of a socket permits clamping by the envelope which is desirable both for the rigid support and the thermal conduction it provides. Examples of cradle-type tube shields used for many subminiature tubes are shown in figure 3-18. The tube is held rigidly to the chassis or mounting plate, and mechanical resonance of the tube (as a unit) is unlikely. Good heat transmission from the tube clamp to the chassis is not ensured by riveting. Preferably, the clamp should be soldered to

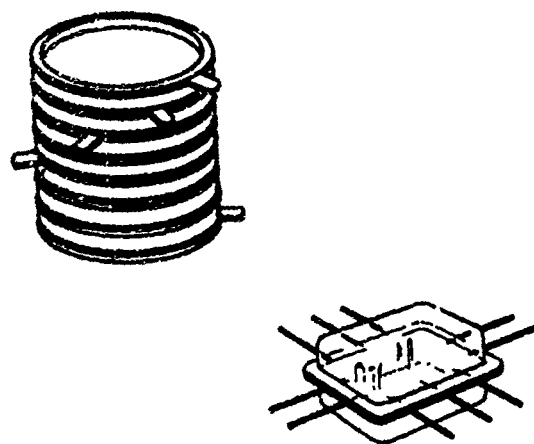


Figure 3-13. New Types of Tubes

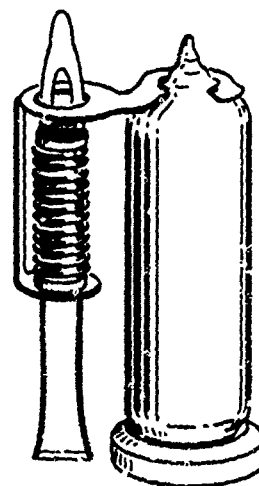


Figure 3-14. Hat and Post Type Tube Clamp

the heat sink as well. It has been suggested that silicone grease between the surfaces being riveted would increase the heat transfer rate.

3-50. An unconventional approach to electron tube clamping is illustrated in figure 3-19. Each tube is held in a hole in a solid metal block. The tubes are held tightly in the holes by hexagonal beryllium copper shims. This mounting method requires very rigid chassis construction. The design provides the chassis with an excellent heat sink and helps reduce tube-envelope temperatures to fifty percent of what may be expected from the average, standard tube shield. Tubes held in this particular chassis were subjected to an environment of 16g from 20 to 1000 cps for a period of 15 minutes in each of three mutually perpendicular planes, and no tube failures occurred. However, the environment in which a particular tube will function must be thoroughly outlined and determined so that equipment is not overdesigned, embodying excessive cost and weight.

### 3-51. TRANSISTORS.

3-52. Transistors, considered apart from mounting techniques, are rigid bodies, resistant to failure due to shock and vibration. Recent tests showed that they do not malfunction when exposed to vibrational stress of 15g through a frequency range of 55 to 2000 cps.

3-53. Although inherently resistant to shock and vibration, transistor failure can still occur through the use of improper mounting techniques. Figure 3-20 shows

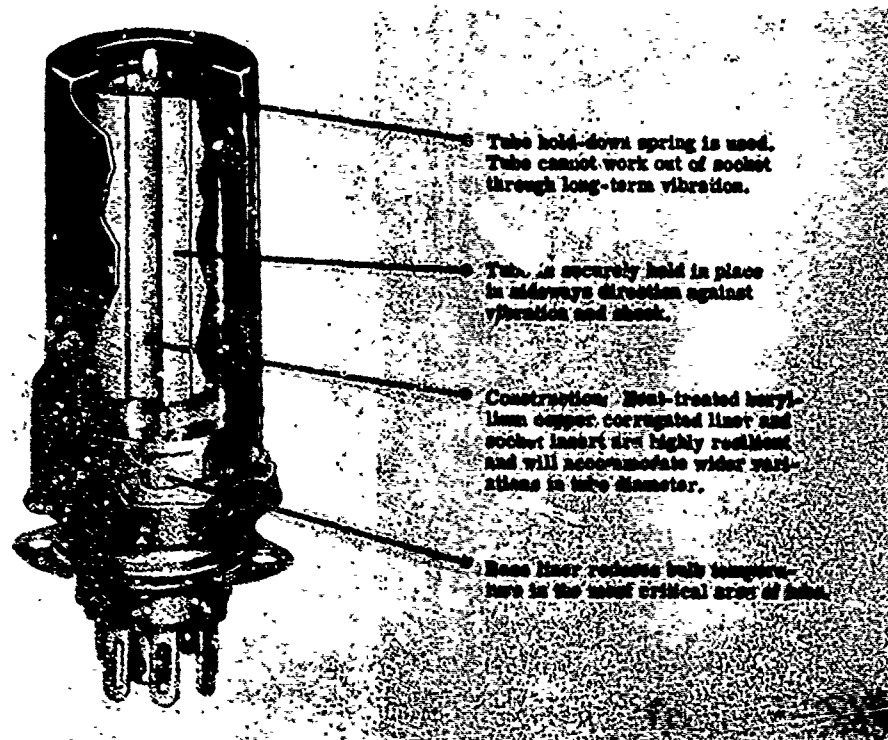


Figure 3-15. Tube Shield Designed to Protect Tube Against Shock and Vibration

five types of transistors which are mounted in various ways in an equipment. The transistor shown in figure 3-20a is held to the chassis with screws. The type shown in figure 3-20b has a threaded stud which screws into a heat sink (figure 3-31). From a shock and vibration standpoint, these two types are obviously desirable because they fasten securely.



Figure 3-16. Tube Socket and Shield with Air Passage for Cooling

3-54. The transistors illustrated in c, d, and e of figure 3-20 are not designed to fasten as securely, and supplementary measures must be employed to ensure their resistance to shock and vibration. The transistor shown in figure 3-20a mounts in a socket in the same manner as electron tubes. For airborne applications, this type must be mounted with some form of retainer such as that shown in figures 3-31 and 3-32. Strips extend across a row of plug-in transistors and are bolted to the chassis. Rubber grommets are used at the points of contact of the transistor and strip.

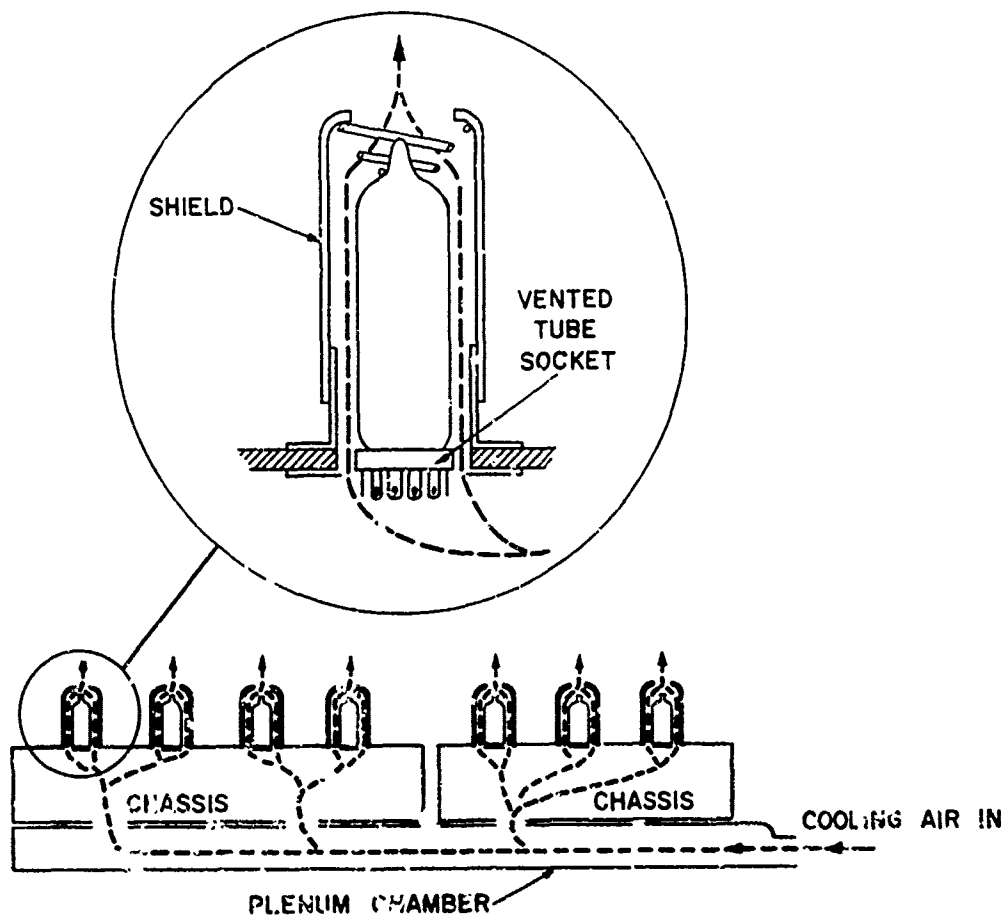


Figure 3-17. Forced Air Cooling of Shielded Tubes

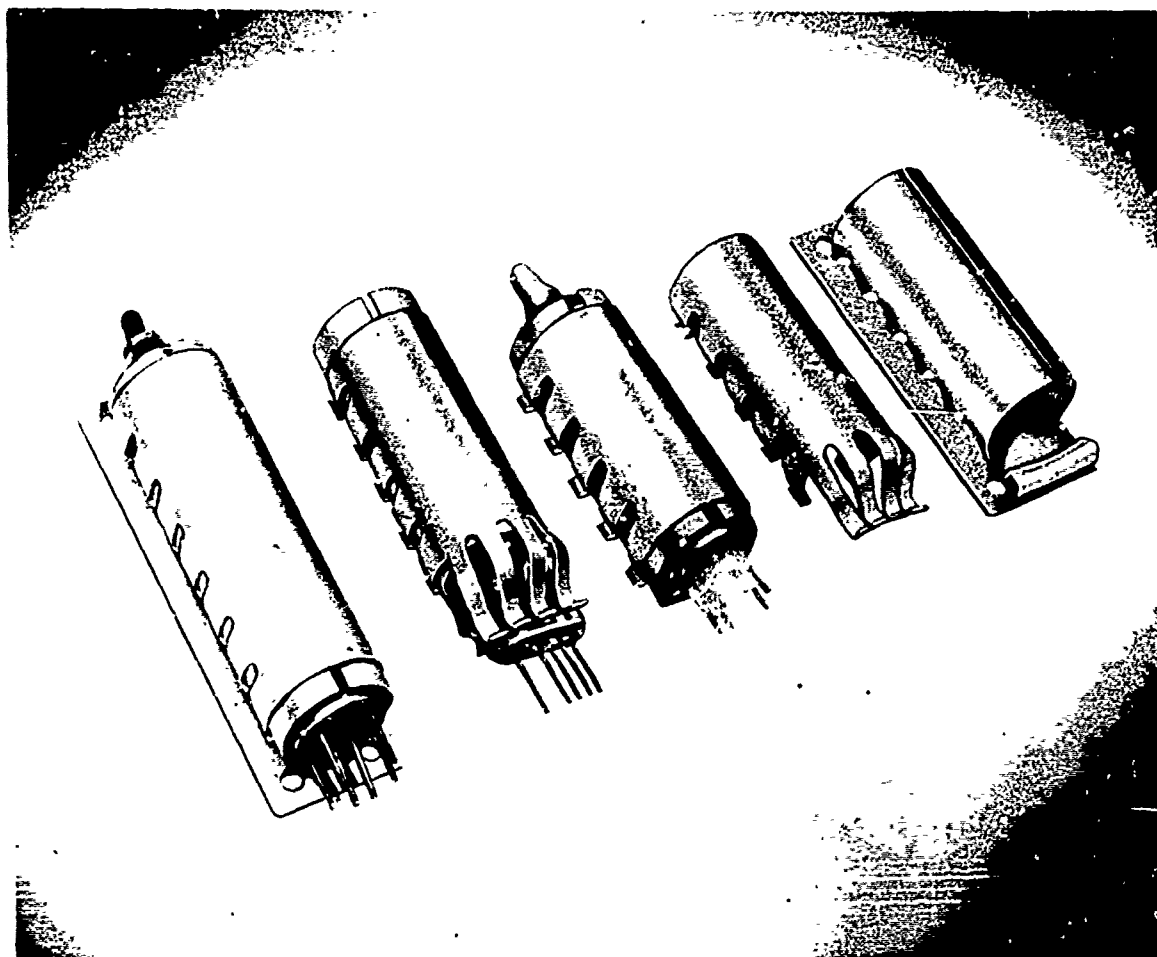


Figure 3-18. Cradle-Type Tube Shields for Subminiature Tubes

3-55. The transistors shown in d and e of figure 3-20 have no visible means of support except their leads. The long type (d) is usually mounted on printed-wiring boards with the long side flat against the board. They are held to the board by spring clips or wire straps. The flat, button type (c) may be used without retainers if mounted on a printed-wiring board with the leads holding the base tight against the board. Both of these latter types of transistors are less susceptible to failure if conformal coating is used to make the board and component parts rigid. The conformal coating also acts to adhere the transistor to the board.

#### 3-56. TRANSFORMERS.

3-57. Transformers are protected against shock and vibration by the nature of their construction. Their internal wiring is tightly bound around a rigid core or bobbin and is impregnated with a moisture-resistant insulating material, usually varnish. Both the firm winding and the hardened varnish rigidly anchor the core and wire assemblies. In addition, encapsulation is often used to seal the entire transformer assembly (figure 3-21). The transformer is coated by dipping or molding it in a compound, usually elastomeric, which is then cured or hardened by an oven-heating process. This gives the transformer a moisture-resistant coat and adds further to the transformer's ruggedness.

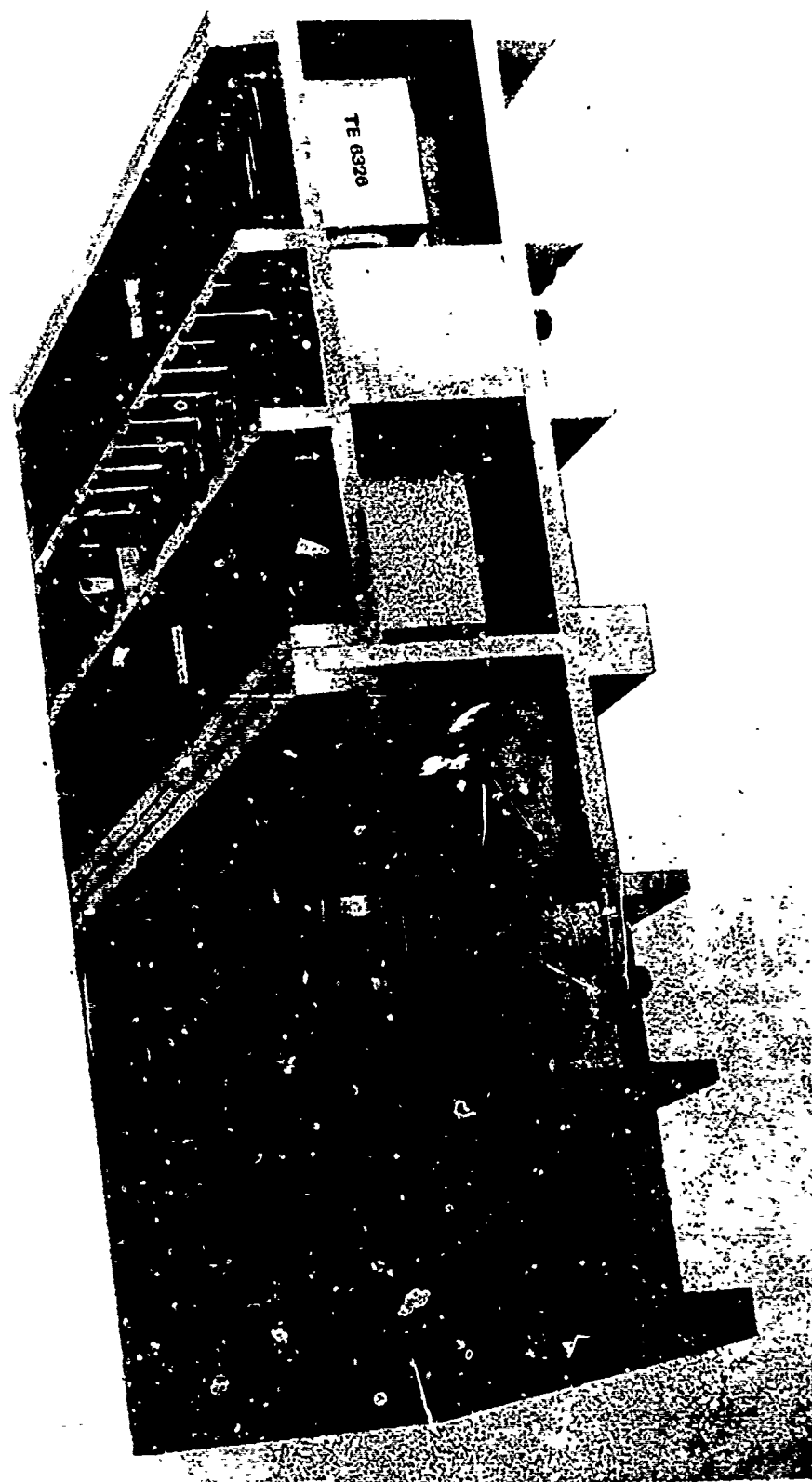


Figure 3-19. Chassis Design Provides Holes for Rigid, Close-Tolerance Mounting of Tubes

3-58. As with most other airborne electronic components, difficulties involving transformers resulting from exposure to shock and vibration environment occur because of poor or inadequate provisions for mounting. In many cases, the transformer, which probably is the heaviest component on the electronic chassis, is held by frail brackets or legs, or undersized bolts. Inadequate support may result in failure or, if the transformer breaks loose, severe damage to the equipment. Frequently, transformers are mounted on a relatively flimsy chassis which lowers the chassis resonant frequency to a point where it is in the environmental spectrum. Thus, input accelerations will be amplified by the chassis resulting in destructive forces on the mounting brackets, bolts, or on the chassis itself.

3-59. The transformer's terminals are considered second to the transformer's mounting in vulnerability to shock and vibration. Unsupported lead lengths should be kept to a minimum to reduce the forces imposed on the terminals.

3-60. Certain types of transformer assemblies have definite points or areas which are susceptible, by the nature of their construction, to shock and vibration. The transformer shown in figure 3-22 exemplifies this point. Having originally been fixed to a chassis only by a steel tape through its base, the transformer was vulnerable to lateral excitation. Adding a support bracket between the base and the core raised the natural frequency of the transformer and distributed the strain rather than concentrating it at the intersection of the base and the tape.



Figure 3-21. Transformer Encapsulated in Silicone Rubber

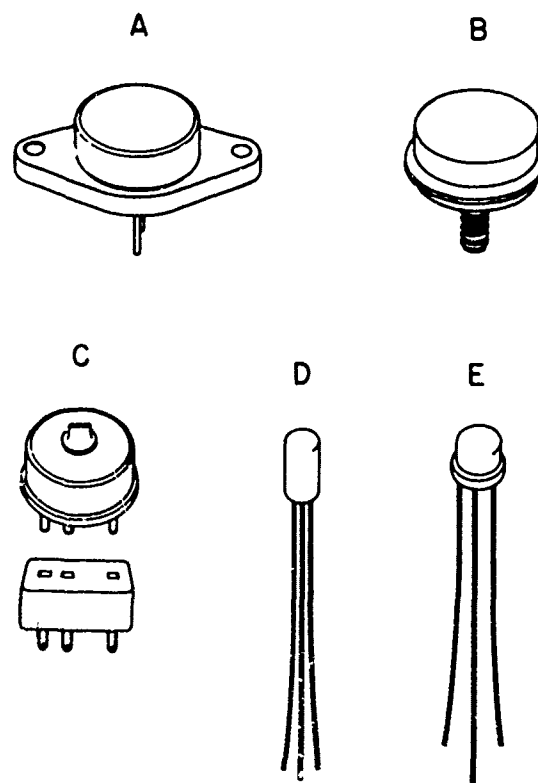


Figure 3-20. Five Types of Transistors

3-61. What may ordinarily seem to be insignificant construction details may mean the difference between satisfactory performance and complete failure. Figure 3-23 shows two transformers which, although of similar construction, are different in that one incorporates specific features which protect the component against damaging shock and vibration.

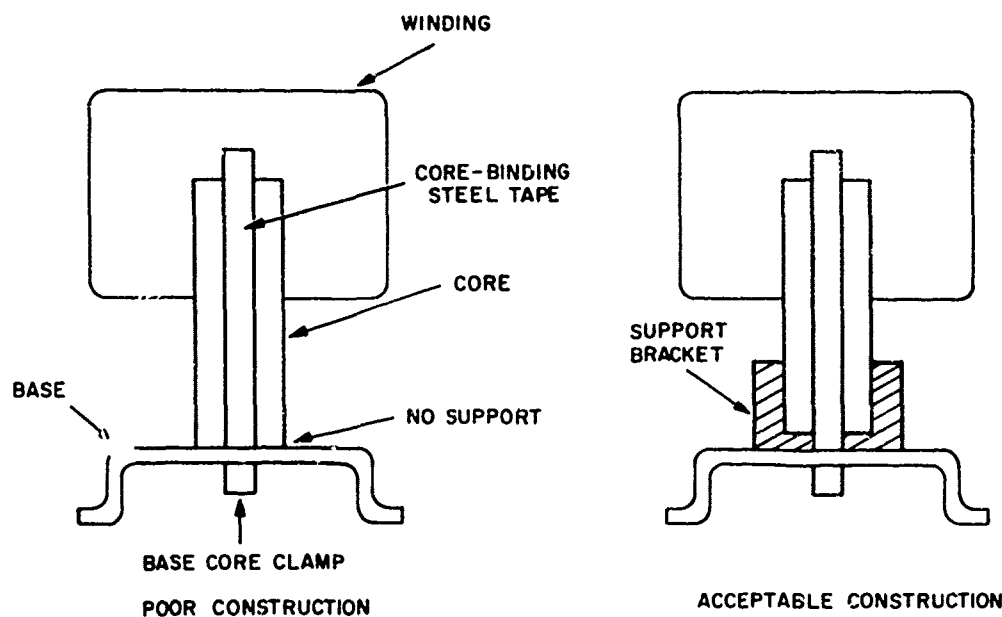


Figure 3-22. Transformer Design to Resist Shock and Vibration

### 3-62. SWITCHES AND CIRCUIT BREAKERS.

3-63. In general, switches are affected little, if at all, by shock and vibration in airborne electronic equipment. Certain types of lever-operated toggle circuit breakers and switches, however, must be used with caution. In a severe shock environment, the unbalanced mass of large, manual actuating levers used in some breakers and switches may have sufficient inertia to trip the breaker. Where this possibility exists, other lever designs must be used. Miniaturization of these normally unbalanced members of a component part will decrease inertia forces under shock.

3-64. Tests have indicated that the commonly used bat-handle toggle switches have sufficient spring pressures to ensure satisfactory circuit contact, either in shock or in vibration environments. Rotary switches, where contacts are of a sliding knife-clamp configuration, are generally found to be trouble-free from accelerations (either linear or rotational), providing their actuating levers and contact-positioning detent devices are balanced or are of very low mass.

### 3-65. CAPACITORS AND RESISTORS.

3-66. Mounting capacitors and resistors by their leads is a simple and economical installation. This method, however, should be restricted to installations where the lead-mounted capacitor or resistor will not be excited at resonance.

3-67. Components mounted by their leads are masses supported by complex resilient beams with little damping. Should this type of assembly become excited at its resonant frequency, the stress on the leads could easily become great enough to cause them to break. Tests on lead-mounted components indicate that the leads fatigue quickly when the component is excited at resonance, but seem to last indefinitely when excited at other frequencies. For example, excitation of 2g at reso-

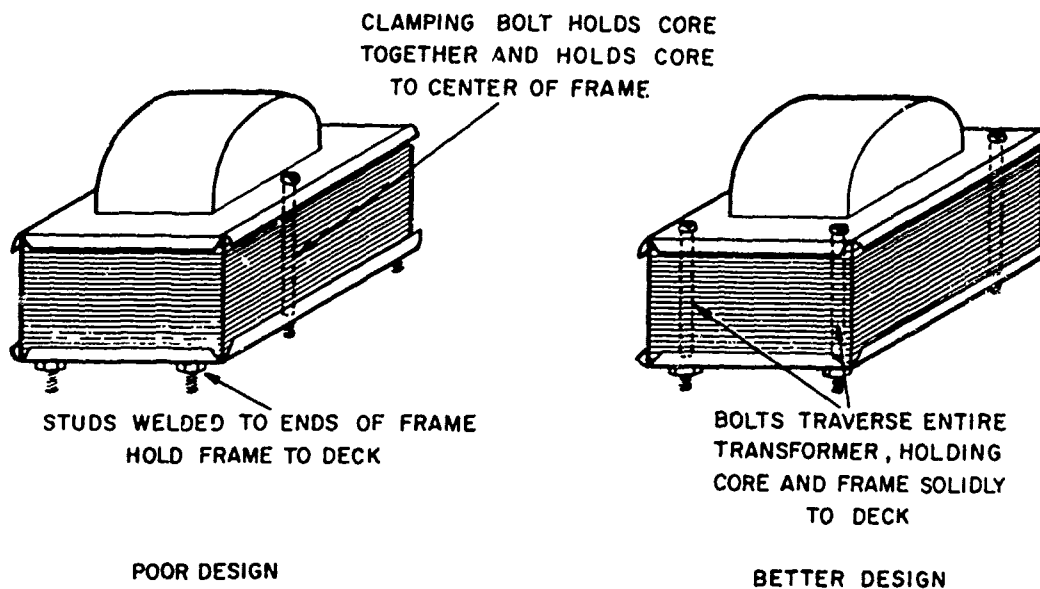


Figure 3-23. Transformer Designs Showing Good and Bad Practices

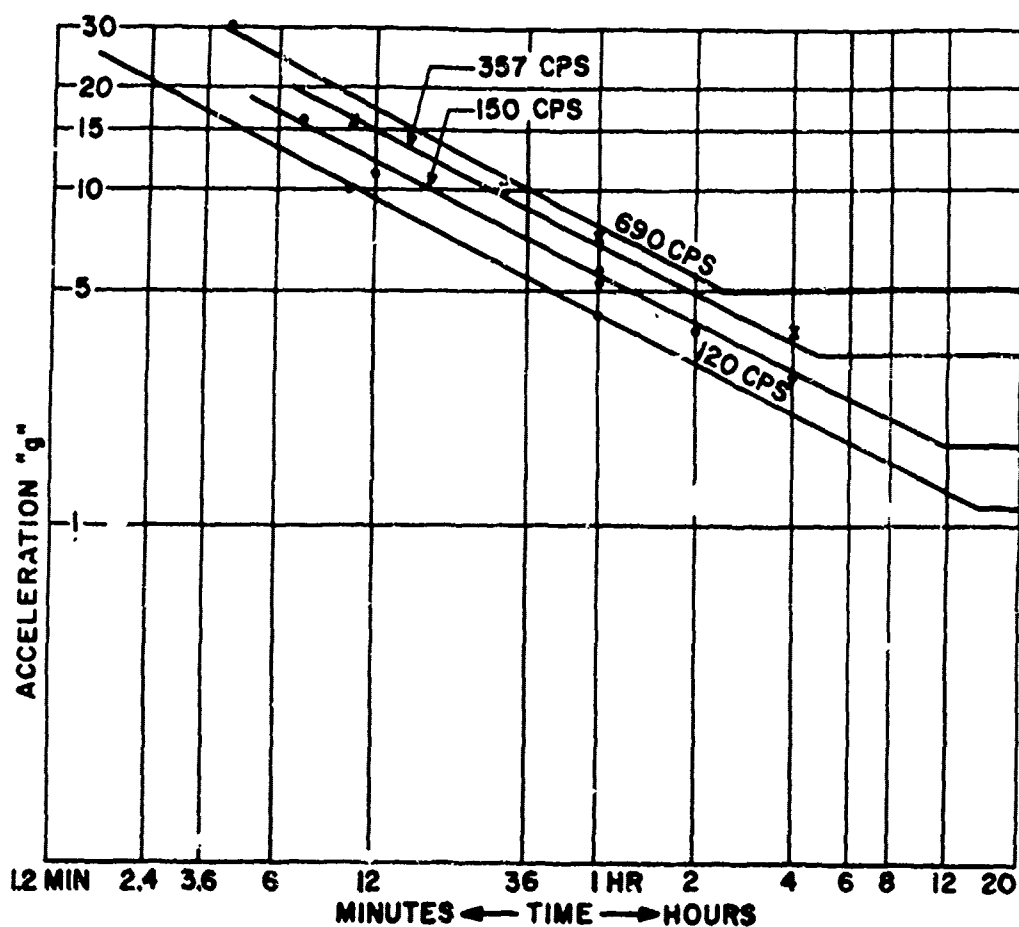


Figure 3-24. Fatigue Life of a Lead-Mounted Resistor Versus Natural Frequency

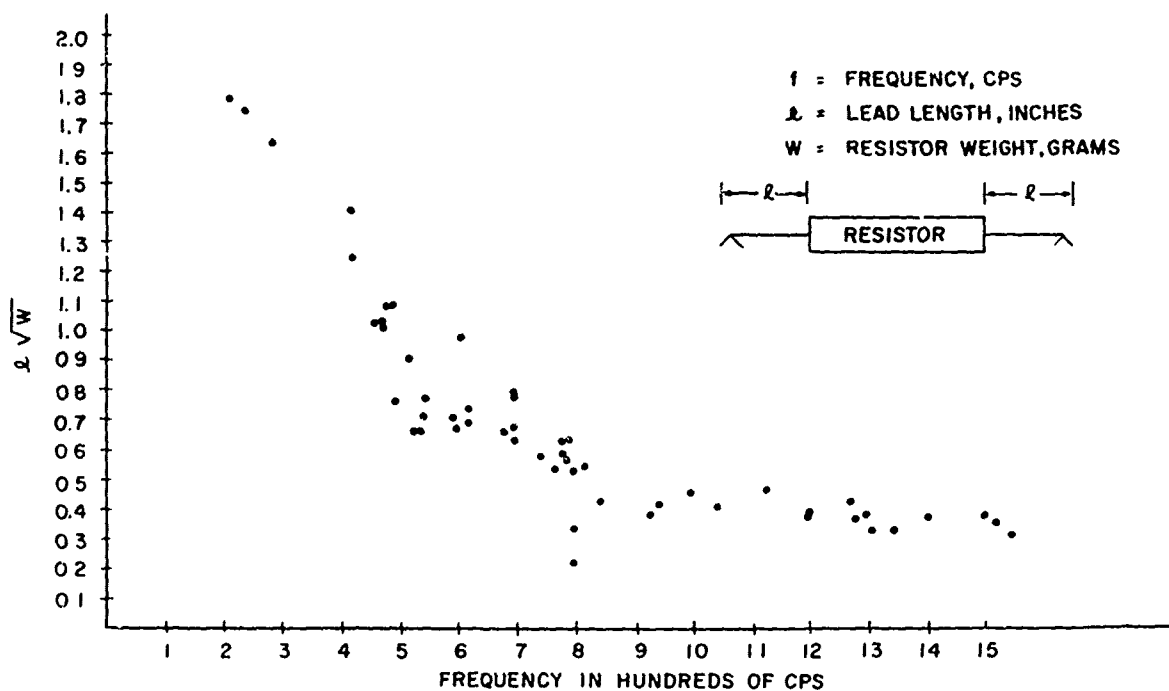


Figure 3-25. Natural Frequencies of Lead-Mounted Composition Resistors as a Function of Weight and Lead Length

nance can cause failure in a few minutes, but tests at 20g for 50 hours at nonresonance produced no failure. Tests show no tendency toward failure of lead-mounted capacitors and resistors under conditions of shock or acceleration (50g acceleration; 50g shock with 10 millisecond duration). This is obviously a result of the light weight of the components. Although lead-mounted components fail much more quickly when excited at resonance than when excited at other frequencies, the life of a component excited at resonance may be increased by increasing the natural frequency. Figure 3-24 shows the results of tests on a particular resistor size and type and illustrates the increased life which can be obtained by increasing the natural frequency by shortening the lead lengths. The curves are averages of the results of several test specimens which were excited at resonance. The curves should not be used as a source of quantitative information on the life of lead-mounted resistors since it represents the results on only one resistor type and size.

3-68. By limiting the lead length and the size (mass) of the component, the mechanical resonant frequency may be kept relatively high. However, the mass of a component is fixed by the required electrical characteristics and there is a practical limit on the shortening of lead lengths. There is, therefore, an upper limit on the natural frequency of a lead-mounted component when using components now available. (Larger lead sizes for a given component weight would give higher natural frequencies.) Figure 3-25 is a plot of experimental data relative to lead-mounted composition resistors. The plot indicates the dependence of natural frequency on lead length and weight. It also illustrates the practical limit on natural frequency, since a simple calculation shows that a mass of 2.86 grams or larger, having a lead length of 0.125 inch or more, will display a natural frequency of 500 cps or lower. The natural frequency of capacitors plotted in this manner would be

still lower, since manufacturers generally place leads with smaller diameters on capacitors than they do on resistors of the same weight.

3-69. It is obvious, then, that the method of mounting components by using short leads as their supports, or by using lighter components, may be valid when applied in environments such as propeller-driven aircraft (or isolation-mounted equipment) where the high-frequency excitation is of low magnitude. However, when the method is applied in a guided missile with rigid mounting, for example, where chassis vibration of appreciable magnitude may exist for long periods at frequencies coincident with the natural frequency of the lead-mounted component, it may become necessary to clamp or cement the component to the chassis as shown in figure 3-26. Attempts have been made to produce a formula based on mass and lead length which would indicate the natural frequency of a lead-mounted component. These attempts have been unsuccessful. As indicated by figure 3-25, for a given combination of lead length and component mass, the variables introduced through mounting give a wide range of natural frequencies.

3-70. If capacitors and resistors are to be lead mounted (in isolator-mounted equipment), the selection of the correct capacitor or resistor will prolong the life of the installation. For example, certain molded tubular capacitors and resistors which hold a portion of their lead length in molding plastic were compared to units in which the leads are terminated with a rivet or some clamping device. Test results showed that embedding the lead in molding plastic greatly reduced the number of lead failures.

3-71. Proper mounting techniques are likewise important. Tests showed that mounting capacitors or resistors to flexible terminal boards by their leads is un-

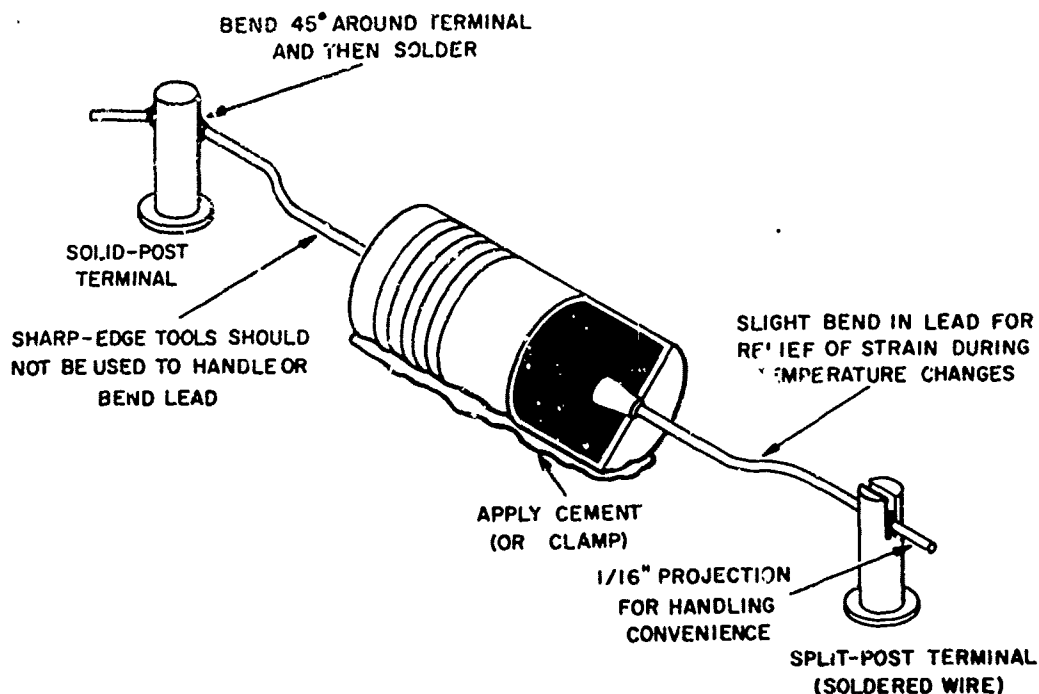


Figure 3-26. Proper Mounting Technique for Lead-Mounted Components Used in Guided Missiles

sound because the resulting lead flexing causes eventual failure. Any installation where the capacitor or resistor leads are subject to flexing is poor.

3-72. Failures of lead-mounted components may sometimes be attributed to shock or vibration when, in truth, this is not the primary cause. For instance, if the leads of these components are wrapped around the terminal, the winching action of wrapping may leave the leads with a high residual tensile stress which may cause the lead to break under the additional stress imposed by differential thermal expansion. Tests of lead-mounted components show that lead failure invariably occurs in the lead and not in the solder joint, even though the joint does not involve wrapping of leads. These tests throw doubt on the wisdom of wrapping leads to secure a good mechanical joint prior to soldering.

3-73. Proper mounting techniques for both fastened and lead-mounted resistors and capacitors incorporate many minor but important considerations. For example, as indicated in figure 3-26, the lead should be handled carefully with the proper tools in order to prevent nicking or scratching, and the lead should be given a slight bend to allow for temperature contractions of the wire between component and terminal.

#### 3-74. TERMINALS.

3-75. The rigidity of a terminal in its mounting hole determines to some extent the ability of the attached lead to withstand mechanical excitation. If the terminal becomes loose and vibrates, the wire in turn becomes susceptible to vibration forces. A terminal may become loose if it is overheated or jarred while a wire is being soldered to it; it may loosen if it is not tightly swaged, or if an attached heavy wire is drawn so tightly that the terminal is strained against its mounting hole. By vibrating and bumping the edges of their mounting holes, loose terminals cause extreme wear around the edge of the hole and become looser. Under these conditions, both the wire and the terminal are endangered. Therefore, several steps must be taken to obtain a reliable terminal function. The insulating board must be of such composition that it will not readily wear or crack around the terminal mounting hole. The terminal itself must be made of such material that the terminal shank will not crack excessively when it is swaged. Careful swaging of the terminal shank, using proper techniques, will increase the gripping power of the terminal.

3-76. Tests show that knurling the terminal enables it to grip the mounting board securely and thus resist turning in the hole. Terminals with knurled shanks generally perform better than those with radial knurling (figure 3-27). Figure 3-28 shows the comparative torques required to turn knurled and unknurled terminals. These curves also show the effect of terminal-to-hole fit. As would be expected, tighter fits provide better turning resistance.

#### 3-77. PRINTED WIRING BOARDS.

3-78. Printed wiring boards generally are smaller, lighter, and cheaper to manufacture than conventional circuits. Even in short production runs, standardized circuits and standardized tooling techniques can afford a substantial savings in the cost of the equipment.

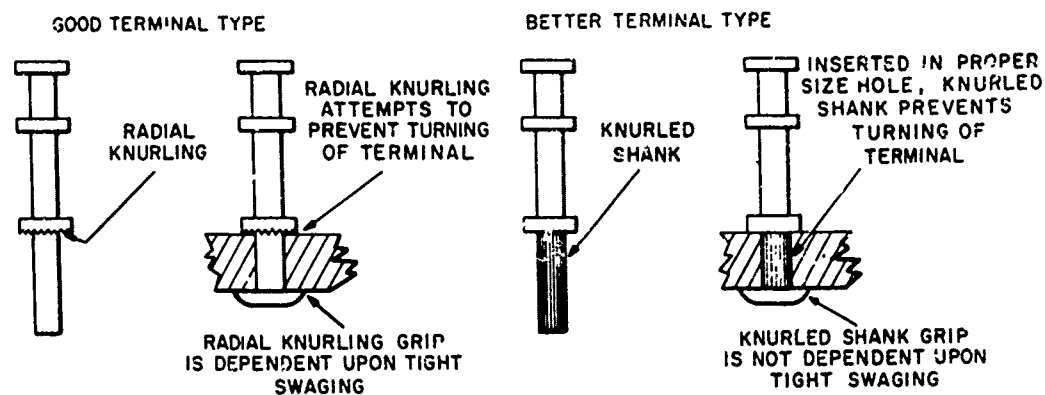


Figure 3-27. Proper Knurling of Terminal Posts

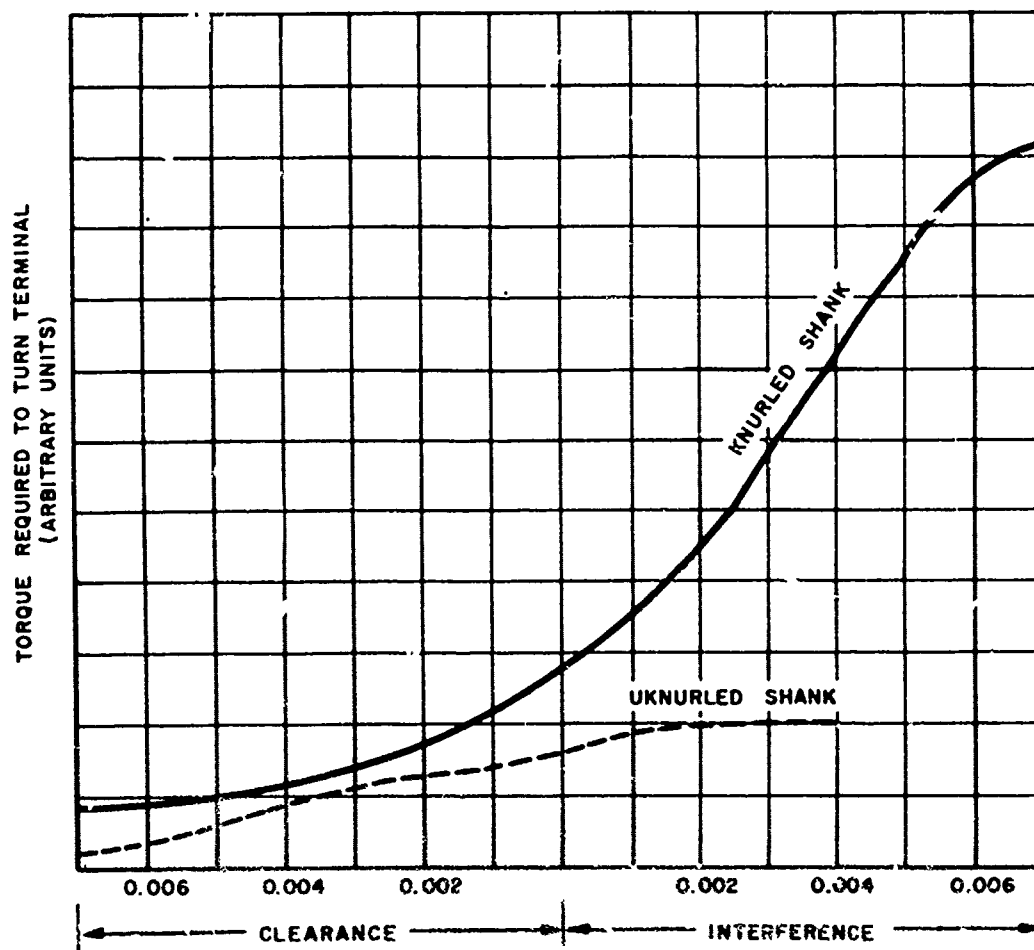


Figure 3-28. Comparative Torques Required to Turn Knurled and Unknurled Terminals

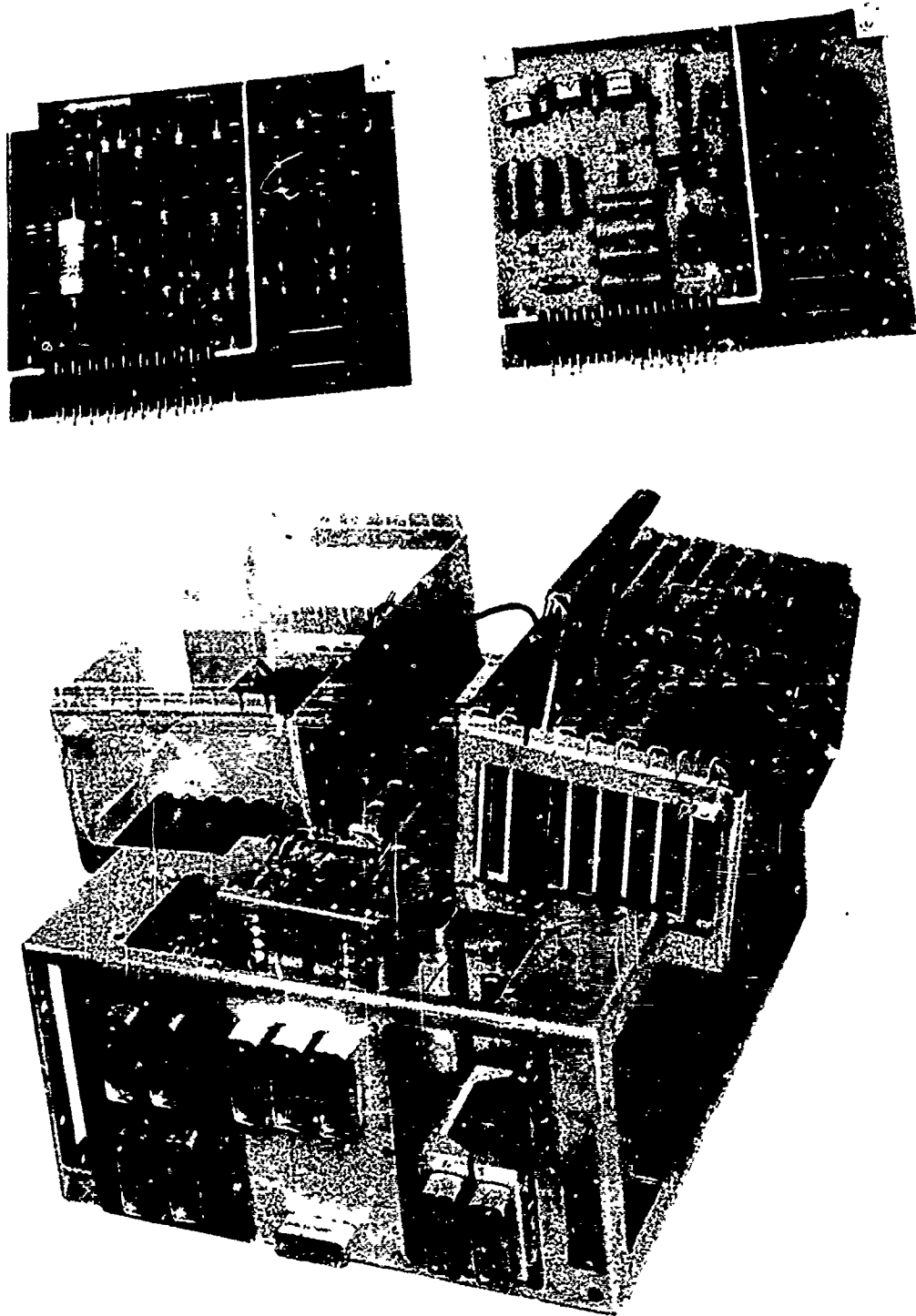


Figure 3-29. Printed Wiring Boards and Method of Packaging

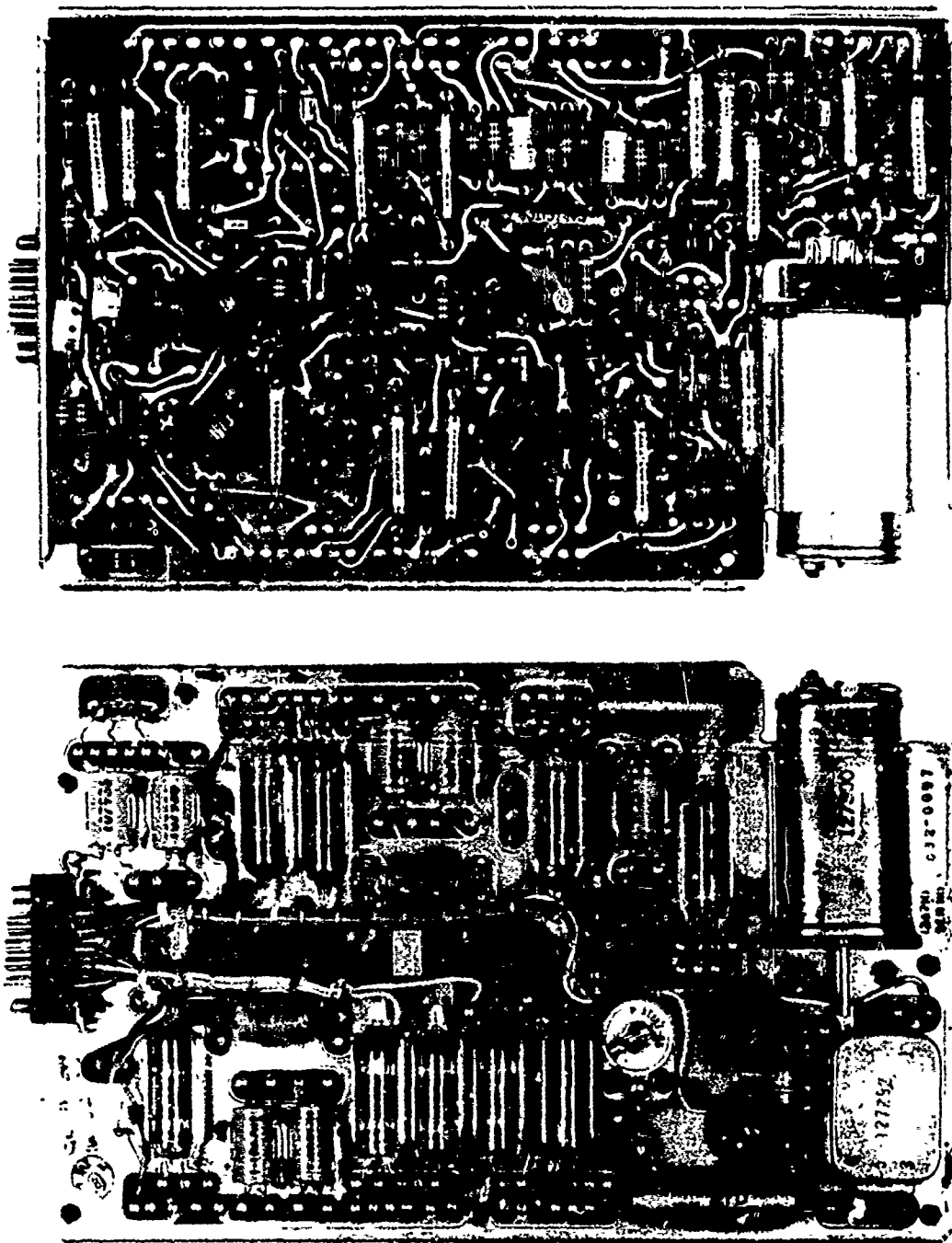


Figure 3-30. Both Sides of a Printed Wiring Board  
Supported by a Sheet Metal Chassis

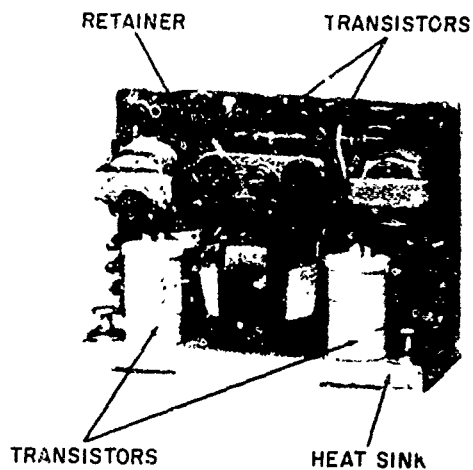


Figure 3-31. Printed Wiring Board Treated With Conformal Coating

3-79. Printed wiring boards usually are thin and flexible and require some form of stiffening structure to prevent low resonant frequencies. The stiffening may consist of a metal chassis, conformal coating, or complete potting in plastic. The exceptions to this rule are small wiring boards, which, because of their size, are rigid enough to be used without additional stiffening in mild vibration environments.

3-80. The board illustrated in figure 3-29 is designed for use in a mild vibration environment; that is, in an equipment that is mounted on isolators. The printed board is stiffened by a header, a connector block, and two bridging strips. Resistors, capacitors, and

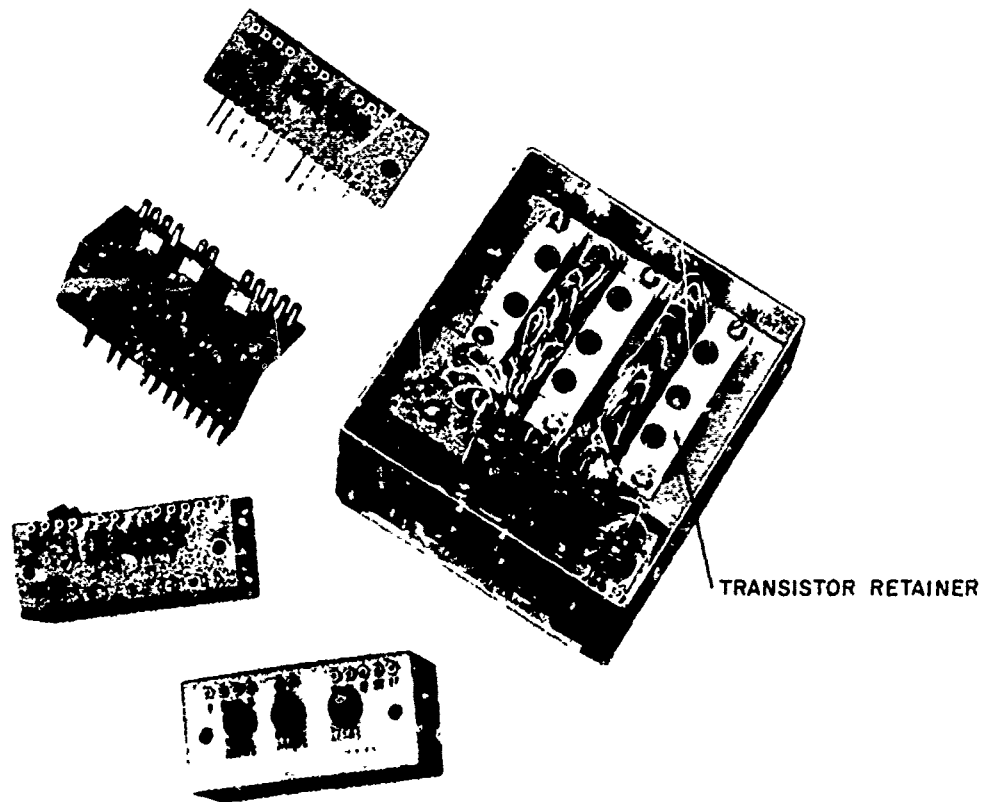


Figure 3-32. Printed Wiring Boards Formed into Potted Modules

diodes are held to the board solely by the leads; the transistors are held by leads and by wire straps. When the boards are packaged as shown in figure 3-29, they are supported only at the connector block and at the header. Were this equipment required to withstand a more severe environment (rigid mounting) the entire perimeter of the board would require support. The capacitors, resistors, and diodes would be clamped or cemented to the board.

3-81. Figure 3-30 shows both sides of a printed wiring board supported by a sheet metal chassis. All heavy parts are mounted to the chassis by screws or spring clips, while the light ones are mounted directly to the wiring board by their leads. A layer of insulating paper is used between the board and the chassis, and the assembly is fastened together around the perimeter and also in the center area. The metal chassis acts as a stiffener for the board and gives structural support to the heavier components.

3-82. Figure 3-31 shows a printed wiring board treated with conformal coating. The component-part side of the board is dipped into a clear plastic material that hardens to form a rigid assembly. Resistors and capacitors are mounted to the board by the leads, and the transformers and other heavy parts are screwed or riveted directly to the board. This technique gives a thick, hard plastic coating and is not to be confused with the usual fungicidal or tropicalizing dips.

3-83. Printed circuit boards can also be made rigid by complete potting in plastic. Figure 3-32 illustrates a technique in which the wiring boards are formed into potted modules. Three boards are assembled at right angles to each other and joined by wire stitching which connects the sections electrically. The assembled boards are then potted with the terminals and the surface of the transistor sockets exposed. The potted units are bolted into the equipment and the connections made to the terminals. The transistors are held down by retainers which are fastened by the same bolts that hold the units.

3-84. Figure 3-33 is an example of microminiature construction using printed wiring boards. All component parts are mounted by the leads between two boards. The boards and parts mutually support each other to form a compact and rigid unit. It appears that this construction would offer more vibration and shock resistance than the same circuitry on a flat wiring board.

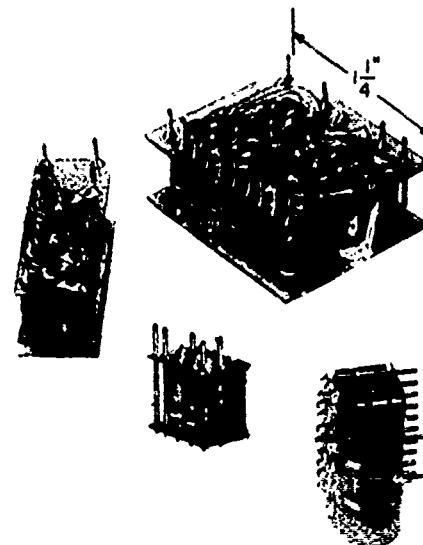


Figure 3-33. Microminiature Construction Using Printed Wiring Boards

## SECTION IV

### RACK AND CHASSIS DESIGN

#### 4-1. GENERAL.

4-2. The primary objective of rack and chassis design for electronic equipment is to mount component parts so they will perform electrically in a mechanical environment. Secondary objectives are ease of maintenance and operation. Rack and chassis design can control the transmission of excitation by fixing the relationship of the excitation reaching the components to the input excitation reaching the rack or chassis, with respect to both frequencies and amplitudes.

4-3. Structurally, the rack may vary from a simple mounting base to a complex construction that integrates many equipments; the chassis may vary from a sheet-metal box to a complex casting. No matter what form the rack or chassis may take, the design of these structures is essentially a matter of providing the best flexibility. A rack or chassis that is too flexible may amplify vibration excitation; one that is too rigid may transmit an unnecessary amount of the shock excitation.

#### 4-4. DESIGN FACTORS.

4-5. Generally, rack and chassis designs are determined by the component parts required for the equipment and the environment in which the equipment must operate. Specifically, the designer must decide (1) whether to design for isolation or resistance, (2) what natural frequencies are desired for the rack and chassis, and (3) component part arrangement.

4-6. COMPONENT PARTS. In some cases, the choice of component parts has to be a compromise between electrical efficiency and mechanical ruggedness. With desired electrical efficiency established in advance of construction, mechanical weaknesses can occur if no rugged component part is available. Circuit redesign may be necessary to permit the use of more rugged parts.

4-7. The rack and chassis (and isolation system, if any) should be designed so as to minimize the effects of component part resonances. Selected component parts should be tested to determine their natural frequencies. Since these natural frequencies are fixed, the excitation reaching the component parts at these frequencies must be of a sufficiently low magnitude. The excitation reaching the component parts is a function of the environment existing at the airframe, the flexibilities of the rack and chassis, and the damping incorporated into the rack and chassis.

4-8. MINIATURIZATION. Miniaturization of components, leading to smaller and lighter chassis and equipments, produces systems with ruggedness and rigidity. The decreased dimensions (beam lengths) and the smaller masses of the miniaturized elements lead to higher natural frequencies. Since an acceleration from the environment produces a force on a component dependent on the mass of

the component, the acceleration forces on lesser weights (miniaturized equipment) are less than on larger weights. Miniaturization can also result in other advantages, such as higher equipment densities and smaller equipment size and weight.

4-9. ENVIRONMENT. The anticipated environment is the determining factor in designing for natural frequencies of the mounting system and equipment. For setting the natural frequencies of the rack and chassis, one can use the following rule-of-thumb: the natural frequencies should be no less than two times and preferably three to four times the highest frequency of the excitation source. This source will be either the isolators or the airframe, depending on whether flexible or rigid mounting is used. Frequently, especially in missiles, the excitation frequencies range so high that it is impossible to design the rack and chassis for a rigidly mounted equipment so that the natural frequencies fall above the excitation frequencies. In this case, the chassis and rack natural frequencies should be set in a range where the acceleration levels are suitably low, and the chassis and rack should be damped to limit resonance responses.

4-10. The natural frequencies for the rack and chassis can be readily fixed if the environment has been accurately measured and clearly stated in terms of the point of attachment of the equipment. The data, however, may not always be so explicitly stated. It may be given for general airframe locations, or for unloaded structures. When the data is given for general airframe locations, the designer will have to account for flexibilities in the mechanical system between the point of measurement and the point of attachment. When it is given for unloaded structures, the problem of estimating the environment may become even more complex. Neglecting the loading of the structure at which the environment is measured can give rise to serious errors. Airframe sections which deflect 1/16 inch under the weight of an equipment are as flexible as many vibration isolators.

4-11. The designer should attempt to minimize the number of structures in the mounting system and equipment. Determining the exact effects of a number of structures can be quite difficult. Simplification of design may eliminate such structures as brackets and terminal boards; making the remaining structures as rigid as possible will further reduce alteration of the environment. Each flexibility eliminated by redesign eliminates a degree-of-freedom from the mechanical system. This not only simplifies design problems, but increases the likelihood of producing a successful equipment.

4-12. NATURAL FREQUENCY OF STRUCTURES. The following discussion gives stiffness and frequency calculations for simple structural members. While complex structures are not covered, the knowledge that can be gained from these calculations can assist in making intelligent guesses for complex structures.

4-13. The natural frequency of an undamped single-degree-of-freedom system is proportional to the static deflection in a linear spring system, as shown in the following calculations:

$$f_n = \frac{\omega_n}{2\pi} \text{ where } \omega_n \text{ is the circular natural frequency.}$$

But  $\omega_n$  was chosen as  $\sqrt{k/M}$  (see paragraph 1-23),

so that  $f_n = \frac{1}{2\pi} \sqrt{k/M}$ .

Since  $M = \frac{W}{g}$ , then  $f_n = \frac{1}{2\pi} \sqrt{\frac{kg}{W}}$ .

Evaluating  $\frac{\sqrt{g}}{2\pi}$  and substituting,  $f_n = 3.13 \sqrt{\frac{k}{W}} = 3.13 \sqrt{\frac{1}{\delta}}$

where  $\delta$  is equal to the static deflection in inches. The natural frequency, then, can be determined from the measured deflection of the structure produced by the load.

4-14. To use the last formula derived above, the deflection must be known. To attempt to determine natural frequencies in the design states of structure, it is necessary to calculate the expected deflection. Figure 4-1 gives formulas for calculating the deflection for the following simple structures: (a) cantilever, point load at end; (b) simply supported beam, point load at center; (c) simply supported beam, point load at any point; (d) fixed beam, point load at center; (e) cantilever, uniformly distributed load; (f) simply supported beam, uniformly distributed load; and (g) fixed beam, uniformly distributed load.

4-15. For calculating the deflection of complex structures, the individual flexibilities are added like resistances (spring constants or stiffnesses are added like capacitances). For series structures, the combined flexibility  $F^*$  (expressed in inches per pound) is the sum of the flexibilities represented by

$$F = F_1 + F_2 \dots \dots \dots$$

For structures in parallel, the combined flexibility is represented by the equation

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} \dots \dots \dots$$

4-16. Calculating the natural frequencies of complex structures is extremely difficult. Any structure that has interconnecting flexible members supporting various distributed masses has many degrees of freedom. The best approach to achieving proper rigidity of structures is to "shake" the equipment under development and to stiffen it through cut-and-try methods, provided that simplified structural analysis is not feasible.

4-17. **STIFFNESS OF STRUCTURES.** The stiffness of a beam is dependent upon the modulus of elasticity of the material and the cross-sectional area and configuration. The modulus of elasticity values ( $E$ ) for several typical chassis materials are given in table 4-1. Typical cross sections of structural shapes are shown in figure 4-2. These configurations are available in a complete range of sizes and materials. Their design is a result of specialized needs, generally, to give the proper amount of stiffness in each direction of deflection at minimum weight.

\* Flexibility is the reciprocal of spring constant; see footnote on page 6.

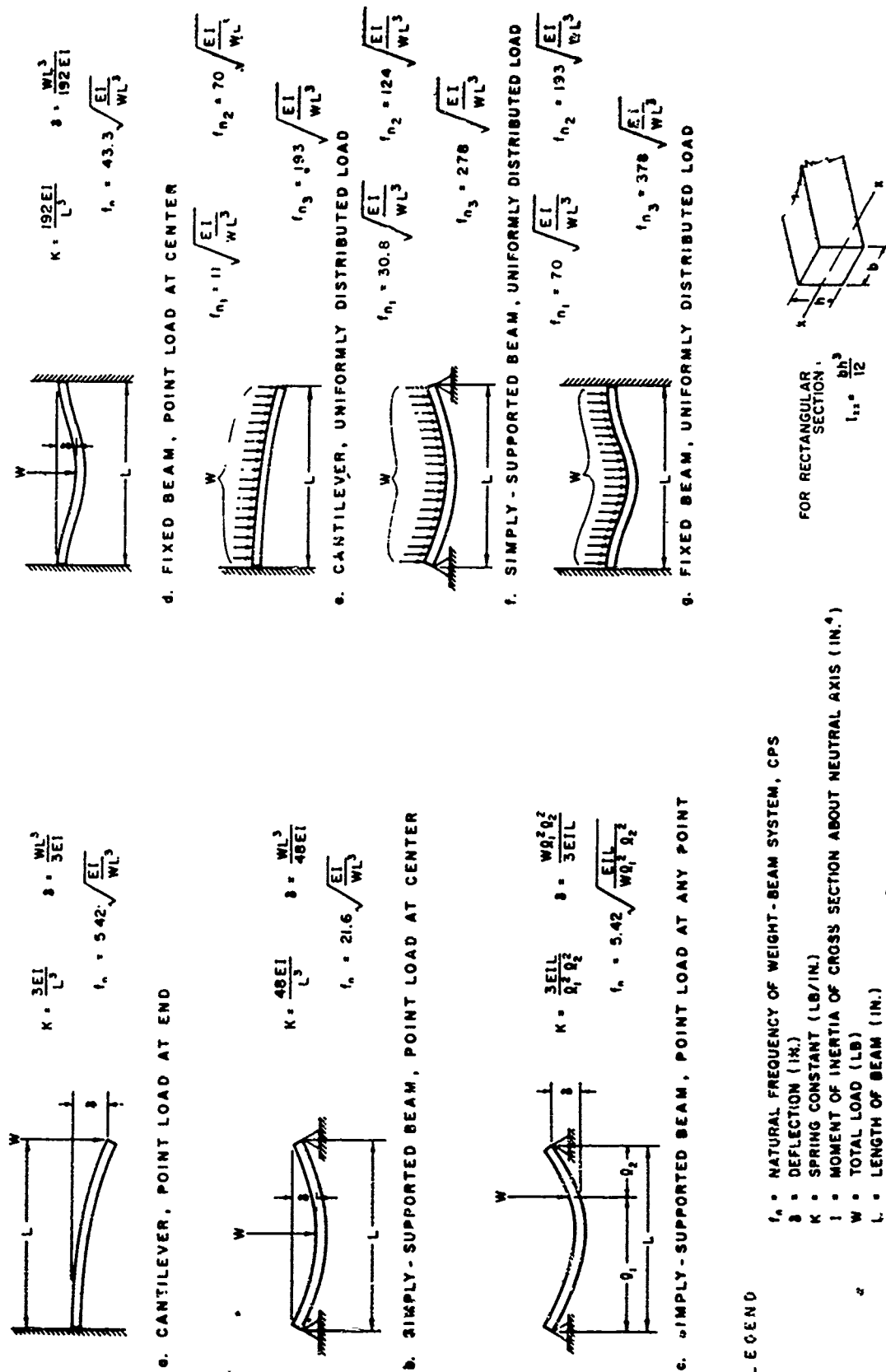


Figure 4-1. Formulas for Calculating the Natural Frequencies in Various Structures

TABLE 4-1. MODULI OF ELASTICITY FOR CHASSIS MATERIALS

Chassis Material	E (Pounds per Square Inch)
Aluminum	$10.2 \times 10^6$
Magnesium	$6.1 \times 10^6$
Steel	$28-31 \times 10^6$
Brass	$13.4 \times 10^6$
Copper	$14.5 \times 10^6$

4-18. If a beam is to be loaded three times as severely vertically as it is horizontally, with equal deflections in each direction, then it should be three times as stiff in the vertical plane. This means that the moment of inertia ( $\text{in}^4$ ) in the vertical plane must be three times that in the horizontal plane. A "square" H-beam will fulfill these requirements. Other structural shapes satisfy other ratios of vertical-to-horizontal stiffness.

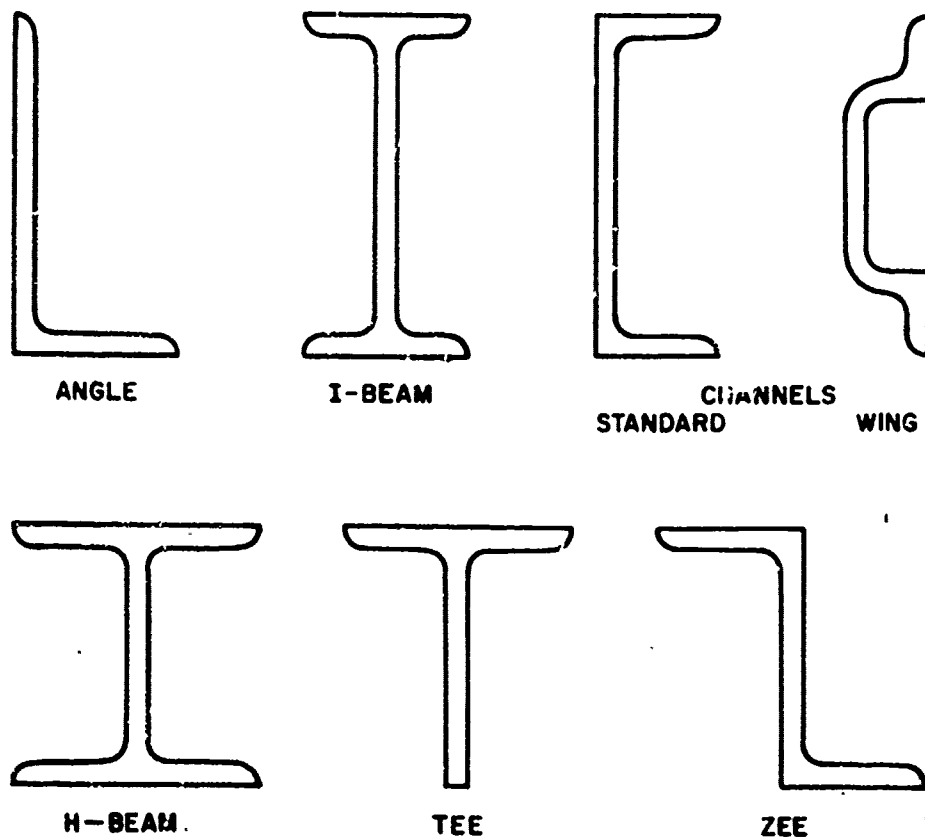


Figure 4-2. Typical Cross Sections of Structural Shapes

4-19. These structural sections should be oriented so that the maximum stiffness occurs in the plane of maximum excitation. Since a standard chassis is, in section, similar to a channel, it follows that if the major excitation is in the vertical axis, the best chassis orientation probably is vertical. However, the effect of vertical chassis orientation on component parts must also be considered; in all probability, they would be excited as cantilevers, if normal mounting techniques were used.

4-20. In designing any structure, a safety factor is used. The expected loading is multiplied by this factor and the structures are designed to hold this overload. The safety factor should increase in proportion to the criticalness of the structure and decrease in proportion to the exactness of the environmental data.

#### 4-21. STRESS DISTRIBUTIONS.

4-22. The stress distributions within the materials forming racks or chassis will have a marked effect on the service life of these structures. To visualize stress distribution, a plate under tension can be imagined to have unbreakable lines across it parallel to the direction of stress. In a plate that has no bends, cutouts, or sudden changes in cross-section, these lines will be equal distances apart. If the plate is bent, perforated, or drawn in being formed into a chassis, these lines will have to pass through or around these deformations. This may result in bunching of these lines, or, more precisely, stress concentrations, since the stress at any point in the plate will be proportional to the spacing between the lines. Although this can only be imagined for metals, such lines and stress concentrations can be seen with polarized light in stressed plastics.

4-23. Since stress concentrations will have deleterious effects on rack or chassis service life, every effort should be made to minimize them. Sheet metal bends should be generous and properly formed; bend-relief should be provided at corners; changes in cross-section should be filleted; sharp-cornered cutouts and notches should be avoided; and hole shape and location should be determined with a view toward their effects on chassis flexibility and strength. Figure 4-3 shows a fractured cantilever bracket damaged because of spot-weld failure and stress concentration at a 90-degree cutout at the bend of the L.

4-24. SHEET METAL BENDS. Large bend radii reduce the likelihood of stress concentrations and of surface cracks due to forming. For practical reasons, bend radii cannot always be made large, but they should be greater than certain minimum values. A safe rule-of-thumb for the more common sheet metal stock, such as aluminum and steel, is to make the bend radius about four to six times the thickness of the material. The "minimum" radii for specific materials are stated in tables of sheet metal bend radii, such as table 4-2, in the metals handbooks. These values, however, are for static applications; for dynamic loadings, bend radii should be made at least twice the stated values if failure is to be comfortably avoided.

4-25. Enlargement of the ends of slots (figure 4-4) prior to bending the corners into the chassis reduces stress concentrations at the bent corners. This reduces the chances of cracking at the corners, especially where two bends are to be made close to each other.

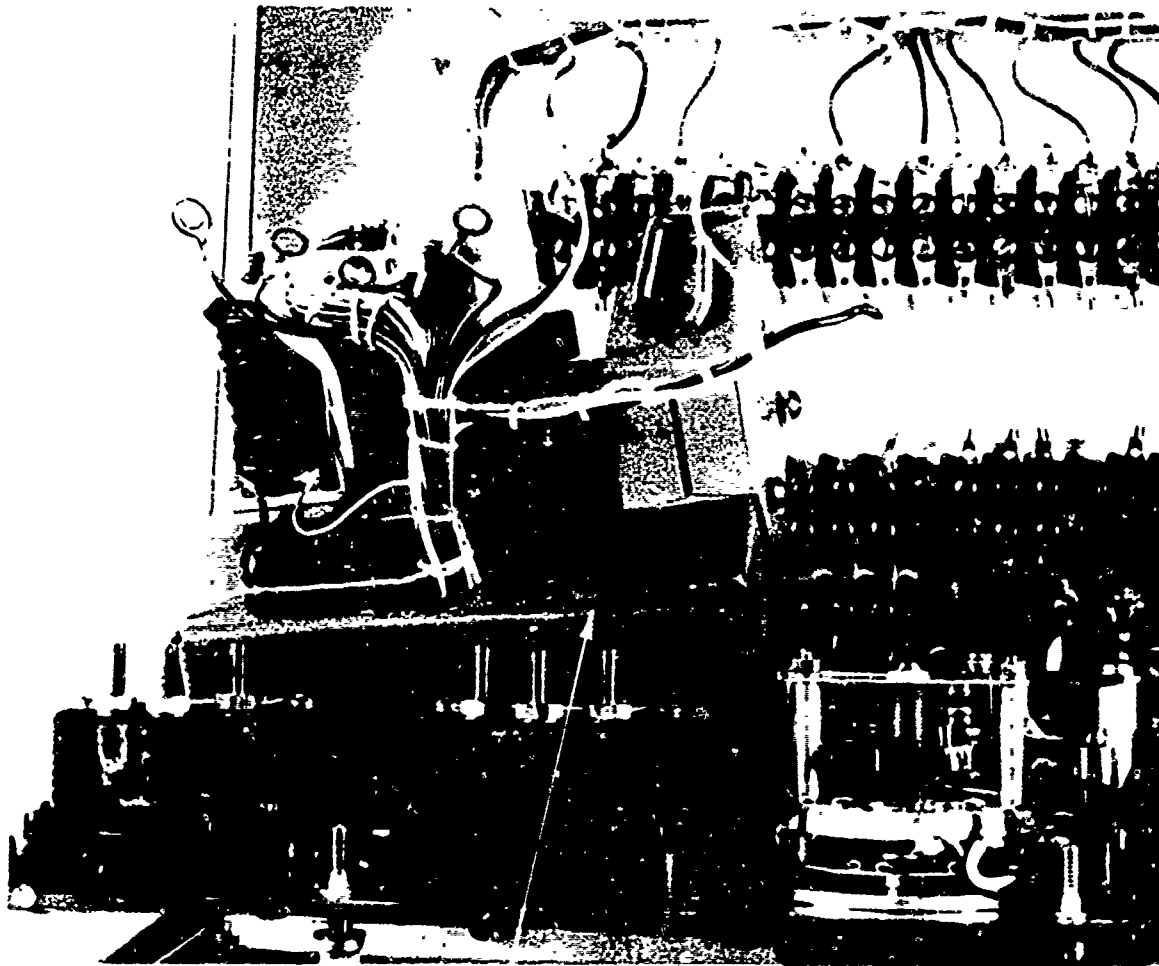


Figure 4-3. Fractured Cantilever Bracket


4-26. **FORMING TECHNIQUES.** If a chassis is formed on a brake, the method of gripping the metal will determine the unit strain produced by bending. The vises for a typical brake are shown in figure 4-5a. When the sheet is bent, stretching occurs principally along the surface indicated. An alternate vise configuration, shown in figure 4-5b, uses inserts to restrain but not to grip the sheet. The same radius is obtained on bending; stretching, however, has been distributed over a greater length, thus lessening the chance of cracking at the outside surface of the bend.

4-27. Another method of shaping chassis is hydroforming. Pressure behind a diaphragm forces sheet stock into a die. The hydraulic method ensures equal pressure at all points, thus distributing stresses uniformly over the sheet.

4-28. **CUTOUTS.** Weakening of the chassis by cutouts for mounting tubes or other small components will be negligible if only a small part of the chassis metal is removed. However, the holes should not be aligned in such a way as to "hinge" the chassis, thus making it more flexible, nor to perforate it for easy tearing.

TABLE 4-2.

SHEET METAL BEND RADII FOR EXCLUDED ANGLES FROM 45° TO 120°


 Excluded angle

Material	Nominal Thickness	Inside Bend Radius	Radius Tolerance
5052-H32 5052-H34	.012 - .064 .072 - .091 .102 - .125 .156 - .188	1/16 1/8 3/16 1/4	±1/64 +1/32 -1/64 +1/32 -1/64 +1/16 -1/32
Aluminum  8061-T4 6061-T6	.012 - .032 .040 - .064 .072 - .091 .102 - .125 .156 - .188 .250	1/16 1/8 1/4 3/8 3/4 1	+1/32 -1/64 +1/32 -1/64 +1/16 -1/32 +1/16 -1/32 +3/32 -1/16 +1/8 -1/16
Brass Yellow brass (85-35), 1/2 hard	.010 - .032 .040 - .091 .102 - .125 .156 - .250	1/16 1/8 1/8 1/4	±1/64 +1/32 -1/64 +1/32 -1/64 +1/16 -1/32
Bronze* Phosphor bronze, Grade A, spring temper	.005 - .010 .013 - .022 .025 - .032	1/32 1/16 1/8	±1/64 +1/32 -1/64 +1/32 -1/64
Copper Electrolytic copper, soft (.030 mm)  Beryllium copper 1/4 hard	.011 - .108 .129 - .259  .005 - .032	1/32 1/16  1/32	±1/64 +1/32 -1/64  ±1/64
Steel Cold-rolled, carbon steel  AISI 302, annealed stainless steel	.012 - .120 .134 - .188  .010 - .062 .072 - .125 .134 - .250	1/16 1/8  1/32 1/8 1/4	±1/64 +1/32 -1/64  ±1/64 +1/32 -1/64 +1/16 -1/32
*Bend-line perpendicular or diagonal to grain direction.			

#### 4-29. JOINING RACK AND CHASSIS MEMBERS.

4-30. The structural members of racks or chassis may be joined by a variety of methods. If the members are to be joined permanently, welding or riveting may be best. If the members are to be separable, they can be held together with screws or bolts combined with appropriate locking devices.

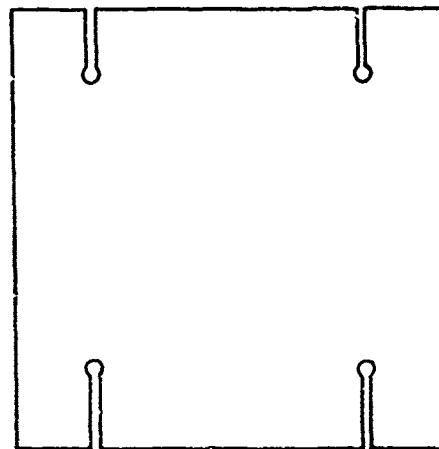
4-31. **FUSION WELDS.** Welding is the method of joining rack and chassis members which produces the fewest shock and vibration problems. A welded joint is permanently tight. However, welding is limited to certain materials and can be used only where permanent joints are desired. Stress concentrations may be produced in welding. These are avoided by making full-depth welds and by ensuring good fusion at the bottom of the weld. Short, intermittent welds are undesirable since each "end" is a stress concentration. After welding, joint strengths can be increased by heat treatment to relieve residual stresses; this extra step, however, may make this a costly joining process.

#### 4-32. RIVETS AND SPOT WELDS.

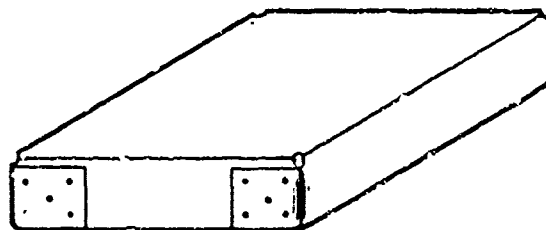
Riveting is a satisfactory method of joining rack and chassis members but must be of the highest quality. Rivets will not hold up well under repeated shocks if they have not provided a tight joint. Loose rivets cause pounding of the members during excitation, which imposes severe stresses on these members. If the rivets themselves are improperly fabricated, severe residual stresses may be left in the rivet heads, and they may snap off under tension.

4-33. If a lap joint is to be loaded in shear, a combination of riveting and spot welding may be used. The joint is spotwelded; the ends of the joints are fastened by rivets to protect against any possible tension forces. If the joint is to be loaded primarily in tension, spotwelding should be avoided and only rivets used. If a great deal of tension is expected, the members should be fastened with bolts and locknuts.

4-34. **SCREWS AND BOLTS.** The most common detachable fasteners are screws and bolts. When carefully selected and used with appropriate locking devices, they are of value for joining rack or chassis members. (Their use in rack construction is limited; welding or riveting is usually used.) Sheet metal screws (those that are not set into a threaded fitting) are not recommended. Vibration excitation can cause them to spin or tear out of their holes.



BLANK



FORMED CHASSIS

Figure 4-4. Enlargement of Slot Ends to Reduce Stress Concentrations at Corners

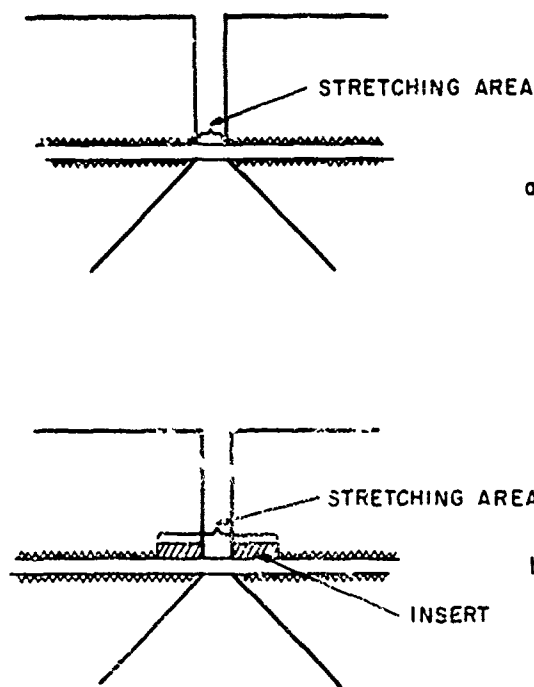


Figure 4-5. Insert Distributes Bending Stresses Over a Greater Surface

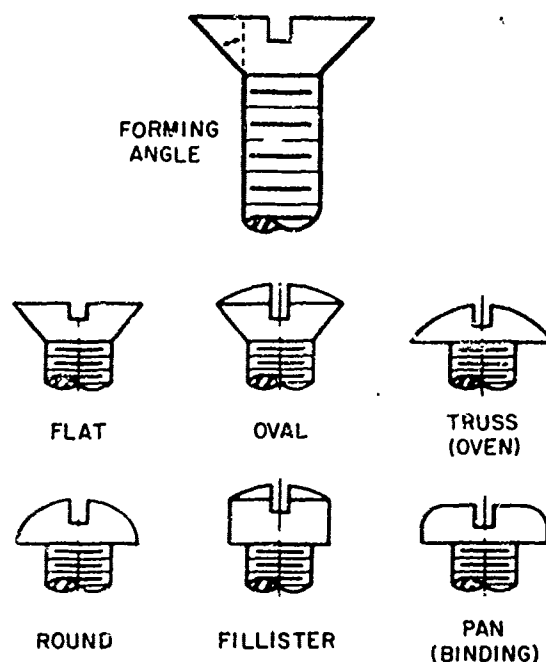


Figure 4-6. Screw and Bolt Heads

4-35. The geometrical configuration of bolts and screws should be considered when selecting these fasteners. For cold-headed screws and bolts, the greater the angle between the lower surface of the head and the screw or bolt body (figure 4-6) the greater was the distortion of the metal in forming, and hence the greater is the residual stress. If the head is thin, the bolt or screw will be structurally weak. With these considerations, of the various types shown in figure 4-6 the best would be the flat or oval head, the worst the binding or oven head. No matter which fasteners are selected, the designer should make sure that they have had suitable heat treatment for reduction of residual stresses.

4-36. Where screws must be locked into tapped holes, those screws containing nonmetallic inserts may provide better and more convenient locking than could be obtained with lockwashers. The inserts, however, should not be used more than once; and if the screw is removed, the insert should be replaced.

4-37. Any stress concentrations are likely to be proportionately greater in small bolts and screws. For this reason, smaller fasteners have relatively less strength than larger fasteners. For the best use of these fasteners, the designer will recognize that a screw or bolt has greater strength when loaded in tension than when loaded in shear. By orienting bolts and screws so that their axes are parallel to the direction of vibration or shock, the danger of fracture of these fasteners is minimized.

4-38. **LOCKWASHERS.** Of the common type of lockwashers shown in figure 4-7, the recommended lockwasher for use in a moderate or severe mechanical environment is the split-ring type. The split-ring washer is reasonably effective in preventing the loosening of bolts due to vibration and shock and is not vulnerable to shock loads. This washer retains at least partial effectiveness when the bolt is

elongated by shock. The principal disadvantage of this, or any, lockwasher is that, should the washer fracture, the bolted members become excessively loose.

4-39. Tooth-type lockwashers have either external or internal teeth which engage the nut or bolt head and the structural member. Tooth-type lockwashers with external teeth are preferred over the type with internal teeth since the external teeth act on a larger radius. These lockwashers are adequate for vibration protection, but somewhat unsatisfactory for shock. They become ineffective with slight bolt elongation. They have been known to break under repeated shock forces.

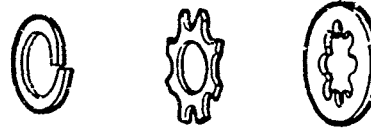


Figure 4-7. Common Lockwashers

4-40. Tooth-type lockwashers are used for making electrical contact. (The teeth cut through anodic and other insulating coatings.) Repeated use of these washers, however, will result in excessive abrasion of soft metals. If both electrical contact and locking are desired, a combination of a tooth-type washer and a locknut is preferred.

4-41. Washers and screw heads are sometimes covered with "Glyptal" or a thick paint. The paint serves as an inspection mark to indicate a secure fastener, but contributes only slightly to the locking action.

4-42. **LOCKNUTS.** The preferred way for holding a screw or bolt and nut secure is by means of a locknut. The locknut develops friction between the bolt threads and the nut. In one type, the friction is obtained by forcing an unthreaded nonmetallic insert in the nut onto the bolt, figure 4-8. In another, the tapped hole (of the nut) is distorted, and considerable torque is necessary to turn the nut on the bolt. In still another type, the face of the nut is irregular; when the nut is tightened against a shoulder, the nut is distorted with consequent binding of the threads. Of the various types of locknut, the elastic-insert locknut, shown in figure 4-8, is preferred (if the temperatures of the environment permit). All-metal locknuts tend to remove the plating from the bolts, which may result in subsequent corrosion problems.

4-43. Locknuts are quite effective in preventing loosening of bolts under vibration. They will also hold joint tightness under shock forces unless the bolt is permanently stretched, and they are not easily damaged. Locknuts cannot be used repeatedly, since the friction members wear with use.

4-44. Where various types of screw and lockwasher (or locknut) assemblies fail to provide the necessary binding, the failure may be due to insufficient tightening. To function properly, the fastener must undergo a certain amount of torque. The use of a torque wrench or a preload-indicating washer is recommended, since a screw-nut assembly not sufficiently tightened may vibrate loose and result in damage to equipment.

#### 4-45. RACK DESIGN.

4-46. Racks are divided into two general classes depending upon whether they provide individual or integrated mounting of equipments. Individual mounting provides for a rack for each black box; integrated mounting is the grouping of several black boxes on or within a single rack. Figure 4-9 illustrates a rack that will accommodate several chassis. Individual mountings are likely to be used where an equipment is to be used in several different planes with different space allocations, etc; integrated mountings are more apt to be used where an equipment is tailored for a particular plane or missile.

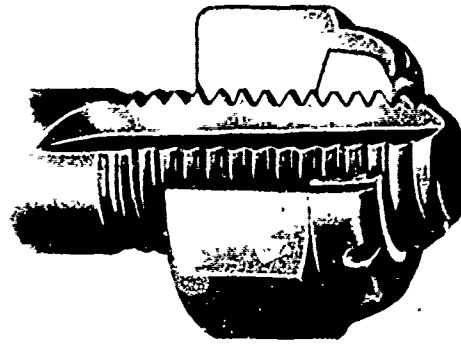


Figure 4-8. Locknut

4-47. Racks are mounted to the airframe either directly or through isolation mounts. At the mounting points, the rack should be sufficiently rigid so that no motions are amplified by flexing.

4-48. The location of the mounting bolts, for both rack to chassis and rack to airframe or shock mounts, requires careful consideration. The farther apart they are placed, the longer will be the moment arm around the center of gravity of the equipment. Thus the restraining action of the mounting bolts or guide pins will have greater leverage in restraining the equipment under excitation.

4-49. A soft metal rack should have steel inserts to receive the guide pins of the chassis, so that the holes are not enlarged by shock and vibration motions. Such devices as roller tracks should not be relied upon to constrain chassis movement; the clearance required for smooth working of these devices may allow chatter which would cause the enlargement of clearances and eventual failure.

4-50. The designer should make full use of the structural nature of a rack. If it is formed of tubing, for example, this tubing can be used as a passage for cooling air, pressurization, or for cabling.

#### 4-51. INDIVIDUAL MOUNTING.

4-52. Equipment designed for use in several different aircraft or missile types is usually provided with separate mountings for the various components. These individual mountings can vary from the relatively simple underneath mounting base shown in figure 4-10 to the more complex center-of-gravity mounting base pictured in figure 4-11. This latter mounting is used in jet fighter aircraft and incorporates electrical terminals and receptacles.

#### 4-53. INTEGRATED SYSTEMS.

4-54. Integration groups several black boxes in a single rack. These black boxes may be components of an equipment, or they may be equipments comprising a com-

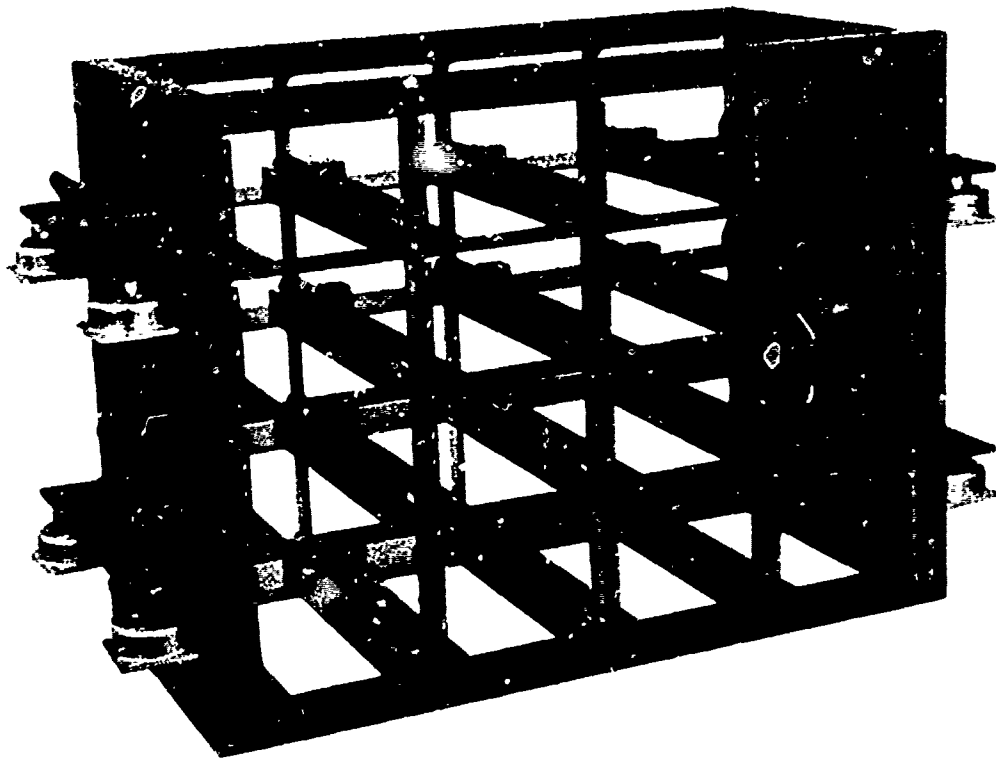


Figure 4-9. Rack Designed to Accommodate Several Chassis

plete electronic system. The individual black boxes are designed to blend dimensionally with each other and with the airframe. Engineered component arrangement and elimination of sway space through rigid mounting to the rack add to compactness. For effective engineering of an integrated system, one organization should be responsible for combining the units and installing them in the airframe. This control may be direct or may take the form of specified weights and dimensions for the modules. If the system is isolator mounted, isolator excursion is lumped into a single system (figure 4-12). Having the equipments in a group also simplifies the problems of cooling, cabling, and pressurization. Servicing is facilitated with modular construction, a feature of most integrated systems. Any module in the system generally can be removed without disturbing the others.

4-55. A disadvantage of the integrated system is that it can be modified only with great difficulty. New parts may not fit, they may not receive adequate support, or they may throw the system out of balance. This latter point, the unbalancing of the system, is extremely important when the system is isolator mounted. The isolators are engineered to hold a certain part of the weight of the system and to have certain natural frequencies when loaded. Since the removal of an equipment, or even a change within a component, may significantly alter the total weight and balance of the integrated package, the isolator mounts might have to be modified to handle

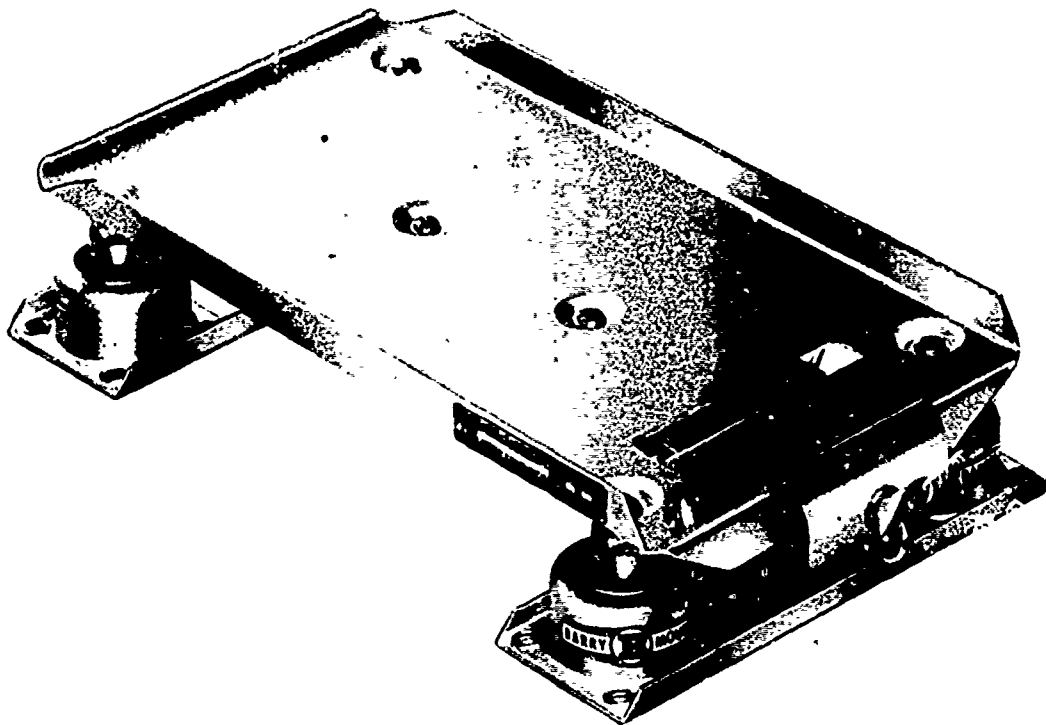


Figure 4-10. Underneath Mounting Base

the "new" system. If the mounts are not altered, the system may develop undesirable coupled degrees-of-freedom, resulting in a relocation of natural frequencies and possibly causing the equipment to resonate in its environment.

4-56. "ELECTRONIC EQUIPMENT RACK SYSTEM." The "Electronic Equipment Rack System" is a method of integrating equipment proposed by the Electronic Components Laboratory of the Wright Air Development Center, and described in WADC Technical Note WCLC 53-8. The system specifies case dimensions and methods of mounting the cases in a console rack. Rectangular slots are provided in the rack. Pressurized cylindrical cases, if any, are fitted into the rectangular system by a rectangular panel attached to the front face of the cylinder, and are supported by a cradle base. In this rack system, equipments of different sizes are grouped so that the total racked equipment conforms to the contour of the airframe. Individual equipments are held in the rack by guide pins in the rear and quick-release fasteners on the case fronts; this arrangement provides optimum rigidity and simplifies removal for servicing or replacement.

4-57. THE FILE-DRAWER CONFIGURATION. As presently developed, the file-drawer configuration defines the weight and dimensions of each drawer. The drawers, weighing about 150 pounds when loaded and measuring approximately 6 by 12 by 24 inches, slide into (and are tightly fastened to) slots in the airframe. Each drawer contains several subassemblies. A drawer may contain a complete equip-

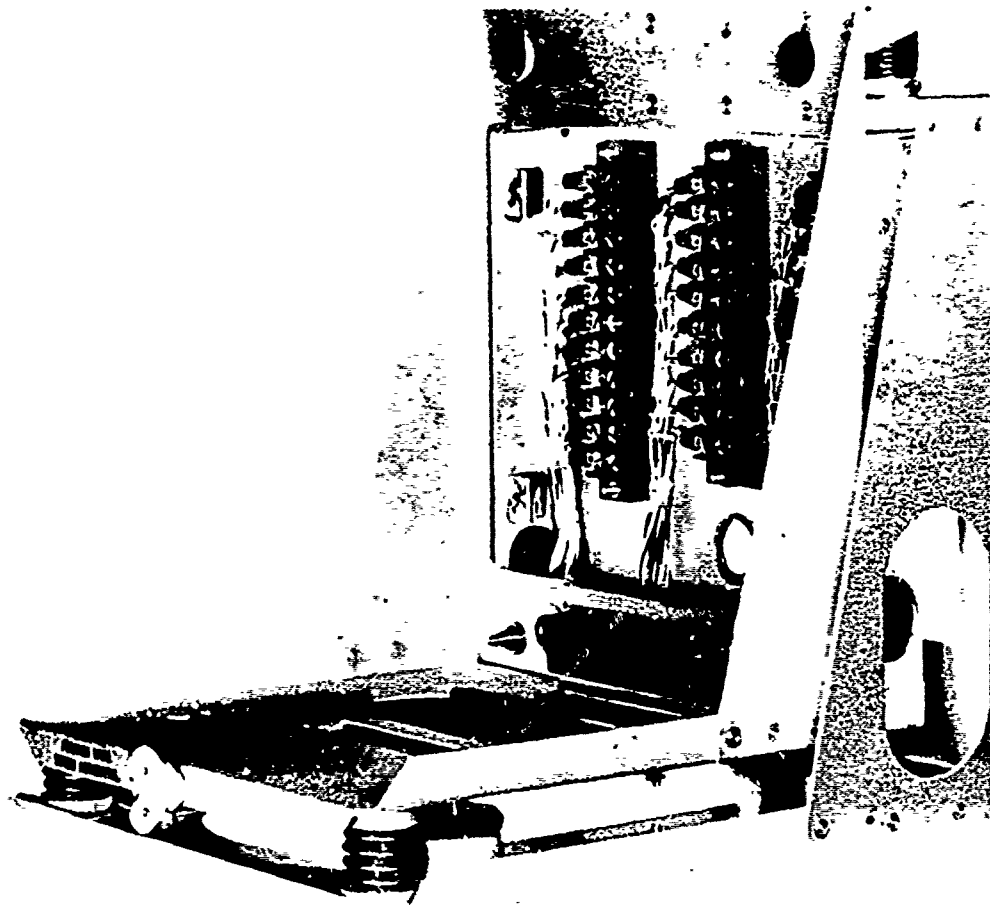


Figure 4-11. Center-of-Gravity Mounting Incorporating Electrical Terminals and Receptacles

ment or a part of an equipment.

4-58. The rigid mounting of the equipments eliminates sway space, allowing more equipments per unit volume. The assembly of several functional units in a drawer permits easy removal for servicing and allows shuffling of equipments for the electronics arrangements of different aircraft.

4-59. "ACCESSIBLE MODULAR CONSTRUCTION." Accessible modular construction, developed by Navy Electronics Laboratories, is a method of integrating equipments which provides ready access for maintenance. The components are mounted on the inside of a cabinet door. When the door is closed, the components are held fast within the cabinet by guide pins. Figure 4-13 illustrates the accessibility of modular-type construction; figure 4-14 shows the construction of modules used in the system.

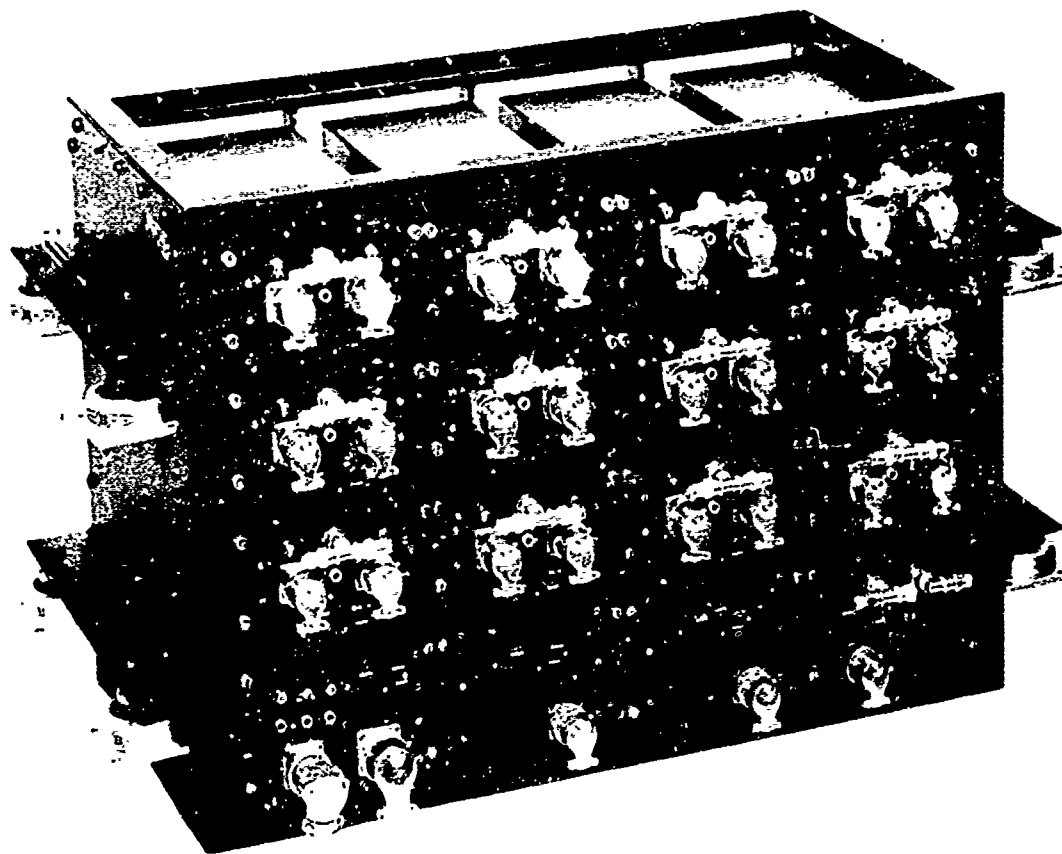


Figure 4-12. Integrated Mounting System

4-60. The design principles for this system are applicable to any integrated system. Subassemblies are used, making the location and replacement of faulty circuits less difficult. The individual subassemblies can be removed for repair without disturbing the others. Each subassembly includes circuits which work together to perform a complete function; and, if any subassembly fails, the others are still operable. Active heat-generating elements (such as tubes) are limited to about three per subassembly. Subassemblies are standardized when possible so that they may be interchangeable in other equipments, thereby reducing the number of spares required. Accessible test points are included in each equipment. Originally developed for shipboard use, this system may be applicable to certain aircraft electronic systems.

#### 4-61. CHASSIS DESIGN.

4-62. The mechanical design of a chassis should provide the optimum degree of flexibility (or rigidity) plus adequate strength to prevent damaging electronic components by amplifying excitations or by shearing of chassis members. The designer must (1) choose the best chassis material, (2) properly arrange and attach

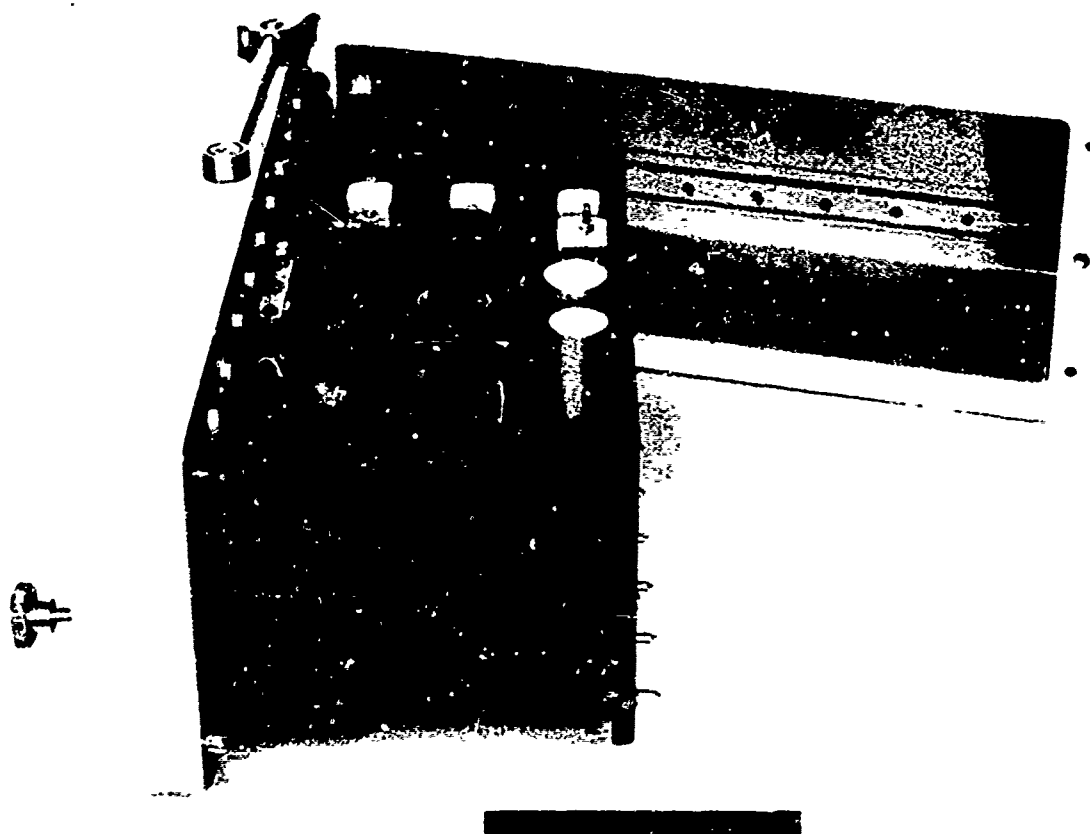


Figure 4-13. Accessible Modular Construction

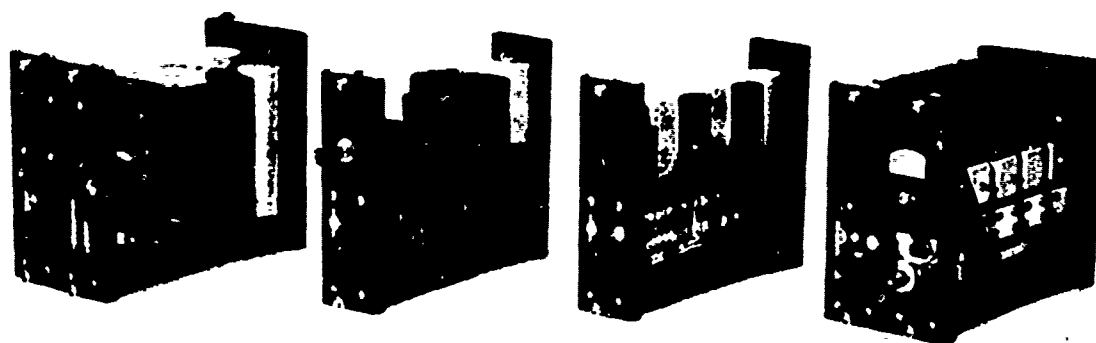


Figure 4-14. Modules Used in Accessible Modular Construction System

the components to the chassis, (3) make suitable provisions for attaching the chassis to the rack, and (4) consider the possibility of introducing chassis damping.

**4-63. CHASSIS MATERIALS.** The cost of materials used in aircraft equipments should be of only minor concern to the designer. The greatest expense factor is labor -- both for construction and for maintenance. If the equipment is easy to build and has such reliability that maintenance time is minimal, it will be the most economical.

**4-64.** The materials used in chassis can be divided into the categories of sheet, plate, castings, honeycomb, and plastics. Sheet metal is the traditional chassis material; although it is suitable for many electronic equipments, its use in some instances may be dictated by habit and not by sound engineering practice. Plate material is, in effect, thicker sheet; it is fabricated according to standard methods and provides greater rigidity. Castings may be preferred over sheet or plate for equipments that are to be produced in large quantity. Some chassis have been cast of magnesium, which, when compared with conventional formed aluminum chassis, provided greater rigidity, more damping, and a larger heat sink. Figure 4-15 is an example of a cast chassis showing both an interior and an exterior view. Although not evident, the chassis has ribs to which the chassis subassemblies are fastened, providing for overall rigidity. Honeycomb materials have often been considered for chassis constructions but are rarely used. The prefabricated honeycomb materials are much lighter than solid stock of equal stiffness and also provide some internal damping (to reduce the amplitudes at resonance). If they are constructed to resemble corrugated paper, the internal spacing can be used to carry cooling-air. Plastics, especially those used in printed-circuit and terminal boards, are coming into wider use as chassis sections. These materials are inexpensive, but often require special bracing. The excitation resistance of boards will depend on how they are mounted. A 1/8-inch terminal board can take up to 500 cps without failure if it is held to the chassis by fasteners which are no more than 3 inches apart. For thinner boards, such as 1/16-inch printed circuits, the fastener spacing should be proportionately smaller. Damping and rigidization can be provided for these boards by a sponge rubber pad backed by a rigid plate.

**4-65. CHASSIS FLEXIBILITY.** Determining the chassis flexibility in advance of construction is difficult but often necessary. The natural frequency of the chassis should not be higher than that which is necessary to protect it against resonance effects, since, in most cases, any flexibility will reduce the shock forces on component parts. It may be possible to make tentative strength and flexibility calculations of the principal structural members and components while the equipment is in the design stage. For practical purposes, it is usually necessary to consider in the calculations only the heavier components such as transformers and chokes; such things as vacuum tubes, small condensers, etc, will have little effect on the natural frequency of the system. The strength and flexibility values determined, however, are not conclusive; it will still be necessary to conduct shock and vibration tests to confirm the calculated values. It is unnecessary to finish and completely wire the equipment to test for strength or for chassis natural frequency (flexibility). (The designer should start testing his chassis as soon as he has something to test.) If the heavy components are mounted on the chassis, the strength and natural frequency of the resulting assembly will be very near that of the finished assembly, assuming that the addition of the lighter parts and the wiring does not affect the vibration in some unusual manner.

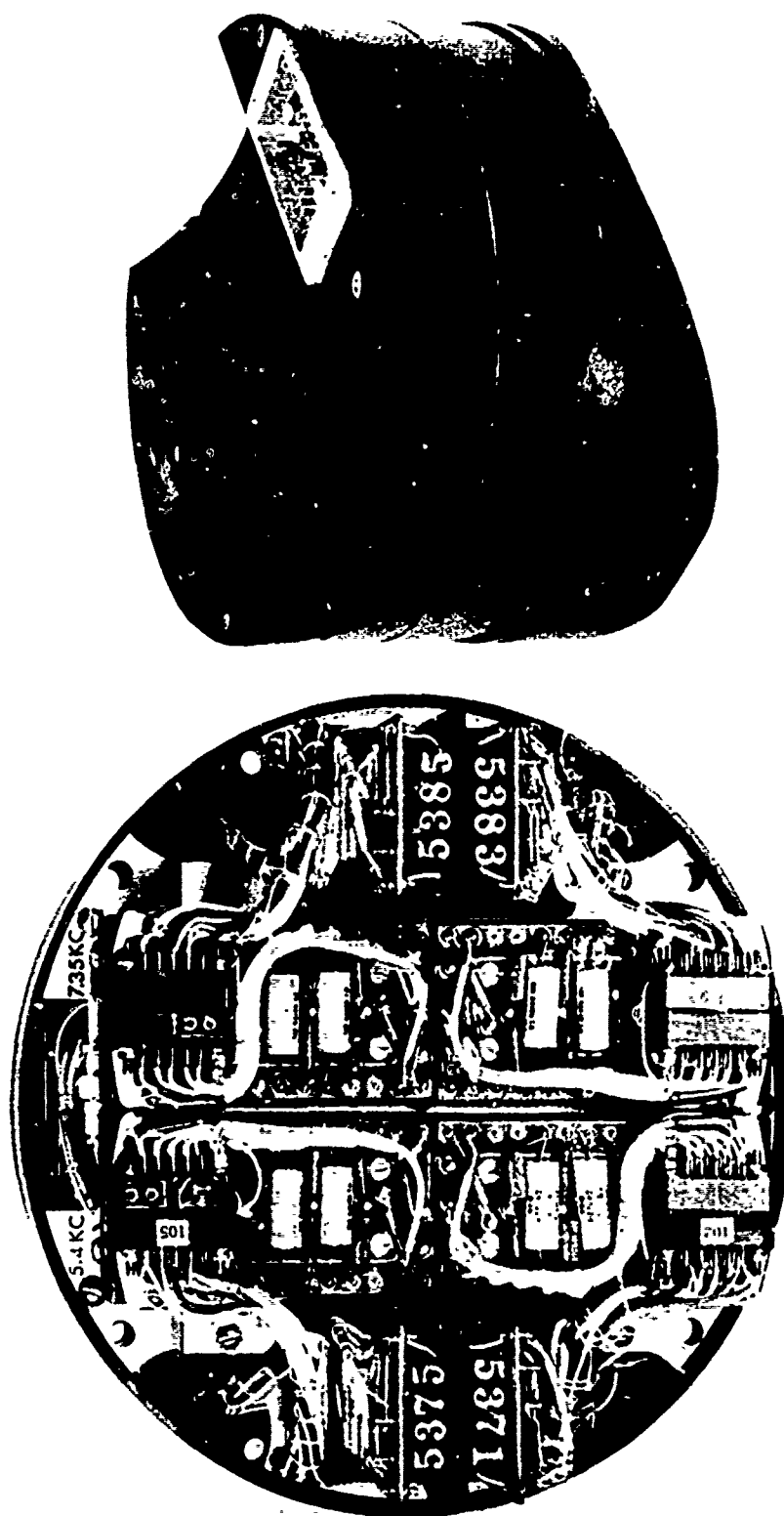


Figure 4-15. A Cast Chassis Showing Both an Interior and Exterior View

4-66. **ARRANGEMENT OF COMPONENT PARTS.** A degree of shock and vibration resistance is obtained through suitable location of components on the chassis, since the location may affect the natural frequency of the chassis and the degree of shock protection provided. Such placement takes into account both the weights of the components and their motions in response to excitation. In most cases, it will not be practical to calculate the best component arrangement, since the time spent in making such calculations would be greater than the time necessary to test and modify a chassis built by trial-and-error methods.

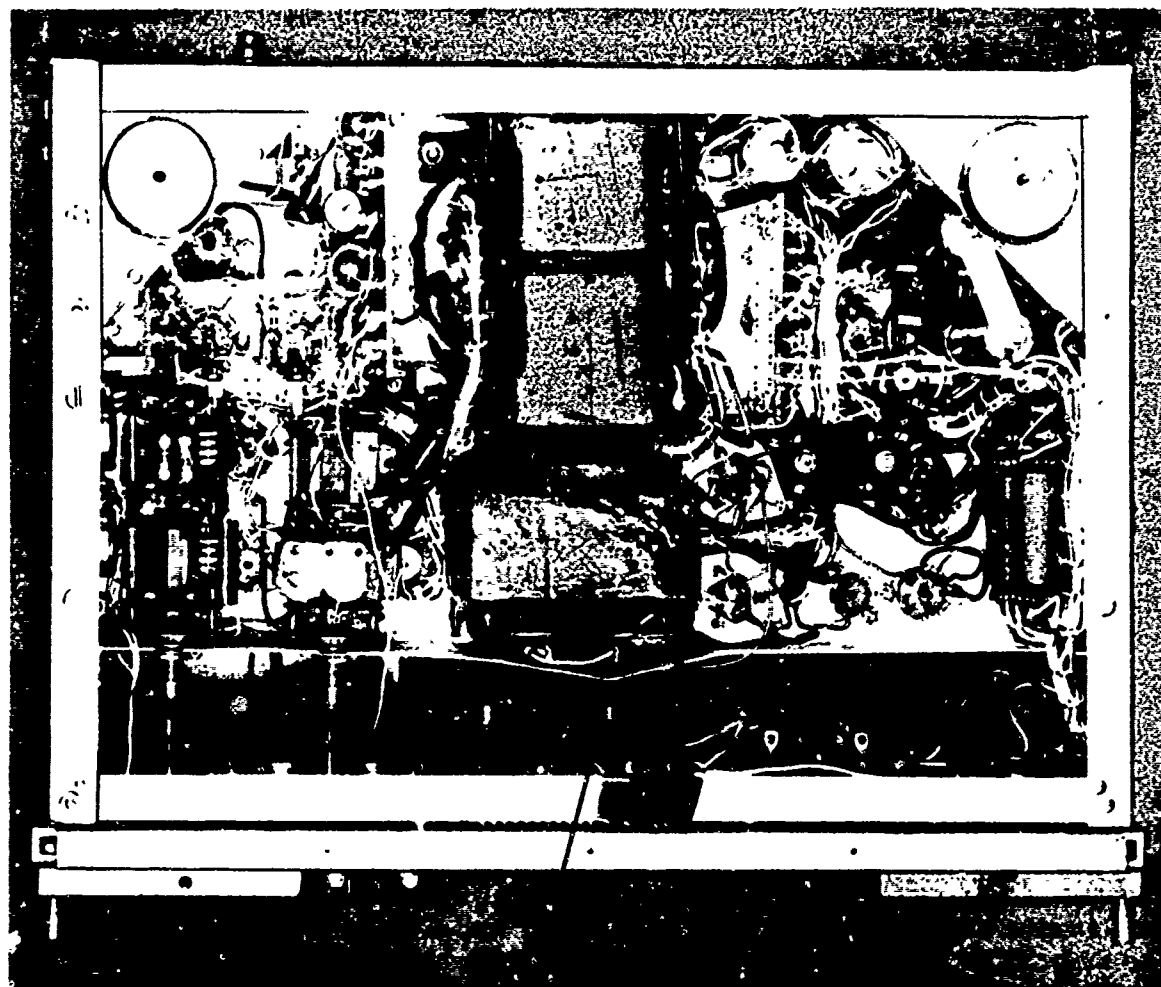
4-67. Common-sense placement of components to satisfy simple design principles can go a long way toward producing a satisfactory end product. Maximum support should be given to components of maximum weight; space should be allowed for the excursions of the components under excitation; and the centers of gravity of the components and of the equipment should be kept close to the plane of the mounts. In conventional pan type chassis, heavy components, such as transformers, should be located at the corners or at the sides of the chassis, where greater support is provided; lighter components, such as tubes, should be centrally located. The closer a heavy weight is located to the center of an unbraced chassis, the greater is the chassis deflection. The chassis shown in figure 4-16 was deformed under vibration because of the low resonant frequency produced by the placement of a heavy transformer and choke at the center of an inadequately supported chassis. Perhaps the best example of bad shock and vibration design is the placement of a heavy transformer in the center of a deck, with vacuum tubes grouped around the transformer.

4-68. If a heavy component has a rigid base, mounting it at a corner of the chassis will not only provide it with support but may raise the natural frequency of the assembly, since the base can serve as a cross-member for the chassis. As the component is moved away from the corner of the chassis, the chassis-component combination becomes more flexible. Lighter component parts such as small tubes and capacitors contribute little toward lowering the chassis natural frequency when mounted in the central chassis area. In most cases, these lighter parts benefit from the central chassis location since the chassis flexibility affords them some measure of shock protection.

4-69. More sway space is required between components on the more flexible parts of the chassis. If possible, the components should be distributed about the chassis so that the chances of their striking one another are minimized. Considerations other than mechanical ones may prevent these requirements from being fully met, but they should be carried out as nearly as possible without sacrificing electrical efficiency.

4-70. Another general rule for the arrangement of component parts is that mass remote from the plane of the mounts should be minimal. Heavy components should be kept low in a bottom-mounted piece of equipment to keep the center of gravity low. In like manner, the arrangement of any group of components should be such as to place the heaviest components as close to the mounts as possible. The closer to the mounts heavy components are, the less the risk of large excursions under excitation.

4-71. Although neatness and tradition dictate that the components should be arranged in an orderly fashion, there is reason for believing that an asymmetric



**Figure 4-16. Chassis Deformation Resulting from Inadequately Supported Components**

(irregular) arrangement may be of value. Such arrangement should help keep everything from resonating at the same frequency and direction of vibration input.

#### **4-72. USE OF STIFFENING MEMBERS.**

**4-73.** Stiffening members in a chassis will increase the chassis natural frequency and increase its structural strength. Stiffening members can be both integral and external. Integral stiffeners are formed into the chassis when it is made. The stepped chassis shown in figure 4-17 is an example of integral stiffening; the riser between the two steps is bent from the top plate of the chassis and acts as a stiffening member across the width of the chassis.

**4-74.** External stiffeners are beams fastened to a standard chassis, as shown in figure 4-17. They may be located as needed and may be added during later design and testing stages. Figure 4-18 is a photograph showing the use of stiffening members for both a chassis and subchassis. The holes in the stiffening beams reduce weight without appreciably affecting rigidity.

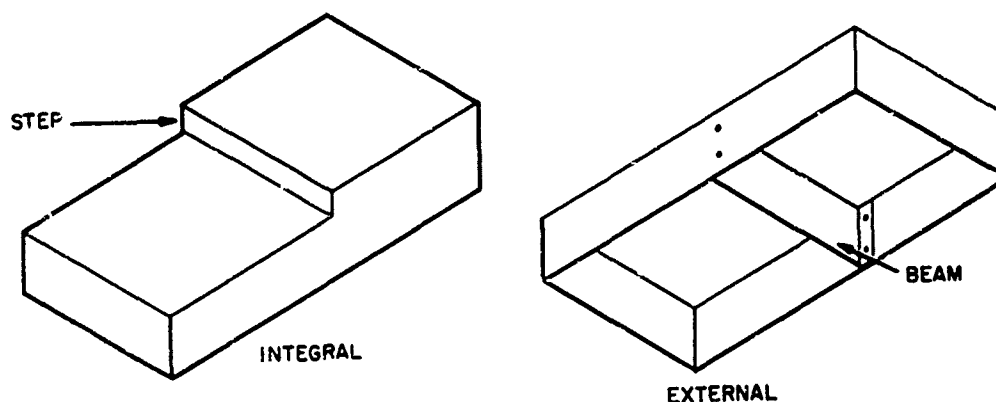


Figure 4-17. Chassis Stiffeners

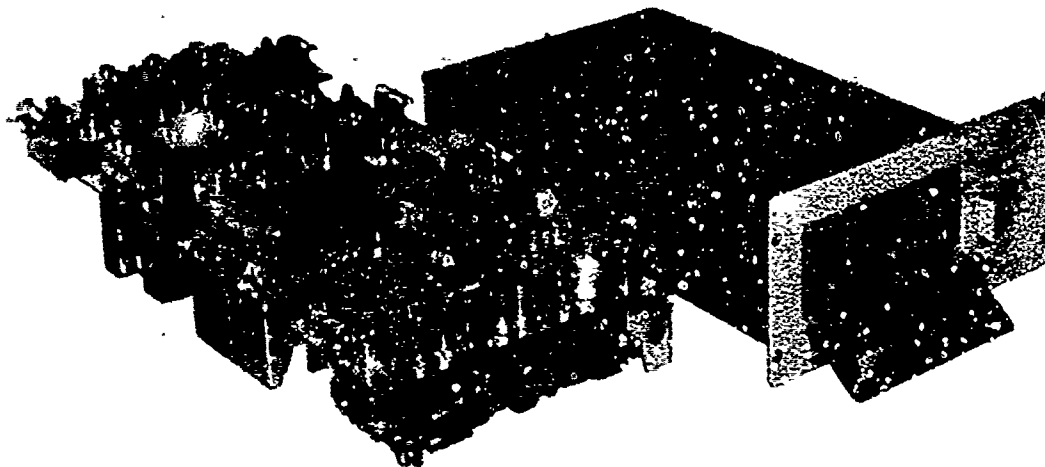


Figure 4-18. Stiffening Members in a Chassis and Subchassis

4-75. The simplest stiffener is a beam placed across the width of the chassis on either the underside or the topside, and fastened to the chassis along the beam's length. The point of maximum static deflection for a uniformly loaded or center-loaded chassis is its center. A beam across the center of the chassis would provide additional stiffness and reduce the static deflection.

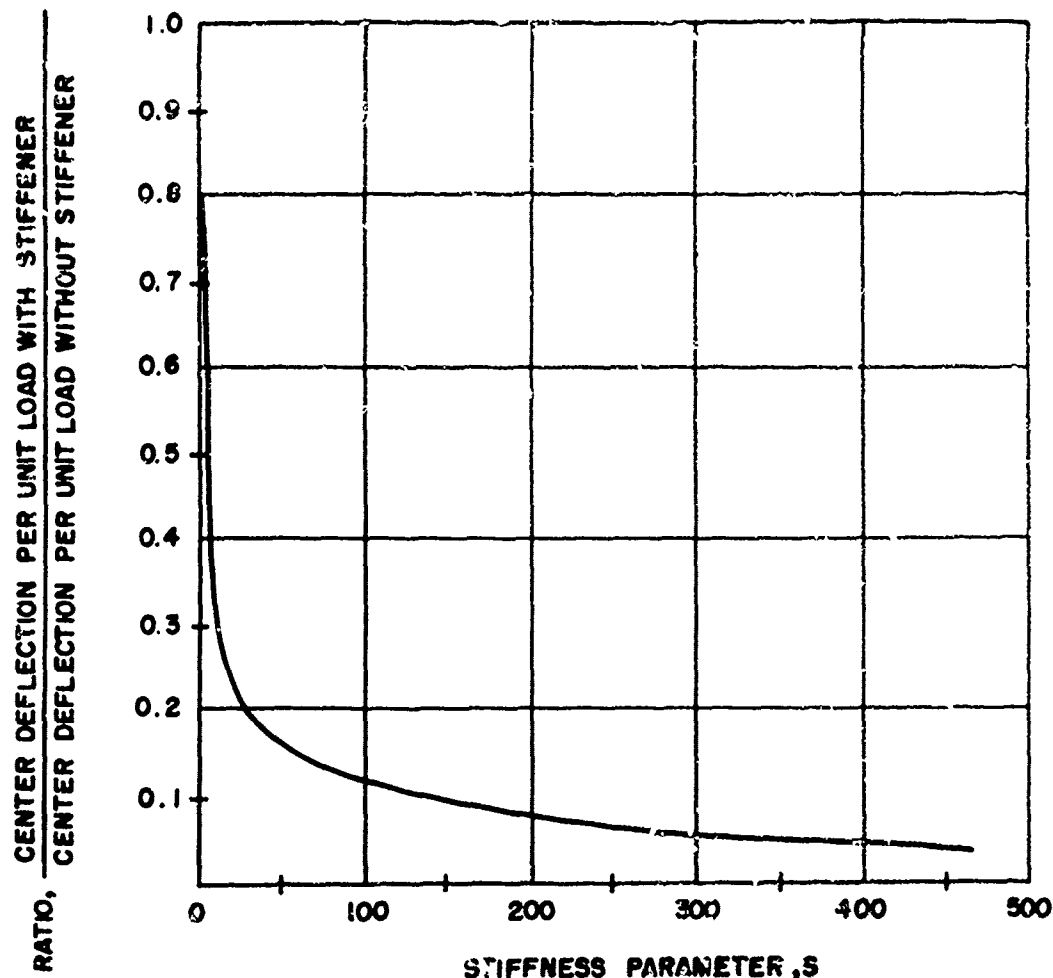


Figure 4-19. Effect of Transverse Stiffener on Box-Type Chassis

4-78. The amount of reduction in static deflection with the addition of a transverse rod to a chassis can be estimated by determining the stiffness parameter (see below) of the chassis and beam, and referring this stiffness parameter to a chart (figure 4-19). This chart is a plot of the stiffness parameter versus the ratio of deflections for the stiffened and unstiffened chassis. The curve in figure 4-19 was obtained experimentally by measuring the static deflection of chassis with and without a stiffener under a given load.

4-77. The stiffness parameter (S) is a dimensionless quantity and is determined by a formula which was derived empirically, as was the curve of figure 4-19. The formula is

$$S = \frac{E_s (I + Ac^2)}{Db}$$

where S = stiffness parameter (dimensionless)

$E_s$  = modulus of elasticity for the stiffener (lb/in.<sup>2</sup>)

$I$  = moment of inertia of the cross section of the stiffener about an axis through its centroid ( $\text{in}^4$ ) and parallel to the top of the chassis

$A$  = cross-sectional area of the stiffener ( $\text{in}^2$ )

$e$  = the distance from the centroid of the cross section of the stiffener to the center of the thickness of the top of the chassis (in.)

$D$  = rigidity of the top of the chassis ( $\text{lb in.}$ );  $= \frac{E_c h^3}{12 (1 - \sigma^2)}$

$b$  = width of the chassis (in.)

$E_c$  = modulus of elasticity for the chassis ( $\text{lb/in}^2$ )

$h$  = thickness of the top of the chassis (in.)

$\sigma$  = Poisson's ratio for the chassis material (dimensionless).

4-78. Stiffening a structure to raise its natural frequency is not a cure-all for unwanted resonances. This measure may lower the equipment's resistance to shock excitation, so that a compromise may be necessary between stiffening and, perhaps, damping.

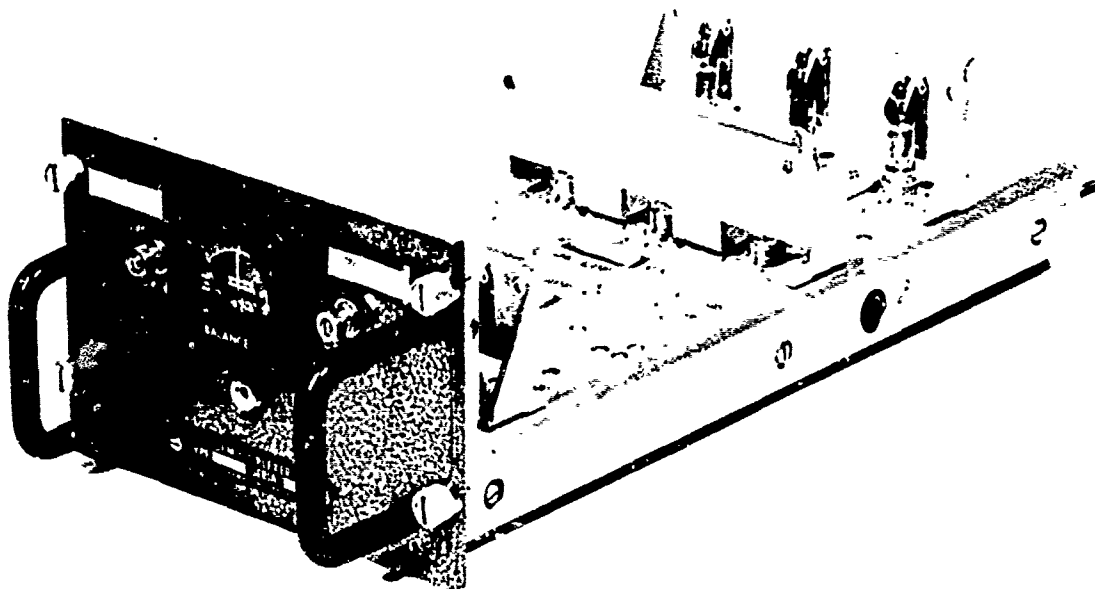


Figure 4-20. Brackets Resistant to Mechanical Excitation

**4-79. BRACKETS AND HARDWARE.** Brackets securing component parts to the chassis must be rigid with high natural frequencies to prevent excessive displacements of the mounted parts during excitation and must possess sufficient strength so as not to break or deform permanently under environmental loadings. The rigidity of a bracket depends on its geometry, modulus of elasticity, and loading. The natural frequencies for brackets fabricated from sheet stock tend to be low while those for cast and forged brackets are relatively high. The configuration of the bracket will have a great deal to do with its stiffness. Cantilever brackets, in general, should be used only for holding very light parts. The brackets shown as tube supports and heat sinks in figure 4-20 would not be expected to produce destructive resonances in most environments.

**4-80** Chassis hardware should be rugged, and the chassis itself must be particularly sturdy where this hardware is attached. Guide pins should be located out near the corners of the chassis where greater rigidity is provided. Latches of the turn-to-release type should be provided with detents so that they will not vibrate loose or jounce open under shock. Spring-loaded, quick-fastening devices, such as Dzus fasteners, have proved satisfactory for limited use in airborne electronic equipments. It is important that the size of the Dzus be properly related to the amount of load it will be expected to carry. Fasteners of this type may be affected by shock and vibration if the acceleration forces are sufficient to overcome the spring loading.

## SECTION V

### DAMPING

#### 5-1. GENERAL.

5-2. Damping, in a vibrating mechanical system, reduces displacement amplitudes by dissipating some of the energy by converting it to heat. When the ratio of forcing frequency to natural frequency is below 1.414, damping is almost always desirable because it reduces the magnification of impressed excitation. However, above the ratio 1.414, the displacement amplitude of the equipment mounted on damped isolation mountings is higher than it would be on undamped mountings and presents a greater threat of damage to the equipment.

5-3. Chassis damping is desirable at all frequencies, although the transmissibility curves for with, and without, damping are the same as those for isolation mounted equipment. While the transmissibility will be higher without damping at ratios above 1.414, it is a desirable feature in this situation. By reducing the relative motions of the different parts of a structure, flexing is reduced and the likelihood of fracture is lessened.

5-4. There are four types of damping used to control resonant responses in electronic equipment: (1) viscous, (2) hysteresis, (3) friction, and (4) air damping. Viscous damping results from the displacement of fluids. Hysteresis damping is caused by energy losses in a cyclically stressed material. Friction damping is a result of the resistance to sliding of two contacting surfaces. Air damping is caused by the displacement of air.

#### 5-5. VISCOUS DAMPING.

5-6. Viscous damping is defined as that damping which exists when the opposing force is proportional to the velocity of the body. The term "viscous" is used because the above situation is most frequently and closely approximated by the use of viscous fluids as damping agents\*.

5-7. **VISCOSITY.** All fluids have an internal friction which resists relative motion between its particles. As a fluid flows over a surface, such as the walls of a tube, the fluid particles in contact with the surface remain at rest (within limits) and the velocity of flow increases with distance from the surface. Considering the fluid to consist of layers, there is an apparent friction between adjacent layers. Viscosity is the term used in designating the amount of this internal friction.

5-8. In a viscous fluid, the velocity gradient (rate of change of velocity with distance) is proportional to the shearing stress on the fluid (the force, per unit area, parallel to the direction of flow, which is causing the flow).

\* Although, in most cases, fluid dampers do not exactly produce a force proportional to velocity, this relation is used and defined as viscous damping because it permits a comparatively simple mathematical analysis.

$$\mu \frac{v}{d} = \frac{F}{A} \quad \text{or} \quad \mu = \frac{Fd}{Av}$$

where

$\frac{F}{A}$  = shearing stress on fluid

$v$  = relative velocity between layers

$d$  = distance between layers

$\mu$  = proportionality constant.

The proportionality constant ( $\mu$ ) is called the coefficient of viscosity or the absolute viscosity. If two surfaces are separated by a viscous fluid, then the force necessary to move one surface parallel to the other is directly proportional to the surface area, the velocity, and the viscosity of the fluid, and is inversely proportional to the distance between the surfaces.

5-9. The viscosities of several liquids are listed in table 5-1. The severe changes in viscosity produced by a moderate increase in temperature should be noted. For aircraft use, fluids which experience a minimum change in viscosity with temperature changes, such as the silicone fluids, should be selected.

5-10. **THE DAMPING ACTION.** A viscously damped system is shown schematically in figure 5-1. A dashpot provides the damping force. A piston attached to the mass moves vertically through the liquid in a cylinder fastened to the support. The damping force is controlled by the viscosity of the liquid and the clearance between the piston and cylinder walls.

5-11. The equation of motion for free vibration of the viscously damped single-degree-of-freedom system of figure 5-1 is:

$$M\ddot{x} + c\dot{x} + kx = 0$$

5-1

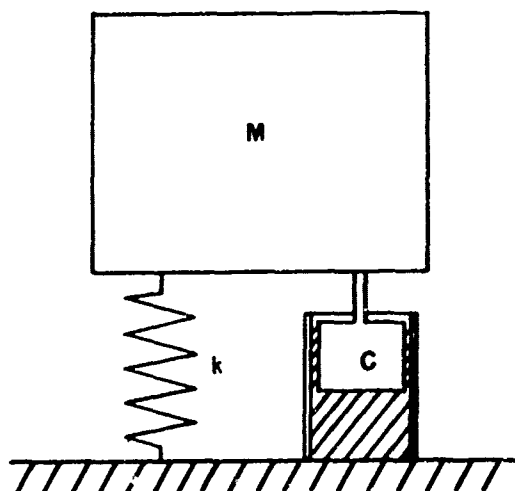


Figure 5-1. Damped, Single-Degree-of-Freedom System

The solution to this equation, and the type of motion described by it, depends on the relative values of the mass ( $M$ ), the coefficient of viscous damping ( $c$ ), and the spring constant ( $k$ ).

5-12. When the mass is released from a displaced position, and simply returns to the equilibrium position without oscillating, the system is said to be over-damped and the motion of the mass is as shown in figure 5-2. This type of motion occurs when the damping coefficient is greater than twice the square root of the product of the mass and the spring constant ( $c > 2\sqrt{kM}$ ).

5-13. If the damping coefficient is less than twice the square root of the product of mass and spring constant ( $c < 2\sqrt{kM}$ ),

TABLE 5-1. COEFFICIENTS OF VISCOSITY FOR VARIOUS LIQUIDS

Liquid	Temperature (°F)	Coefficient, $\mu$ (dyne-sec/cm <sup>2</sup> )
Water	68	0.01
Light machine oil	60	1.14
	100	0.34
Heavy machine oil	60	6.61
	100	1.27
Motor oil, SAE 30	60	3.62
	100	0.89
Glycerine	68	8.5
Castor oil	68	9.86

the mass will vibrate about the equilibrium position when released from a displaced position. The amplitude of vibration will decrease with each cycle and the motion of the mass will be as shown in figure 5-3. The circular frequency of vibration in radians per second is expressed by:

$$q = \sqrt{\frac{k}{M} - \frac{c^2}{4M^2}} \quad 5-2$$

5-14. As pointed out in the two preceding paragraphs, when  $c$  is greater than  $2\sqrt{kM}$  there is no cyclic free vibration and when  $c$  is less than  $2\sqrt{kM}$  there is a gradually decaying vibration. The amount of damping at which the transition takes place is called critical damping and is denoted as  $c_c$ .

$$c_c = 2\sqrt{kM} \quad 5-3$$

Substituting this constant in equation 5-2 leads to the equation:

$$q = \left(\frac{k}{M}\right)^{1/2} \sqrt{1 - \left(\frac{c}{c_c}\right)^2} \quad 5-4$$

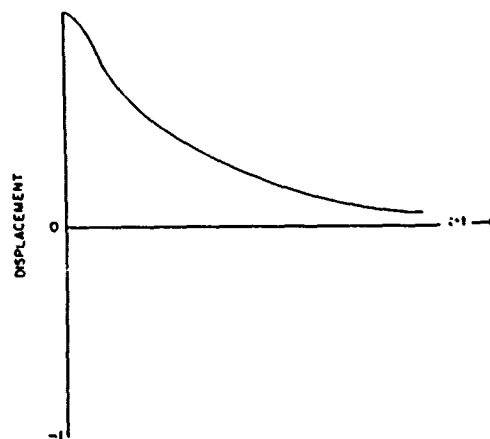


Figure 5-2. Motion of a Single-Degree-of-Freedom System with Damping Greater than Critical

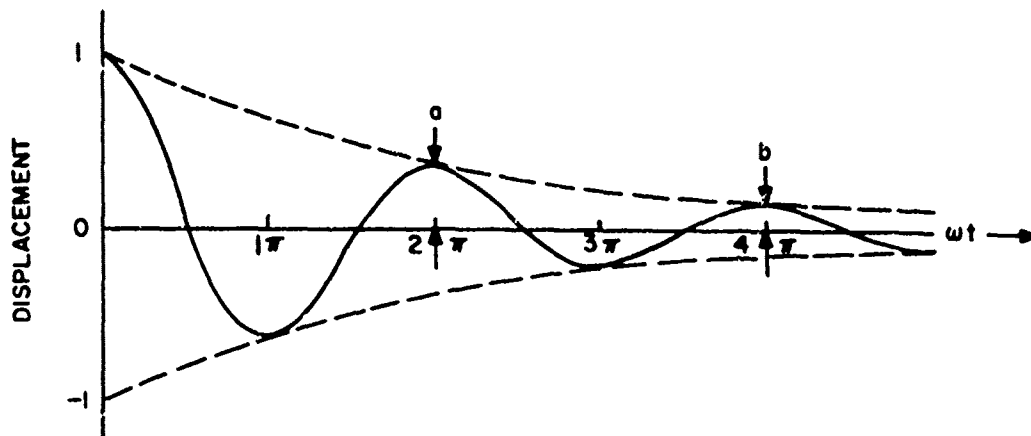


Figure 5-3. Free Vibration of a System with Damping Less than Critical

$$f = \frac{1}{2\pi} \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2} \quad 5-5^*$$

where  $\omega_n = \sqrt{\frac{k}{M}} =$  the undamped circular natural frequency.

The amount of damping applied to or inherent in a system is usually expressed either as the ratio of the damping to the system's critical damping value (damping ratio =  $\frac{c}{c_c}$ ) or as a percentage of the critical damping value (% damping =  $100 \frac{c}{c_c}$ ).

5-15. For damping less than critical (figure 5-3), the envelope of amplitude peaks is a logarithmic curve and the ratio between amplitudes for successive cycles is:

$$\frac{a}{b} = e^{\frac{\pi c}{Mq}} \quad 5-6$$

The value  $\frac{\pi c}{Mq}$  is termed the logarithmic decrement. Substituting the value for  $q$  given by equation 5-4 gives the following expression for the logarithmic decrement.

\* This equation is analogous to the electrical equation:

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \frac{1}{2\pi \sqrt{LC}} \sqrt{1 - \left(\frac{R}{2} \sqrt{\frac{C}{L}}\right)^2}.$$

In this case,  $\frac{R}{2} \sqrt{\frac{C}{L}}$  is the ratio of the resistance in the circuit to the resistance for critical damping,  $R_c = 2 \sqrt{\frac{L}{C}}$ .

$$\Delta = \frac{2\pi \left( \frac{c}{c_c} \right)}{\sqrt{1 - \left( \frac{c}{c_c} \right)^2}}$$

5-7

5-16. Figure 5-4 indicates the effect of damping on the natural frequency of the system. For damping values close to zero, the damped natural frequency is nearly identical to the undamped natural frequency. At damping of 0.5, the damped natural frequency diminishes to approximately nine-tenths (0.875) of the undamped natural frequency. Only at high damping values is the natural frequency of the damped system appreciably different from that of the undamped system.

5-17. The resonant frequency, or frequency at which peak transmissibility occurs, is not identical with the natural frequency of free vibration for a damped system. The effects of damping on the resonant frequency are shown in figure 5-5, where the transmissibility peaks of the more highly damped systems are displaced to the left of the resonant point for the undamped system. Thus, if the resonant frequency for the undamped system is 25 cps, at a damping value of 0.5 the peak transmissibility will occur at 20 cps. The difference between a natural and a resonant frequency is appreciable only with higher damping values. This difference may be seen by comparing frequency ratios for damping values near 1 in figure 5-4 with the frequency ratios for peak transmissibility given in figure 5-5.

5-18. THE EFFECT OF VISCOUS DAMPING ON TRANSMISSIBILITY. As noted previously, transmissibility is the ratio of the maximum displacement of the mounted body to the maximum displacement of its support. The expression for transmissibility of the viscously damped single-degree-of-freedom system shown in figure 5-1 is

$$T = \sqrt{\frac{1 + \left( 2 \frac{f}{f_n} \cdot \frac{c}{c_c} \right)^2}{\left( 1 - \frac{f^2}{f_n^2} \right)^2 + \left( 2 \frac{f}{f_n} \cdot \frac{c}{c_c} \right)^2}} \quad 5-8$$

where

$T$  = transmissibility, a dimensionless ratio

$f$  = the forcing frequency in cycles per second

$f_n$  = the natural frequency of the undamped system in cycles per second

$c/c_c$  = the damping ratio.

(If  $c/c_c = 0$ , this equation reduces to the formula of paragraph 1-24.) Figure 5-5 shows transmissibility curves for damping ratios of 0, 0.05, 0.1, 0.2, 0.5, and 1.0. As the damping  $c$  in-

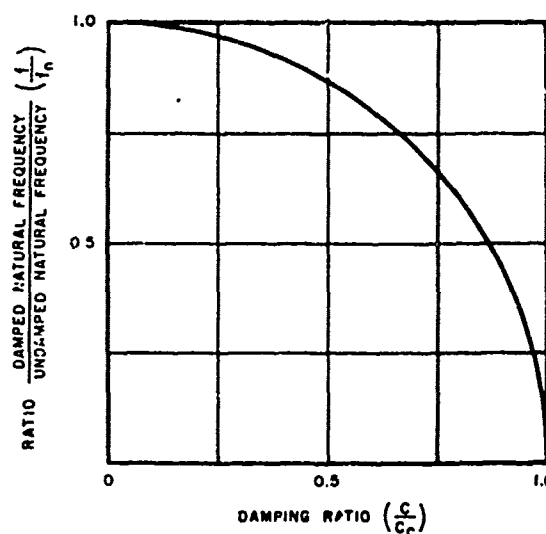


Figure 5-4. Ratio of Damped to Undamped Natural Frequencies, as a Function of Viscous Damping Ratio

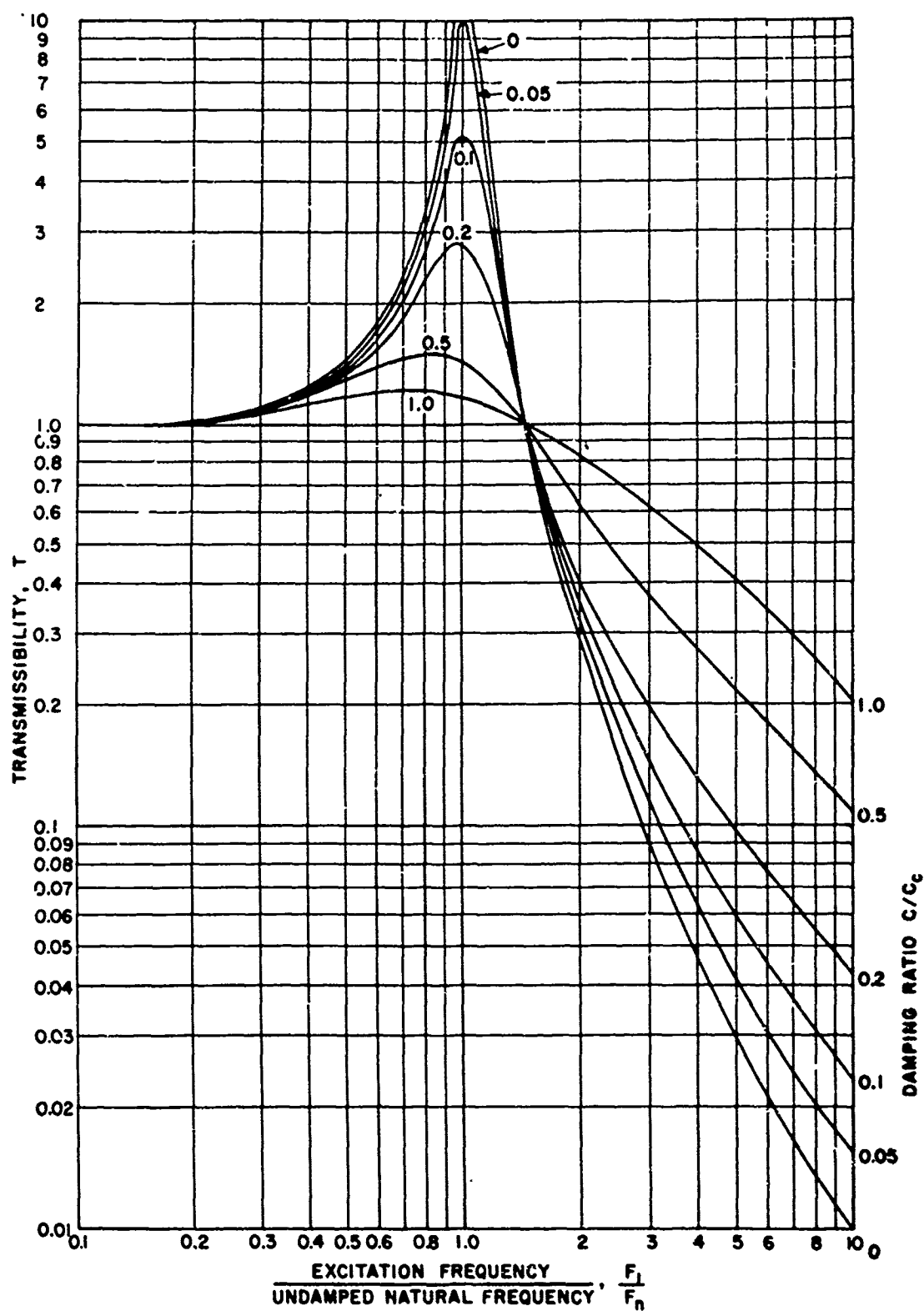


Figure 5-5. Transmissibility Curves for Various Damping Ratios (Viscous)

creases from 0 ( $c/c_c = 0$ ) to the value of  $c_c$  ( $c/c_c = 1$ ), the transmissibility or magnification at resonance decreases from infinity to approximately 1.15. "Infinite damping" is rigid mounting and produces a transmissibility of 1 at all frequencies.

5-19. The frequency ratio at which maximum transmissibility occurs is determinable in terms of the damping ratio, from equation 5-8. If this frequency ratio is substituted in equation 5-8, the maximum transmissibility can be expressed as a function of the damping ratio:

$$T_{\max} = \frac{4 \left( \frac{c}{c_c} \right)^2}{\sqrt{16 \left( \frac{c}{c_c} \right)^4 - 8 \left( \frac{c}{c_c} \right)^2 - 2 + 2 \sqrt{1 + 8 \left( \frac{c}{c_c} \right)^2}}} \quad 5-9$$

Figure 5-6 is a plot of this equation.

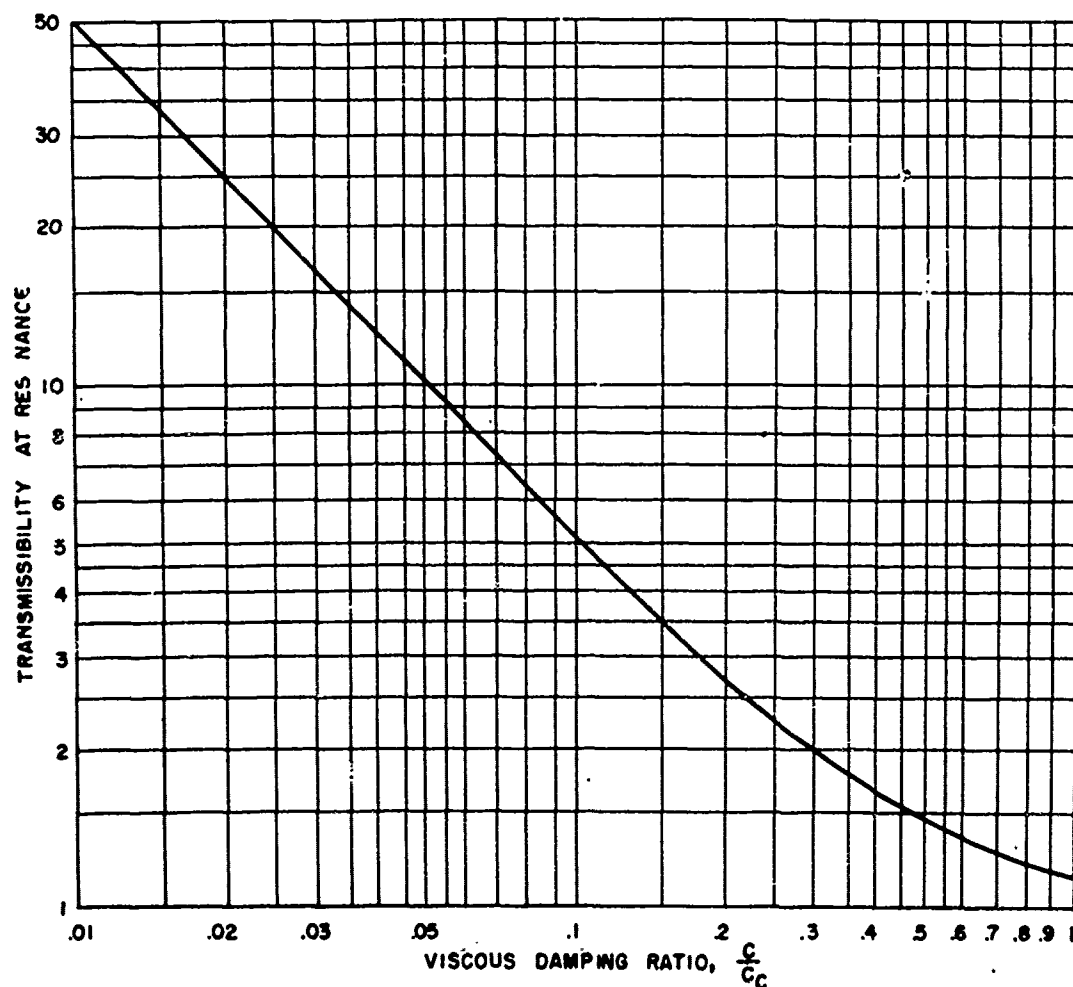


Figure 5-6. Transmissibility at Resonance as a Function of Damping Ratio

5-20. For small amounts of damping  $\frac{c}{c_c} (< 0.1)$ , the frequency ratio  $f/f_n$  at which maximum transmissibility occurs is very nearly one and equation 5-8 can be written:

$$T_{\max} \approx \sqrt{\frac{1 + \left(2 \frac{c}{c_c}\right)^2}{2 \left(\frac{c}{c_c}\right)^2}} \approx \frac{1}{2 \frac{c}{c_c}} \quad 5-10$$

5-21. It is evident, then, that the smallest amount of viscous damping will limit response amplitudes at resonance. This occurs because the energy dissipated in each cycle is a function of the amplitude of the cycle, and this dissipated energy and the energy added to the system balance each other at some finite amplitude, thus imposing a limit on the maximum response amplitude.

5-22. DETERMINING THE APPROPRIATE VISCOUS DAMPING VALUE. Choice of the best damping value for an isolation system is necessarily a compromise between the maximum permissible transmissibilities at resonance and those at frequencies above resonance (the major operating range for the isolation system). From figure 5-5 it can be seen that the higher damping values lower the transmissibility at resonance but increase the transmissibility beyond a frequency ratio of 1.414.

#### 5-23. HYSTERESIS DAMPING.

5-24. Hysteresis damping is the result of the gradual dissipation of energy that occurs within a flexing body due to imperfections in the elastic properties of materials. In a perfectly elastic material, strain would be proportional to stress, and the strain energy would be recovered at each removal of stress. The energy loss due to hysteresis occurs far below the material's fatigue limit, indicating that this damping is not a result of internal structural changes. The gradual cessation of motion of a struck tuning fork is an example of hysteresis damping.

5-25. The damping capacity of a material is the ratio of the energy loss per cycle of stress to the full strain energy at the maximum stress of the cycle. The energy loss per cycle is shown by the difference in amplitude between immediately successive cycles of oscillation (for free vibration). The amount of damping afforded by hysteresis is small and it is common to express it in terms of an equivalent viscous damping ratio. If the transmissibility at resonance is known, the equivalent damping ratio can be obtained from figure 5-6 or from the approximate formula  $\left(\frac{c}{c_c}\right) \approx \frac{1}{2T}$ . This damping ratio can be used in the formulas for viscous damping with only small inaccuracies.

5-26. While it is common to express hysteresis damping in terms of equivalent viscous damping, the viscous damping formulas are good approximations for hysteresis damping only with small damping values. The more exact expressions for transmissibility at resonance and logarithmic decrement are:

$$T_{\max} = \frac{\sqrt{1 + h^2}}{h} \quad 5-11$$

$$\Delta = \frac{2\pi h}{1 + \sqrt{1 + h^2}}$$

5-12

where  $h$  is a hysteresis damping coefficient, and for small values  $h \approx 2 \frac{c}{c_c}$ .

5-27. Table 5-2 lists approximate hysteresis values for various materials. Rubber, particularly synthetic, is a commonly used material in isolation mounts because of its relatively high damping coefficient. Although cork is a good damping agent, it is seldom used in airborne electronic gear since it is a fungus nutrient.

5-28. For many isolator applications, sufficient damping is provided by the damping characteristics of the material. Extra damping, such as friction damping, is required in many instances to dissipate adequately the energy passed into the mounting system.

TABLE 5-2. APPROXIMATE HYSTERESIS VALUES  
FOR VARIOUS MATERIALS

Material	Hysteresis Coefficient (h)	Equivalent Viscous Damping Ratio
Steel	0.01	0.005
Rubber, 30 durometer	0.04	0.02
Rubber, 60 durometer	0.16	0.08
Neoprene	0.12	0.06
Buna	0.40	0.20
Silicone rubber	0.23	0.11
Cork	0.13	0.064

#### 5-29. FRICTION DAMPING.

5-30. Friction is the force which acts between the contacting surfaces of two bodies and tends to resist their sliding motion. If the resistance to sliding prevents motion of one body relative to the other, it is called static friction. If the resistance opposes the motion of two moving bodies, it is called kinetic friction. Friction is either harmful or useful, depending upon the purpose of the mechanism in which it exists. Friction is harmful in bearings because it wastes power; it is useful in brakes or in dampers.

5-31. The following are laws of friction for unlubricated surfaces as determined by experimentation:

- The friction between surfaces depends upon the material and the finish of the surfaces.
- Friction is approximately independent of the surface area.
- The friction force is approximately proportional to the force pressing the surfaces together, whether it is caused by gravity, compression springs, or some other means.

5-32. The force required to overcome static friction is given in the equation for the coefficient of static friction:

$$C_s = \frac{F_s}{F_c}$$

where  $F_s$  = the force, applied in the direction of motion, just sufficient to start the object moving  
 $F_c$  = the force pressing the friction surfaces together  
 $C_s$  = the coefficient of static friction, which is a constant for a given pair of substances under given conditions.

Similarly, the force  $F_k$  required to move an object with uniform speed against friction is defined in the equation

$$C_k = \frac{F_k}{F_c}$$

where  $C_k$  is the coefficient of kinetic (or sliding) friction.

5-33. If  $C_s$  should equal  $C_k$ , the friction force is defined as coulomb friction. Normally, friction materials do not provide a constant friction force since the static friction does not equal the kinetic friction. In the usual mathematical treatment of friction damping, however, it is assumed that the friction force is constant, regardless of the position or velocity of the vibrating mass.

5-34. **THE DAMPING ACTION.** With coulomb damping, the reduction in amplitude in successive cycles is a constant quantity and is given by the equation

$$\Delta X = 4F/k$$

where  $\Delta X$  = the reduction in amplitude per cycle  
 $F$  = the friction force  
 $k$  = the spring constant.

Since the damping force is constant, this type of damping should not be used in a system that is excited at resonance unless the driving force is known to be less than the friction force. If the reverse situation exists, the amplitude will increase in successive cycles, theoretically, to infinity.

5-35. The constant friction of the coulomb damper produces effects quite opposite to those of the viscous damper. When the amplitude or frequency of the vibration is low, the acceleration of the mounted body is low and the resultant inertial force may be less than the friction force of the damper. The damper then functions as a rigid connection, the isolator has no resilience, and the transmissibility is one. It was noted earlier that at low frequencies or amplitudes, the resistance of a viscous damper to motion is low.

5-36. **THE EFFECT OF FRICTION DAMPING ON TRANSMISSIBILITY.** Figure 5-7 shows typical transmissibility curves, determined empirically, for a vibration isolator with no external damping and with three different external friction dampers. (The displacement of the highly damped resonance peaks to the right was not accounted for.) The effect of the friction dampers is evident. With no damper, the magnification at resonance is 8.9\*, and at  $f/f_n = 2$  the transmissibility is 0.35.

\* Indicating hysteresis damping of 5.6%.

The magnification factor is reduced to less than 2 by a 0.4-lb friction-damping force with the transmissibility at  $f/f_n = 2$  increased to only 0.47.

5-37. DETERMINING THE APPROPRIATE FRICTION DAMPING VALUE. From figure 5-8, the amount of necessary damping can be established by plotting the maximum permissible transmissibility desired against the spring constant. For example, if the equipment weighs 20 pounds and causes the isolator upon which it

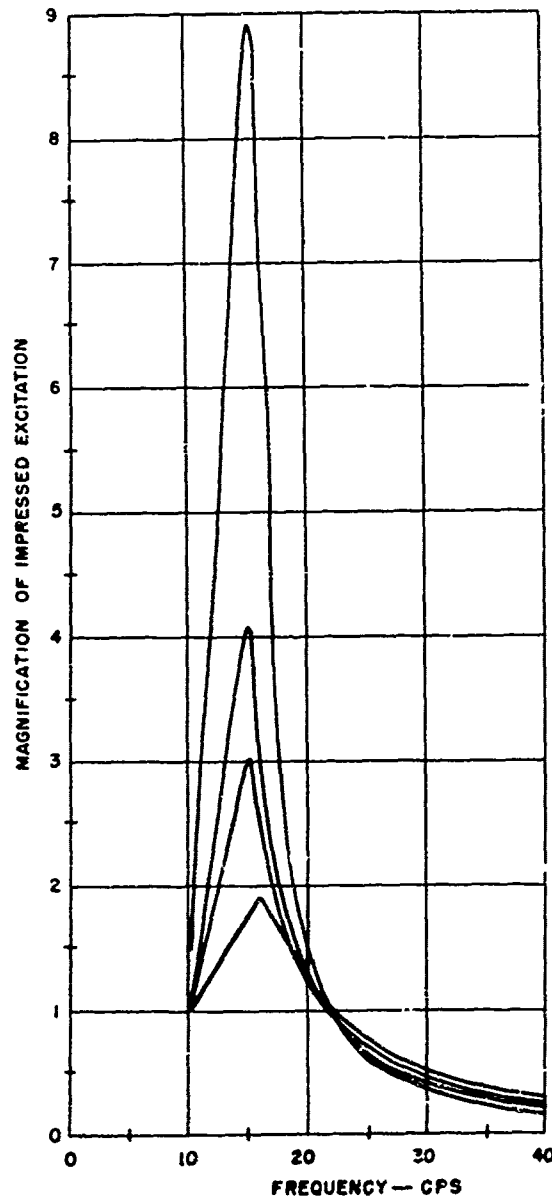


Figure 5-7. Typical Transmissibility Curves for a Vibration Isolator With and Without Friction Dampers

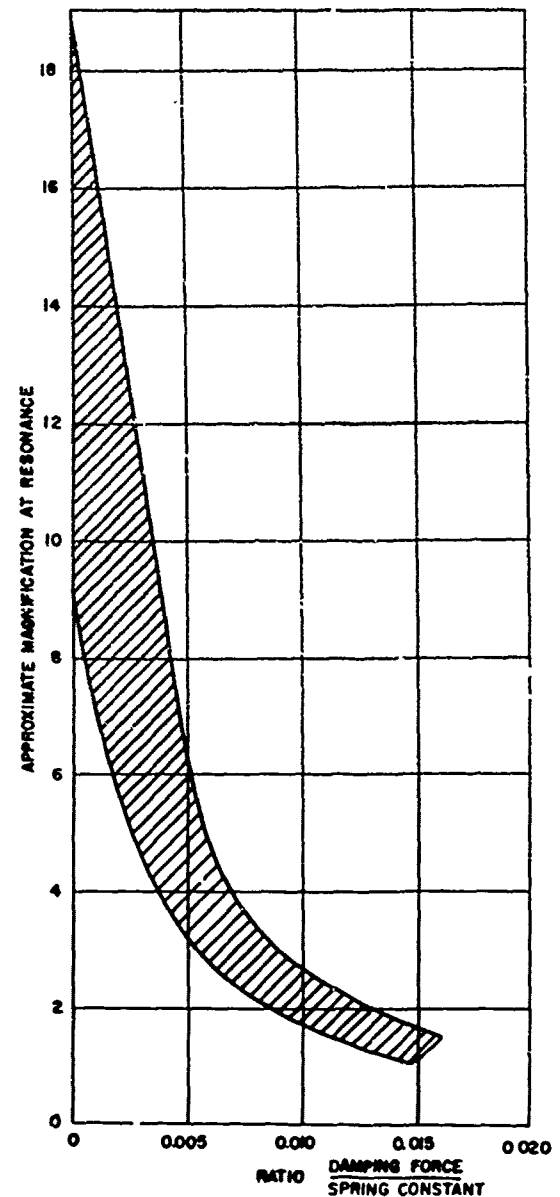


Figure 5-8. Ratio of Damping Force to Spring Constant Plotted Against Transmissibility

is mounted to deflect 1/2 inch, the spring constant ( $k$ ) is 40 pounds per inch. If the maximum transmissibility is to be 2, the ratio of damping force to spring constant must lie between 0.00875 and 0.01275. To provide this ratio, the damping force must be between 0.35 and 0.51 pound. Friction dampers are available which provide a damping force of from 0.20 to 10.0 pounds. The external dimensions of these dampers range from 1.25 by 1.25 by 0.4 inches to 2.4 by 2.4 by 0.9 inches.

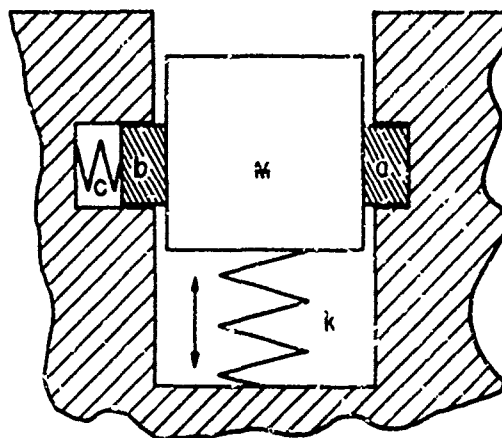


Figure 5-9. Friction-Damped, Single-Degree-of-Freedom System

5-38. **FRICITION DAMPERS AND MATERIALS.** Figure 5-9 is a schematic illustration of a friction-damped, single-degree-of-freedom system. The mass slides between two blocks, a and b, which are forced against it by the spring c. The damping force is the product of the pressure of the blocks against the mass and the coefficient of friction between the mass and blocks.

5-39. A commercially available friction damper provides damping along the vertical axis and along any axis in the horizontal plane. This damper is used with equipment isolators and may be incorporated into the isolator itself. It dissipates vibration energy with sliding friction, vertically, between a steel insert and the inside diameter of a split nylon washer, and horizontally, between a split washer and two nylon retaining discs which can also slide against their contact surfaces. The vertical damping force is controlled by a garter spring, the horizontal force by a finger spring.

5-40. Damping is incorporated in some steel spring isolators by packing the center of the spring with steel wool. This produces damping reported in a range from 15 to 20 percent of critical (transmissibility at resonance of from approximately 2.5 to 4). The friction between the fibers of the distorted wool pad provides the damping.

5-41. At least one isolator on the market obtains a certain amount of friction damping by containing its helical springs (of which each isolator has a large number) in sleeves of polyethylene. The rubbing of the steel springs against the walls of the polyethylene tubing provides the damping.

5-42. Friction surface materials are often the major failing of friction dampers. With improper materials, the starting or breakaway resistance is high and the friction falls rapidly as the velocity increases. Certain nonmetallic brake lining materials in contact with metal surfaces produce a fairly uniform coefficient of friction substantially independent of velocity. These materials are also durable and insensitive to temperature, moisture, etc.

### 5-43. AIR DAMPING.

5-44. Air damping results from the direct transfer of energy from a vibrating system to air (kinetic or heat). Since air is viscous, air damping is similar to viscous damping; however, at room temperature, air has about 1/50th the viscosity of water and the damping force obtained is small compared to viscous damping. Thus, air dampers would be preferred over friction or viscous dampers for isolating lightweight components.

5-45. THE DAMPING ACTION. A vibrating object surrounded by air has forces imposed upon it by the air and these forces are in a direction opposed to the velocity of the object. For free vibration, air damping produces a logarithmic decrement (as do hysteresis and viscous damping). For forced vibration, the damping force is proportional to the square of the velocity of the support.

5-46. Air damping is usually stated, for convenience, in terms of an equivalent viscous damping value. Computation of the equivalent damping of an air damper would be extremely difficult due to the number of variables involved. The equivalent damping value is obtained by measuring the maximum transmissibility at resonance of a vibratory system containing the damper. The greater the damping force, the lower the maximum transmissibility. If the transmissibility value is known, the equivalent damping value can be obtained from the plot of figure 5-6.

5-47. DAMPERS. An isolation system with air damping is shown schematically in figure 5-10. The damper is composed of a piston which fits tight (but essentially without friction) against the walls of a cylinder that has two orifices in its head. When moved by vibration, the piston causes pressure changes within the chamber which force air through the orifices.

5-48. Air damping is incorporated into isolation mounts by means of a bellows which forces air through an orifice as the bellows is distorted by the relative motion between the support and mounted equipment. The force required to move the air through the orifice is lost by the system and limits the amplitude at resonance. A rubber bellows, sealed except for an orifice, is effective for motion in both vertical and lateral directions. As the mass on the damper moves down (or as the support moves up), the bellows flattens and its volume decreases. This increases the pressure inside the damper and air is forced out through the orifice. As the mounted body moves up, the volume of the bellows increases and air is pulled in through the orifice. With lateral movement of the mounted body, the bellows is distorted, also resulting in pressure differentials and the movement of air in and out of the bellows.

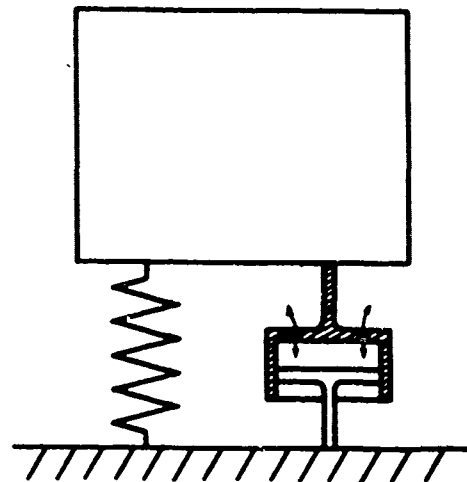


Figure 5-10. Isolation System Using Air Damping

5-49. The volume and the wall thickness of the bellows are critical in the design of an air damper. If the walls of the bellows are thin, they will stretch when there is a pressure buildup. The amount that the bellows will distend determines the ability of the damper to absorb shock. The thicker the walls, the higher the shock-transmission factor.

#### 5-50. DYNAMIC VIBRATION ABSORBER.

5-51. An interesting damping mechanism, although not generally applicable to airborne electronic equipment, is the dynamic vibration absorber. For special vibration problems where the frequency of the exciting force is constant, and where damping or structural change is impractical, the dynamic vibration absorber can be used to advantage.

5-52. The dynamic vibration absorber consists of a small mass-spring system, tuned to the frequency of the exciting force, and attached to the mass from which vibration is to be eliminated (figure 5-11). The absorber vibrates so that its spring force is at all times equal and opposite to the exciting force. Since there will then be no net force acting on the mass  $M$ , that mass will not vibrate. Dynamic vibration absorbers are used only when the main mass is excited at or near resonance.

5-53. When a dynamic vibration absorber, such as is illustrated in figure 5-11, is tuned to the frequency of the main mass ( $\omega_a = \Omega_n$ ), the response of the main mass to input excitation is as shown in figure 5-12. While resonance of the mass has been eliminated at a frequency ratio of  $\omega/\Omega_n = 1$ , two other resonant frequencies now exist. Although figure 5-12 shows these resonances as occurring at frequency ratios of 0.8 and 1.25, these values are not fixed but are dependent on the ratio of the damper mass to the main mass.

5-54. The curve of figure 5-12, showing two frequency ratios for infinite response, is for an undamped system. If small amounts of damping are incorporated into the dynamic vibration absorber, the response at each of these two frequency ratios will be reduced in much the same way as was shown in figure 5-5. As the damping approaches infinity, however, the absorber mass and the main mass are effectively locked together and become a single-degree-of-freedom system with infinite response at a single resonance. Between damping extremes, an optimum point exists where the peaks of the response curve are at their lowest point.

5-55. With moderate amounts of damping, there are two resonant frequencies and, hence, two peaks in the curve. Adjusting the natural frequency of the

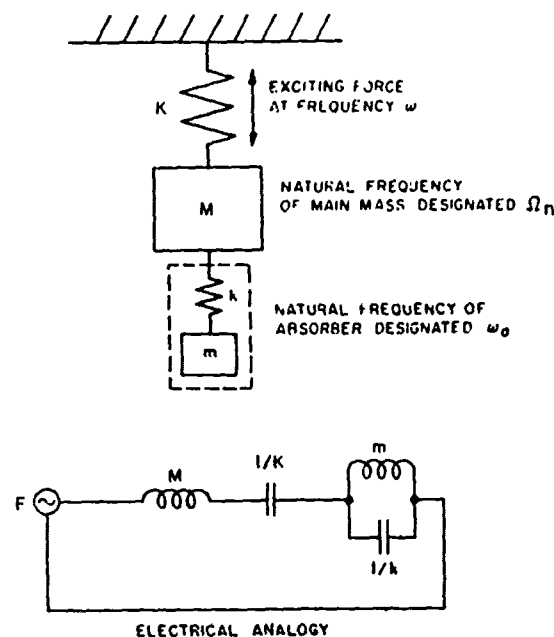


Figure 5-11. Dynamic Vibration Absorber

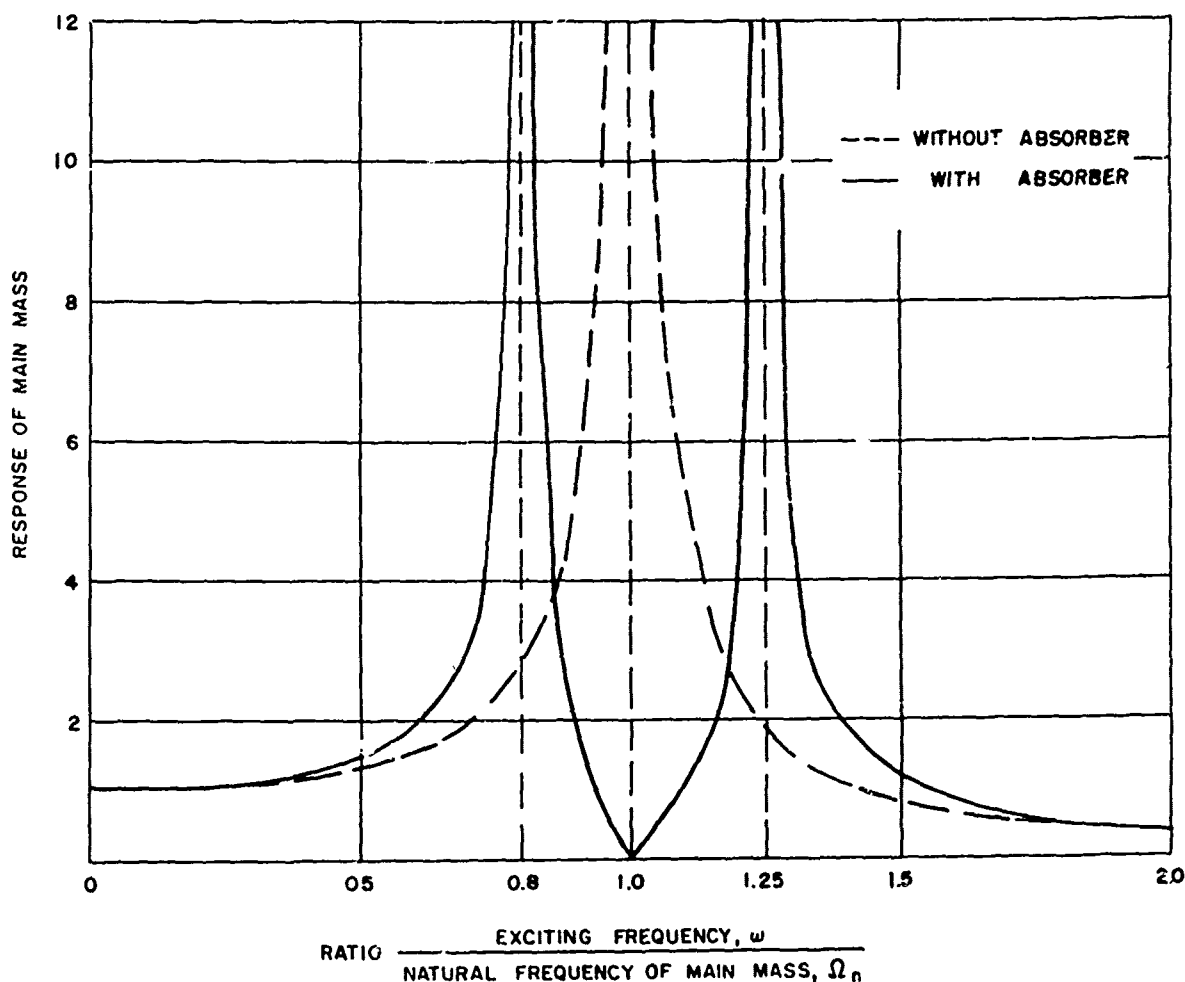


Figure 5-12. Response of a Mass Excited at Resonance With and Without Vibration Absorber

dynamic absorber to equal the natural frequency of the main mass's system ( $\omega_a/\Omega_n = 1$ ) will not result in equal heights nor in minimum values for the two peaks. To achieve this, the absorber has to be adjusted for optimum tuning and optimum damping. For optimum tuning, the ratio of absorber frequency to main mass frequency ( $\omega_a/\Omega_n$ ) should be equal to

$$\frac{M}{M + m}$$

With optimum tuning established, the amount of damping required to establish the peaks at their lowest value must be determined. This optimum damping value is given by the equation:

$$\frac{c}{c_c} = \sqrt{\frac{3 M^2 m}{8 (M + m)^3}} \quad *$$

\*  $c_c$ , in this case, is not the usual critical damping value which would be equal to  $2\sqrt{km}$ .  $c_c$ , as used here, is defined as  $2m\Omega_n$  or  $2m\sqrt{K/M}$ .

5-56. With optimum tuning and optimum damping, curve C of figure 5-13 is obtained when  $M/m$  is equal to 5. Curve A shows the motion of the main mass without the absorber; curve B shows the motion of the mass with an undamped, tuned absorber. It is evident that the excursion is less for the damped, tuned vibration absorber between frequency ratios of approximately 0.8 and 1.2, and that above 1.2 the excursion is just slightly more than that which occurs without an absorber.

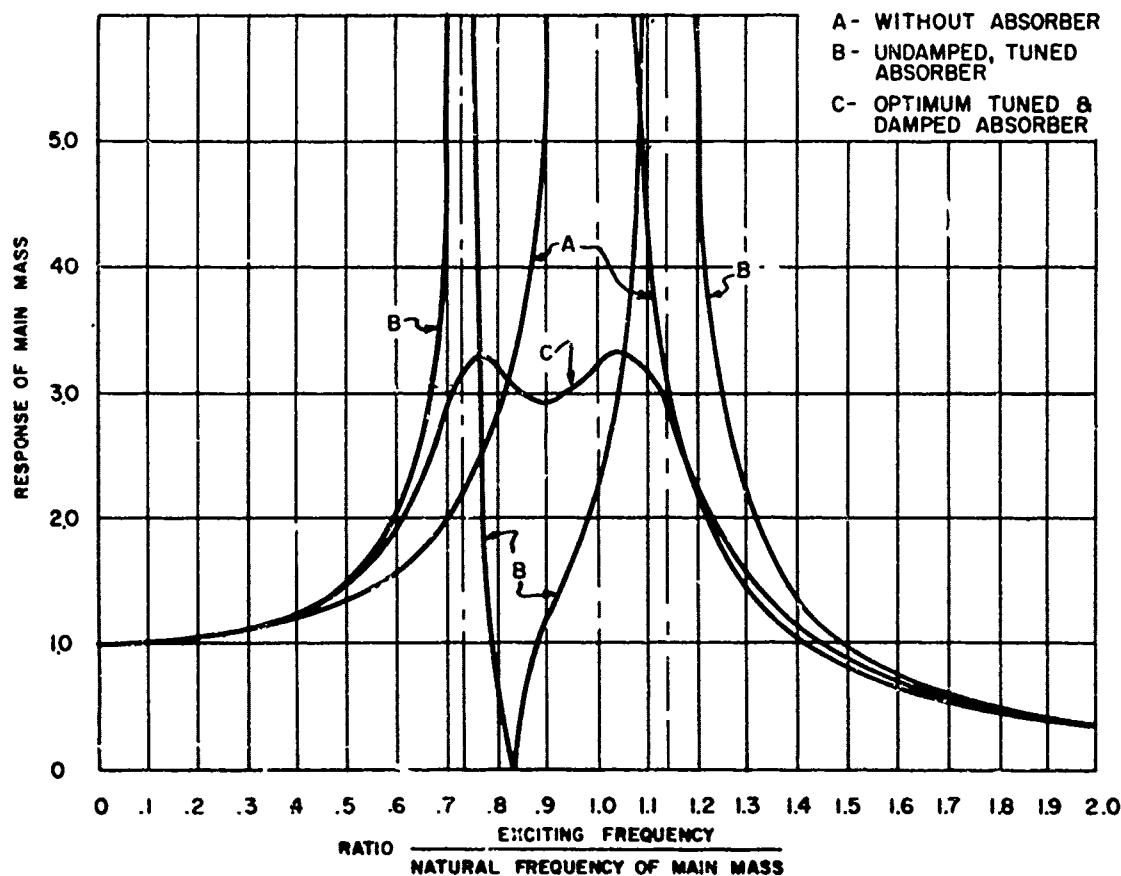


Figure 5-13. Response of a Mass Excited at Resonance (A) Without an Absorber, (B) With an Undamped Tuned Absorber, and (C) With an Optimum Tuned and Damped Absorber

5-57. It is interesting to consider using some part of the electronics equipment as the absorber mass. A transformer, isolation mounted within the equipment (and tuned to the mount frequency), could possibly serve to eliminate the destructive effects of environmental resonances. Of course, the large excursion of the transformer (relative to the chassis) would have to be handled in some way.

## SECTION VI

### EQUIPMENTS FOR SIMULATING AND MEASURING SHOCK AND VIBRATION

#### 6-1. SHOCK SIMULATORS.

6-2. The forerunner of present-day shock simulating equipment was developed by the British Navy during World War II. The British attempted to develop equipment that would resist the damaging shock waves normally expected in combat. A method of testing new designs was a necessary part of the program, and the British equipped a laboratory to mechanically shock equipments. The severity and the methods of creating shocks were varied until the damage approximated that occurring in combat. The criteria used to test new designs proved to be of great value in developing shock-resistant equipment.

6-3. The Hi-Impact shock machines now used by the U. S. Navy are improvements of the early shock tester developed by the British. There are two types: one for testing light-weight equipments (shown in figure 6-1) and the other for medium-weight equipments. (There is a third type under development for testing equipments above 4500 pounds.) Briefly, the test machines contain pendulum-type weights that strike a carriage upon which the equipment is mounted. The light-weight shock tester also has a drop-weight feature. There have been criticisms that this type of shock machine does not produce a shock wave that approximates that occurring in actual equipment use; but the rebuttal to this is that this testing equipment succeeds in its purpose regardless of the theoretical considerations involved.

6-4. With the varied shock environments of aircraft, e. g., landing, buffeting, etc, the need arose for a shock simulator with which the duration of shock could be better controlled. This led to the development of the sand-drop machine by Wright Air Development Center. This simulator used the drop-elevator principle in which the platform containing the equipment fell into sand. By varying the number of wooden blocks in the bottom of the platform, their penetration into the sand could be varied, thus giving control of shock pulse duration.

6-5. There are now many variations for arresting the fall of the elevator and thus for controlling the pulse duration. One of the oldest of these is the use of springs, as in the American Standards Association simulator. (This machine actually preceded the development of the sand-drop machine but it is limited to the testing of small, light-weight equipments.) Other methods include a blade on the bottom of the platform cutting through thicknesses of lead, a rod mounted under the platform piercing lead, and the platform itself compressing lead pellets. A completely different equipment and a recent development is the "Hyge Shock Tester" which utilizes the principles of hydraulics and differential pressure to induce the controlled shock.

6-6. The following paragraphs describe the more important shock simulators for airborne applications in some detail. These descriptions will give the designer knowledge of what his equipment will eventually be subjected to in testing and will enable the designer to develop some understanding of the relationship between the environments produced by the shock machine and the equipment's ultimate operating locale.

6-7. **DROP-TYPE SHOCK SIMULATOR.** The JAN-S-44 (Army-Navy specification) drop-type shock simulator, also referred to as the American Standards Association drop-type shock simulator, is used extensively for testing small electrical instruments. This machine, shown in figure 6-2, has an elevator platform approximately 6 inches square to which the instrument undergoing test is attached. The elevator platform is constrained to move in a vertical direction by guide rods. The platform is lifted to a predetermined height and released. The fall of the platform is arrested when a calibrated spring, attached to the underside of the platform, engages a curved anvil. The elevator is caught on the first rebound to prevent more than a single shock. The anvil, framework, and guide rods are all supported by a solid cast-iron base.

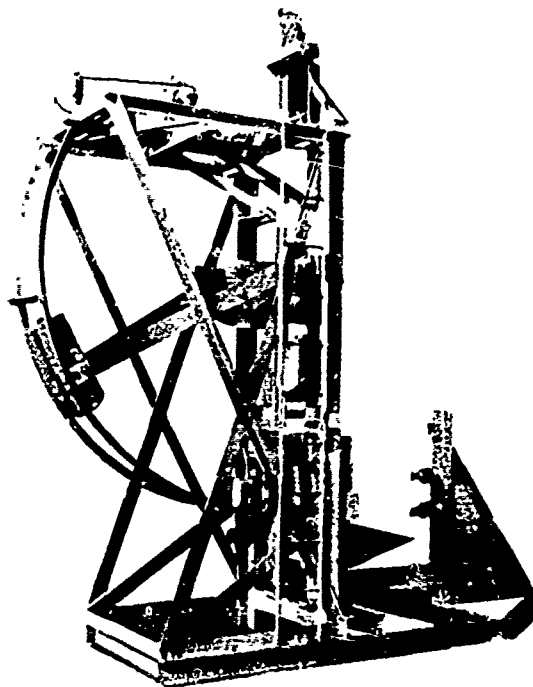


Figure 6-1. Hi-Impact Shock Simulator for Testing Light-Weight Equipments

6-8. The severity of the shock is controlled by the height of the elevator prior to the drop. The duration of the shock can be changed by using a stiffer or softer calibrated spring at the bottom of the elevator. The shape of the acceleration pulse (shock) is a half sine wave. The drop height and spring used are determined by requirements of the test and can be calculated from calibration curves

6-9. **VARIABLE-DURATION, DROP-TYPE SHOCK SIMULATOR.** The variable-duration, drop-type shock simulator is designed to subject equipment and components, up to a weight of 1200 pounds, to shocks of reproducible magnitude. Generally, the simulator consists of a frame, track-wheel guide assembly, elevator assembly, arresting medium, motor and winch assembly, and a base. The arresting medium is, in most cases, impact blocks dropping into sand. A punch penetrating lead is used when a pulse of 8 milliseconds or less is desired and the specimen is 20 pounds or less.

6-10. There are three principal variables which determine the maximum amplitude of acceleration and the duration of the pulse:

1. The combined weight of elevator, dead load, and equipment being tested.
2. The number and arrangement of blocks when sand is used, or the size of the arresting punch when lead is used.
3. The height of the drop.

6-11. Specification MIL-S-4456 (USAF) describes variable-duration simulators of three weight classes: 20-, 150-, and 1200-pound machines. The 20-pound simulator, shown set up for a sand-drop test in figure 6-3, is designed to use either sand or lead. The cradle which appears to be resting on the sand in the illustration supports the lead impact block when the sand and wooden blocks are not used. When sand is used, the wooden blocks are arranged to fall on either side of the cradle. While specified as a 20-pound machine, it will accommodate specimens up to 50 pounds. It will accept equipment measuring 15 inches in each dimension.

6-12. The 150-pound simulator, similar in appearance to the 20-pound machine, is designed to be used with sand only. The elevator will accommodate specimens 30 inches in each dimension and in some models up to 400 pounds in weight. The larger elevator allows more variations in block arrangements for a greater spread of pulse durations. By varying the number of blocks from 6 to 12, the rate of penetration of the blocks into the relatively inelastic sand is changed and pulse durations from 6.5 to 32 milliseconds can be attained.

6-13. The 1200-pound simulator is also exclusively a sand-drop machine. The construction, principles of operation, etc, are the same as those in the 150-pound machine except that it will accommodate larger (about 5 feet in each dimension), and heavier (up to 1200 pounds) specimens.

6-14. **FLAT-SPECTRUM DROP-TYPE SHOCK SIMULATOR.** The flat-spectrum drop-type shock simulator, shown in figure 6-4, has been recently developed to provide a shock motion having a flat response spectrum. The machine was designed to handle equipments up to 400 pounds and to provide 100g minimum shock spectrum over a frequency range of 100 to 700 cps.

6-15. The major components of this simulator are the usual frame, motor, release mechanism, elevator, and anvil. The elevator is of massive, rigid proportions to keep all natural frequencies above 1500 cps. The table is isolated

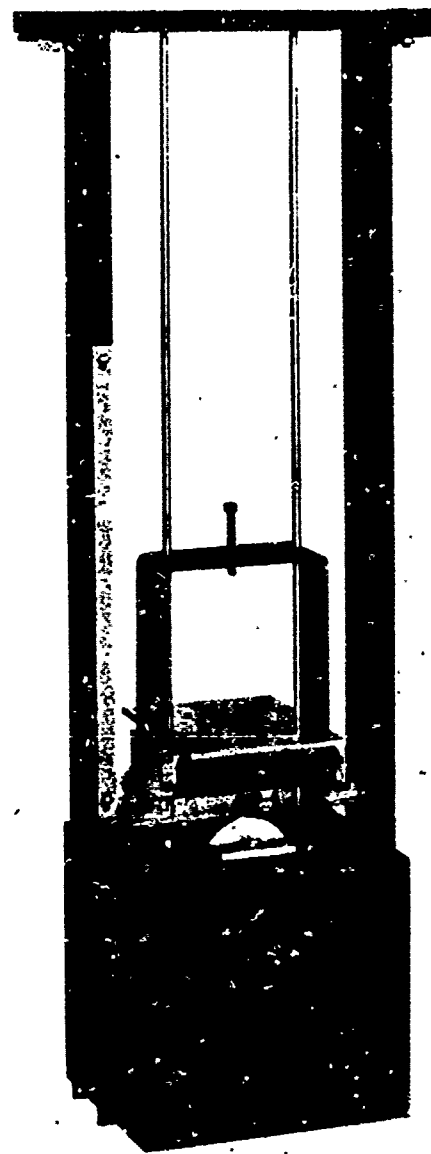


Figure 6-2. The JAN-S-44 Drop-Type Shock Simulator

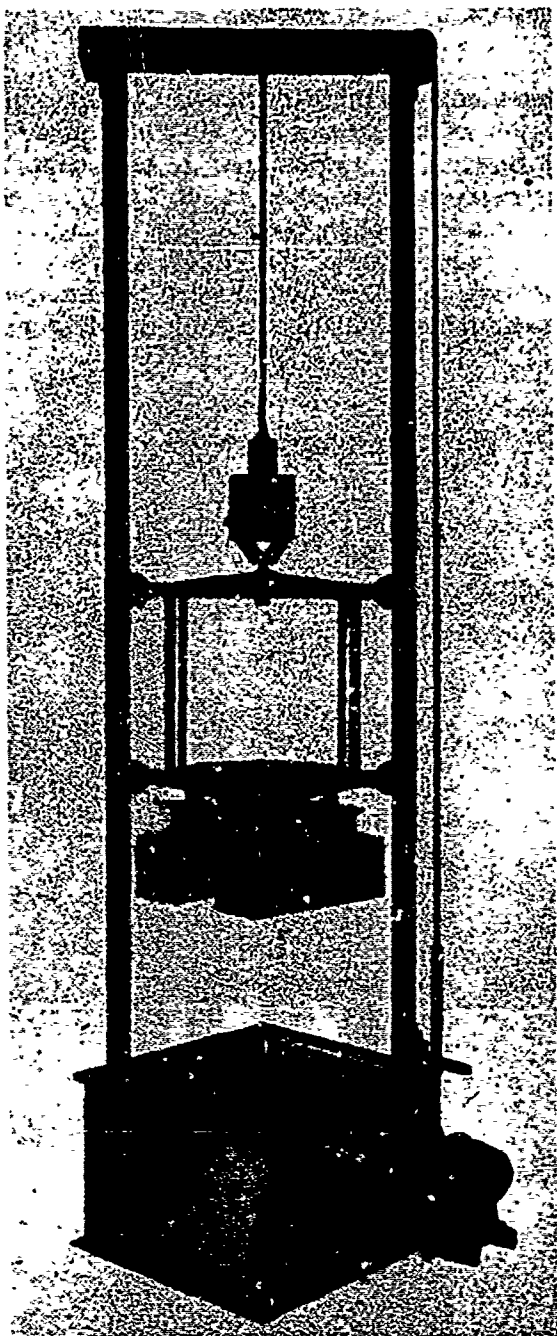


Figure 6-3. Variable-Duration, Drop-Type Shock Simulator, 20-Pound Class

shock simulator, it is possible to test the equipment in both environments simultaneously.

6-20. The Hyge shock tester is reputed to have good waveform repeatability. By varying the metering pin contours, it is possible to achieve different shock wave-

by shear pad isolators to eliminate the effect of structural resonances. The anvil is set in a 6-foot cube of reinforced concrete designed to spread the impact force throughout.

6-16. The arresting medium can be rubber, steel springs, or lead pellets depending upon the shape of the shock pulse desired. The rubber or steel springs will give a reproducible half sine wave; a conical lead pellet will give a reproducible sawtooth pulse.

6-17. **HYGE SHOCK TESTER.** The Hyge shock tester (figure 6-5) subjects equipments to shock by less obvious methods than dropping the equipment or hitting a platform on which the equipment is mounted with a hammer. The machine produces shock by subjecting the equipment to a rapid acceleration as a result of the action of differential pressures on the opposing faces of its thrust piston. The waveform is controlled by the use of metering pins (which control the differential pressures) of different contours.

6-18. The Hyge shock tester can be used with guide rails similar to those used in the drop-type machine. The rails are used to achieve a low-level deceleration. Pneumatically operated brakes in the carriage bring the equipment to the more gradual stop. With rails, the shock tester can be mounted in either a vertical or a horizontal position.

6-19. The size of the Hyge tester makes it adaptable to combined environment testing, which more closely approximates actual operational conditions. For example, by placing a high- or low-temperature chamber over the

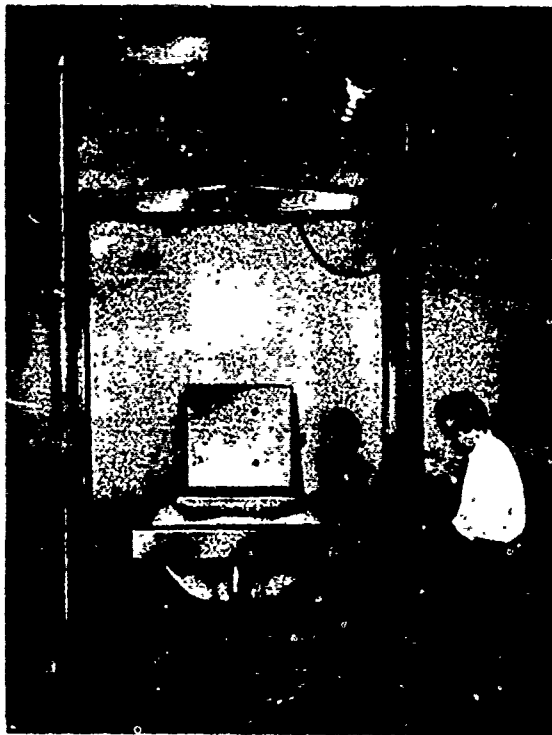


Figure 6-4. Flat-Spectrum Drop-Type Shock Simulator

the boom of the rotary accelerator is driven by a piston rather than by electric or hydraulic motors. A strut is filled with air and, when the proper pressure is reached, the piston is released, thus accelerating the boom rapidly. As the boom is accelerated angularly, the acceleration at the specimen location changes from tangential to radial and, with the specimen fixed, the direction of acceleration relative to the specimen would change. To eliminate this, the mounting platform for the test item turns during the initial acceleration so that the specimen axis is always aligned with the resultant of the tangential and radial accelerations. In this way, the specimen effectively experiences rectilinear acceleration. After the piston arm is fully extended, the rotating boom is free-wheeling, with gradually diminishing acceleration until it is braked to a stop.

6-24. The rotary accelerator uses booms of different sizes, depending upon the size of the object tested. For example, it can accelerate an 8-pound subject at 550g in a buildup time of from 5 to 35 milliseconds using a small boom, and a 100-pound subject to 75g in from 10 to 50 milliseconds using a much larger boom.

6-25. **ROCKET SLEDS.** Rocket sleds are another example of special equipment designed to produce a long-duration shock. They are capable of testing larger equipments than the rotary accelerator. They are probably best known for their use in experimentation on the effects of acceleration shocks on man.

6-26. The principle of operation of the rocket sled is relatively simple. A sled rides on tracks and is accelerated by rockets. The test item is mounted on the sled. The system can be designed for the test period to occur during either acceleration or deceleration. Various methods are used for arresting the sled, depend-

forms. The tester will deliver a thrust of approximately 10,000 pounds to the test specimen.

#### 6-21. LONG-DURATION SHOCK SIMULATORS.

6-22. The previously discussed shock simulators give an acceleration pulse (shock) of relatively short duration; however, there are many environments, such as occur during a jet-assisted or ballistic missile takeoff, where the accelerations are of long duration. To simulate these long-duration shocks (accelerations), many special types of equipment have been designed and built. Representative of these are the rotary accelerator and the rocket sled.

6-23. **NAVAL ORDNANCE LABORATORY ROTARY ACCELERATOR.** The rotary accelerator is similar to the centrifuge (see paragraph 6-28) in that the test item is mounted on the end of a rotating arm and whirled. However,

ing upon the deceleration rate desired and the amount of runway available.

#### 6-27. STEADY-STATE ACCELERATION SIMULATOR (CENTRIFUGE).

6-28. Steady-state accelerations are experienced during aircraft maneuvering or during ballistic-spin missile flight. The only simulator capable of providing steady-state accelerations is the centrifuge. Essentially, the centrifuge consists of a rotating arm, a platform for mounting and securing the test item, a counterbalancing arm and weight, and a driving mechanism. Figure 6-6 shows a simple centrifuge installation.

6-29. The size of the rotating boom can vary from 1 foot to 50 feet or more. The dimensions of the test item determine what size centrifuge is used for a test. If the component is too large in relation to the radius of the rotating arm, there will be considerable variance in accelerations across the area of the test item.

6-30. There are many elaborations on the simple centrifuge. Slip rings are used for supplying power and instrumentation to the test item. Movable weights can be used for counterbalancing the test item. Optical systems or television cameras can be mounted on the centrifuge to permit study of the test item during accelerations. The centrifuge can be installed in a pit or surrounded with a guard to protect personnel against injury from flying objects resulting from accidents. There can be provisions for combined environment testing, such as is provided in the Convair centrifuge.

6-31. The Convair centrifuge is used to test missile components. It is designed to rotate at 121 rpm while testing a 1-ton load. The boom is 40 feet long and weighs 10 tons. A 3-foot-long steel capsule is provided to house the component during testing. The temperature in the capsule can be varied from  $-100^{\circ}\text{F}$  to  $+350^{\circ}\text{F}$ , thus providing the combined environmental testing of temperature and acceleration. (It is planned to combine three environments by adding a shaker mechanism in the capsule.) The centrifuge is powered by three hydraulic motors with a combined capacity of 125 gpm under 5000 psi pressure. Hydraulic pumps are driven by 400-hp electric motors. Braking results from reversing the fluid flow in the hydraulic motors.

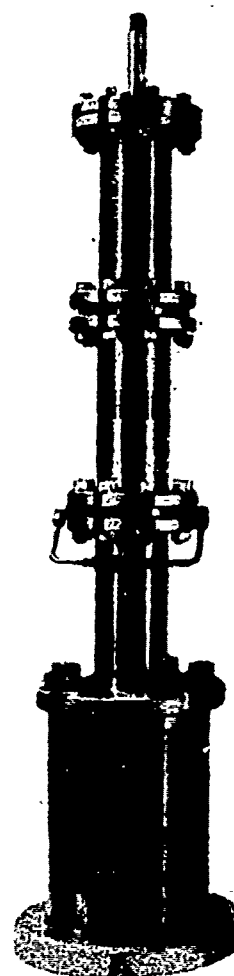


Figure 6-5. Hyge Shock Tester



Figure 6-6. Centrifuge Installation

## 6-32. VIBRATION SIMULATORS.

6-33. Specifications describing test procedures for vibration usually do so by giving the magnitude of two parameters: (1) the frequency and (2) either the total excursion\* in inches or the maximum acceleration in g's. There are several reasons for using either total excursion or acceleration as the second parameter. At low frequencies, an envelope of the environmental extremes can be most conveniently expressed in terms of an amplitude which is constant over the frequency range.

At the higher frequencies, a constant g envelope best describes the normally encountered environments. Also, the excursion is so small to achieve certain g levels at high frequencies that it is difficult to measure or set accurately.

6-34. The type of vibration simulator used is also a determining factor as to whether total excursion or g level becomes the second parameter. Initially, the most common vibration simulator was the positive-drive machine. This machine operates at low frequencies with preset amplitudes, and, as the frequency is changed, the g level varies. If acceleration were the parameter used, each value would have to be converted to amplitude because this is the quantity the machine will accept. An alternative would be to adjust the amplitude while the simulator is operating and, with expensive instrumentation, read acceleration values as they occur. With the advent of the electrodynamic vibration simulator, capable of reproducing high-frequency vibrations, it became more convenient to give the parameter in g's because amplitude is not preset in these machines. Immediate g values are read and the input voltage (force applied to the shaker table) can be adjusted to the desired g value. For the electrodynamic shaker, expensive instrumentation would be necessary to maintain constant amplitudes.

6-35. In addition to the positive-drive and electrodynamic types mentioned above, there are other types of vibration simulators such as the reaction, hydraulic, and pneumatic types. The following is a description of these various simulators that attempt to reproduce the most damaging vibration environment an equipment must be designed to withstand.

6-36. **ELECTRODYNAMIC VIBRATION SIMULATOR.** The electrodynamic vibration simulator (figure 6-7) employs the dynamic loudspeaker method of suspending a coil in the field of a d-c excited electromagnet to produce vibration. If an alternating current is fed to the coil, the coil (and the shaker table) will vibrate at the input frequency (figure 6-8). By varying the input frequency, the shaker table frequency can be controlled. For simulating steady-state vibration, an electronic

\* Total excursion is the distance traveled from one extreme to the other. Amplitude is the distance from the mean position to one or the other extreme, and is equal to one-half the total excursion. Double amplitude, obviously, is equal to the total excursion.

oscillator or alternator can be used to supply the alternating current. For random-type vibration, a random noise generator is used.

6-37. The major advantage of the electrodynamic vibration simulator is its versatility and the ease with which high frequencies can be attained and controlled. Frequencies up to 2000 cps are available in large units, and up to 10,000 cps in small units. Instrumentation (e. g., accelerometers, amplifiers, etc) permits instantaneous monitoring of g levels. Automatic feedback arrangements can provide constant g levels or constant amplitudes over a sweep of frequencies. It is adaptable to such procedures as electronically recording the vibration environments of actual service and then reproducing them in the vibrator.



Figure 6-7. Electrodynamic Vibration Simulator

6-38. The basic moving mass of the electrodynamic vibrator, i.e., the coil and platform, must be kept to a minimum because it is accelerated with the test specimen. This necessarily light structure makes these machines unsuitable for directly vibrating heavy loads. This disadvantage is offset by connecting the vibrator by mechanical linkages to large loads mounted or suspended independently so that the shaker, when carrying only its own weight, can expend all its force output against the object. However, the natural frequency of the system consisting of the mass of the test specimen (and table if mounted) and the elasticity of the mechanical linkage tends to limit the testing frequencies because, as a single-degree-of-freedom system, it will accept little power beyond its natural frequency.

6-39. There are various test setups in which a shaker vibrates a load through a mechanical linkage. Figure 6-9 shows an auxiliary table mounted on vertically oriented cantilever springs. The springs are made from flat stock and thus are soft in the desired horizontal direction of vibration and rigid in the other, to minimize extraneous motions. The table is rigidly constructed so that its natural frequency will not limit the testing frequencies. A mechanical linkage connects the table to the electrodynamic vibration simulator. A disadvantage is that the springs tend to bow when the natural frequency is reached which distorts the vibratory motion. There is also slight vertical distortion because of the turning motion of the table.

6-40. Another method used is to suspend the test specimen on bungee cords and mechanically link it to the vibrator. However, to properly orient the equipment and the shaker is a time-consuming task. A recently proposed system is to shake

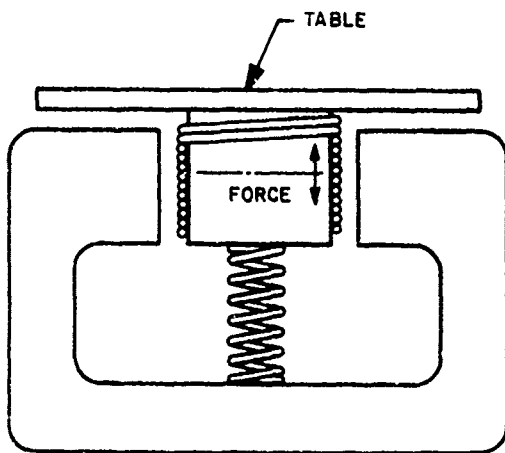


Figure 6-8. Schematic of Electrodynamic Vibration Simulator

combination of equipment used to provide random vibration excitation for laboratory testing is commonly called a complex motion system. The system consists of a signal source, equalizers, a large electronic amplifier, and an electrodynamic shaker. The signal source can be one of two types: magnetic tape or random voltage generator. Magnetic tape, containing the recorded flight vibration data, permits laboratory reproduction of actual flight vibrations. A random voltage generator is used when the testing is done according to a mean square acceleration density spectrum. The spectrum may be representative of flight data from several flights or from several aircraft. Figure 6-10 is a photograph of a random vibration test system capable of a maximum peak instantaneous force of 2700 pounds.

the test specimen using the same system of mechanical linkage; however, in this instance, the test specimen is mounted to a flat surface which is separated from another flat surface (the top of a table) by a film of oil. The system is obviously economical both in the cost of the equipment (metal plates, table, oil, etc) and in eliminating the need for costly equipment orientation such as occurs when using bungee cords. And, according to its proponent, the vibratory motion that is obtained is quite free from extraneous motions.

6-41. The electrodynamic shaker is the only vibration simulator adaptable to random vibration testing. The combination of equipment used to provide random vibration excitation for laboratory testing is commonly called a complex motion system. The system consists of a signal source, equalizers, a large electronic amplifier, and an electrodynamic shaker. The signal source can be one of two types: magnetic tape or random voltage generator. Magnetic tape, containing the recorded flight vibration data, permits laboratory reproduction of actual flight vibrations. A random voltage generator is used when the testing is done according to a mean square acceleration density spectrum. The spectrum may be representative of flight data from several flights or from several aircraft. Figure 6-10 is a photograph of a random vibration test system capable of a maximum peak instantaneous force of 2700 pounds.

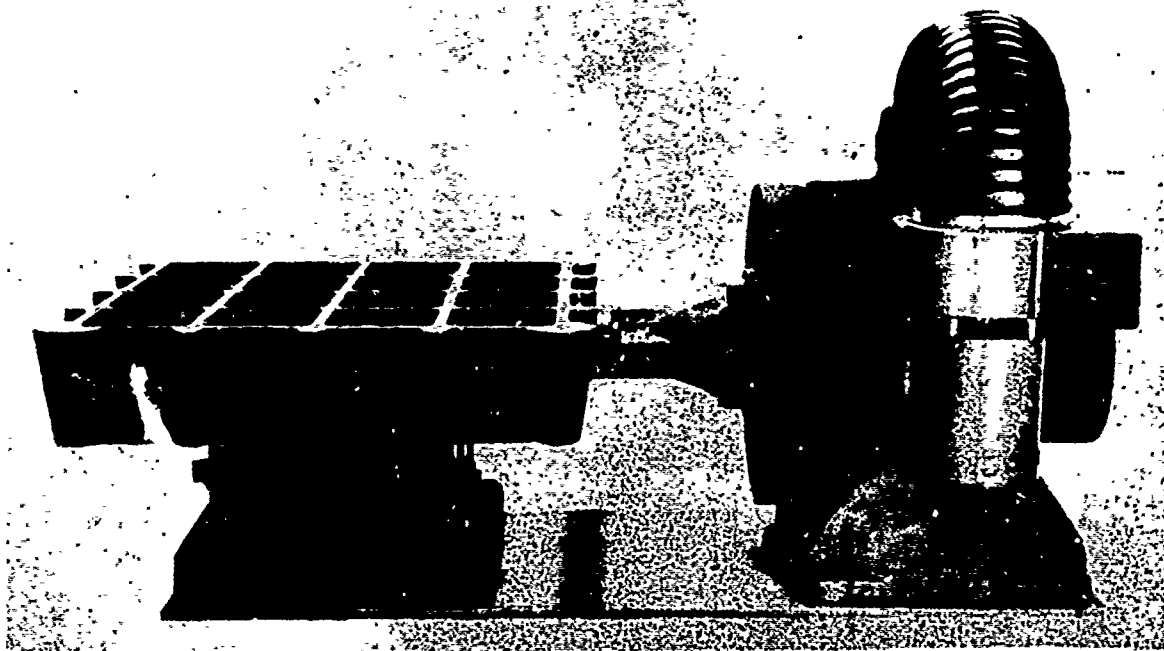


Figure 6-9. Electrodynamic Shaker and Auxiliary Table

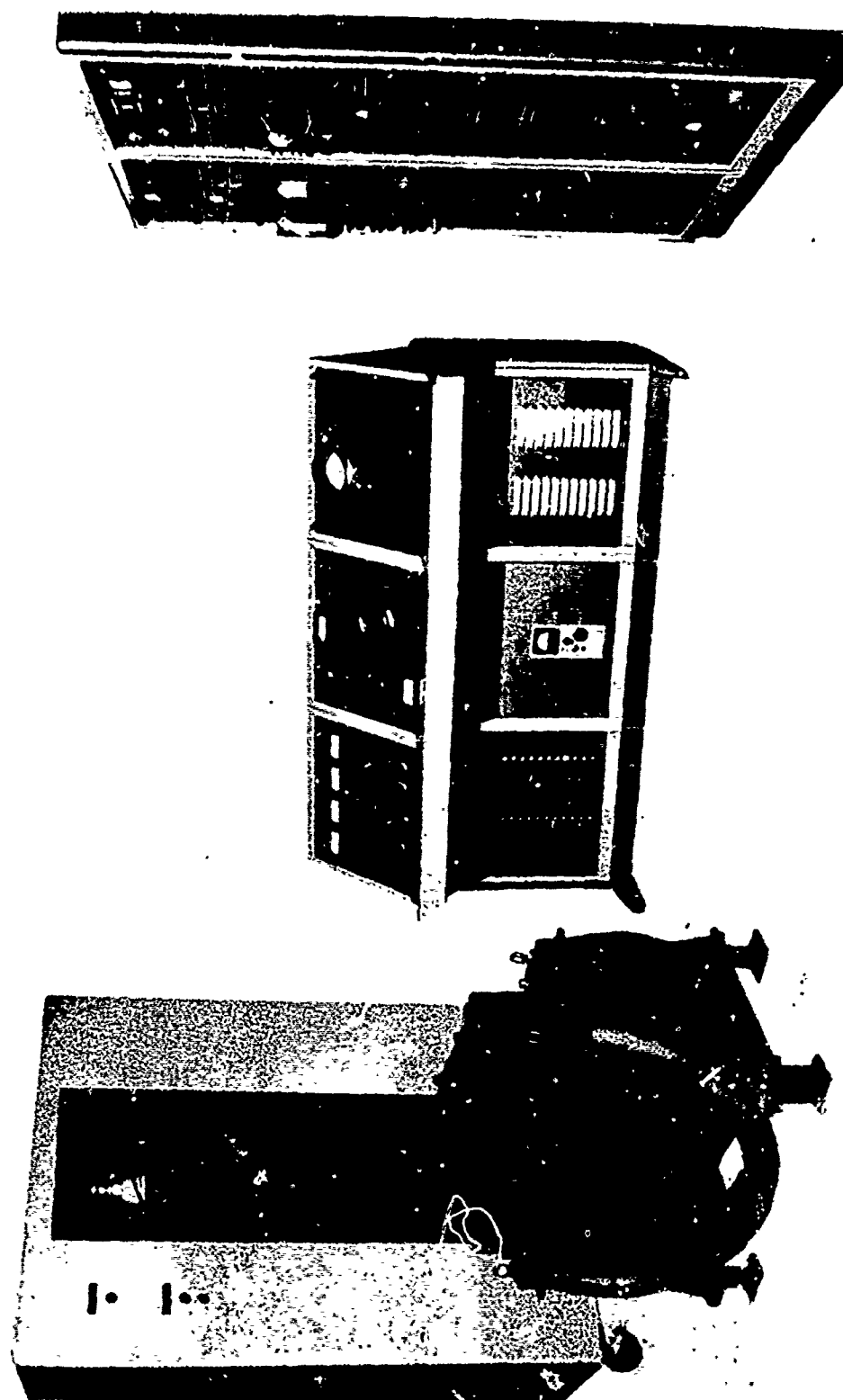


Figure 6-10. Random Vibration Test System

6-42. Whichever type of signal source is used, the equalization circuits are an important part of the complex motion system. It is characteristic of an electrodynamic shaker that, even with no load, the ratio of table acceleration to input voltage varies with frequency. An exciter equalizer adjusts the voltage supplied to the exciter to compensate for this gain variation. The exciter gain curve, and hence the necessary equalization, varies with the load on the table. Also, with a non-rigid load, the gain of the exciter will be affected by the response characteristics of the load. It is customary to use peak-notch equalizers, one for each major resonance, to compensate for gain variations due to resonances within the test item.

6-43. **HYDRAULIC VIBRATION SIMULATOR.** Among the most recent developments in vibration testing equipment is the hydraulically actuated vibrator. It promises greater force outputs than other types of comparative size. The table of the hydraulic vibrator is driven by a piston which is subjected to varying hydraulic pressures. Since the piston-table assembly has relatively low mass, most of the force is available for driving the test specimen. One commercial machine, using servo valves to control the hydraulic pressure, produces a total force output of 20,000 pounds at frequencies up to 2000 cps with a table-piston assembly which weighs only 16 pounds.

6-44. **POSITIVE-DRIVE VIBRATION SIMULATOR.** The positive-drive (mechanical) vibration simulator is a relatively simple device in which motor-driven rotating eccentrics or combinations of eccentrics impart a reversing motion to a mounting platform and test specimen, at a predetermined frequency and amplitude. The majority of machines of this type produce vibration in a single direction: either vertical or horizontal; however, there are machines that produce vibration in two or three directions, either separately or in combination.

6-45. Because the machine frame and foundation must resist the reactive forces of the vibrating test specimen, positive-drive simulators are limited as to the weight of the specimen they can test and the frequency of vibration at which they can test. Positive-drive simulators are now usually designed to test light weights at relatively low frequencies (up to 100 cps), and thus are not suitable for simulating airborne environments.

6-46. **REACTION-TYPE VIBRATION SIMULATOR.** The most basic form of the reaction-type vibration simulator consists of an isolation mounted table which carries a rotating eccentric mass. As the shaft is rotated, the reaction of the table to the rotating unbalanced mass causes the table to describe a circular motion. By gearing two eccentric masses together, rotating them in opposite directions, and properly phasing them, the circular motion will be changed to a vertical reversing motion. By changing the phasing 90 degrees, horizontal reversing motion can be attained. The two-dimensional vibrating machine eliminates the need for additional mounting fixtures. The third direction of vibration (second horizontal direction) is secured by rotating the test specimen 90 degrees on the table.

6-47. The capacity of the reaction-type machines is rated in terms of the weight of the test load. The smallest machines have a load rating of 50 pounds and the largest 10,000 pounds. The majority of these simulators are rated at a maximum acceleration of 10g; some are available with acceleration ratings as high as 20g. Table sizes range from 1-1/2 to 8 feet. A disadvantage to this type of machine is

that the frequency range is limited to approximately 100 cps. An advantage is that the reaction principle eliminates the need for costly, massive foundations to absorb the reactive forces such as are necessary with the positive-drive and electrodynamic types. However, the frequency limitations of the reaction-type machines, as with the positive-drive type machines, makes them unsuitable for testing over the frequency range encountered in airborne environments.

**6-48. MISCELLANEOUS TYPES OF VIBRATION SIMULATORS.** Package testers or bounce testers are used to test the ability of crated and packed equipment to withstand the vibration hazards of transportation. They are designed to test comparatively high loads and simulate large vibration amplitudes at low frequencies. They are usually a positive-drive type of vibration simulator.

**6-49.** Other types of vibration simulators use pneumatic means for generating vibratory forces. One type of machine shakes a freely suspended table structure using the principle of a pneumatic hammer. The frequency of vibration can be varied within limits by varying the air pressure, but no means is provided for varying the amplitude except by adding weights to the table or changing the piston to one of different mass.

**6-50.** A second pneumatic type uses compressed air to spin a steel ball around a circular race. Frequencies as high as 300 cps have been obtained. It is a constant-amplitude machine because there is no provision for changing the mass of the spinning ball nor its radius of gyration. Attempts to produce linear vibrations by synchronizing and phasing two of these devices have proved unsuccessful.

#### **6-51. NOISE SIMULATORS.**

**6-52.** Simulative acoustic energy can be generated in a number of different ways. A jet engine can be used which would provide the most realistic simulation. However, it is a time-consuming and expensive method, necessitating extensive testing facilities. Sirens are frequently used as laboratory noise simulators to produce high-intensity sound fields. The siren, though, is limited in its testing application because, basically, it is a single-frequency device. Installations can use sirens and control them over a range of cycling speeds and frequencies. Probably the most frequently used noise simulator is the loudspeaker, which is relatively inexpensive and covers a wide band of frequencies; however, it is limited in its power output. Air-modulated speakers show promise of providing adequate power over a wide band of frequencies. Further development should result in their acceptance as a noise-simulating device.

**6-53.** Noise simulating devices are used in a variety of enclosures designed to provide various types of acoustic excitation. The noise simulators may be used for progressive wave testing, in reverberant enclosures, or in resonant chambers. Each offers special advantages depending upon the types of acoustic excitation desired, the size of the test object, the range of frequencies in which the test item needs testing, and the sound pressure levels desired. Since the noise simulators can be used in any of the enclosures, the discussion in the following paragraphs is based upon the type of testing rather than the type of noise simulator.

**6-54. PROGRESSIVE WAVE TESTING.** Progressive wave testing may be accomplished either in a free-field or in a plane-wave tube. In free-field acoustic

excitation, the wave fronts are spherical and the sound intensity varies inversely as the square of the distance from the source. Any acoustic excitation that contains these ingredients can be defined as free field. If a point-source noise radiates equally in all directions into free space, it would be free-field excitation. Likewise, if a test facility were designed to approximate these conditions (for example, no walls or sound-absorbent walls), it could be considered to be free-field excitation.

6-55. Free-field testing probably more closely approximates the acoustic environment to which aircraft exterior surfaces are exposed, but it may not be typical of the environment in which electronic equipment will operate. If it were possible to expose a complete aircraft compartment with electronic equipment mounted inside it to free-field acoustic excitation approximately 15 db higher than that desired at the equipment to account for wall attenuation, a test procedure closest to actual conditions could be obtained. This arrangement, however, would be expensive.

6-56. Plane-wave tubes confine the generated acoustic energy within the walls of a tube and are designed so that no energy reflections occur. This approximates free-field acoustic excitation, but, since the energy is confined, much less power is required to produce high-level sound pressures. There is a frequency limitation, however, as the plane waves will occur only when the tube diameter is small compared with the wavelength.

6-57. A recently designed test facility which provides either progressive wave or reverberant (see 6-58) testing is shown schematically in figures 6-11 and 6-12. The sound source consists of two sirens, one low frequency (50-2000 cps) and one high frequency (500-10,000 cps), which may be used independently or in combination. The sirens are coupled to a one-square-foot duct. The duct feeds into an exponential horn which, in turn, feeds into a reverberant chamber. For progressive wave testing the reverberant chamber is internally draped with sound absorbent material to provide an anechoic termination. Progressive wave testing is performed in the duct near the throat of the horn. The facility is designed to provide up to 174 db in the one-square-foot duct.

6-58. REVERBERANT CHAMBERS. Generating high sound intensities in free space or in plane-wave tubes requires considerable acoustic power because all power is absorbed or dispersed and no reflections occur. A sound field of 160 db represents an intensity of 1 watt per square centimeter or 1 kilowatt per square foot; thus 1 kilowatt of acoustical power is required for each square foot of wave front. If an enclosure is used with walls of low absorption (a reverberant chamber), only a small amount of acoustic energy is absorbed per unit area on each reflection, resulting in a buildup of pressure inside the box. Thus, considerably less acoustic power would be required to generate a 160-db sound-pressure level in a reverberant chamber. Reverberant chambers, however, have limitations at low frequencies. As the wavelength becomes larger (with decreased frequency) and approaches the dimensions of the chamber, standing waves become likely.

6-59. The sound field produced in a reverberant box probably more closely resembles that which an equipment will be exposed to in operation, especially if the chamber were made to duplicate the actual compartment. Figure 6-13 shows a

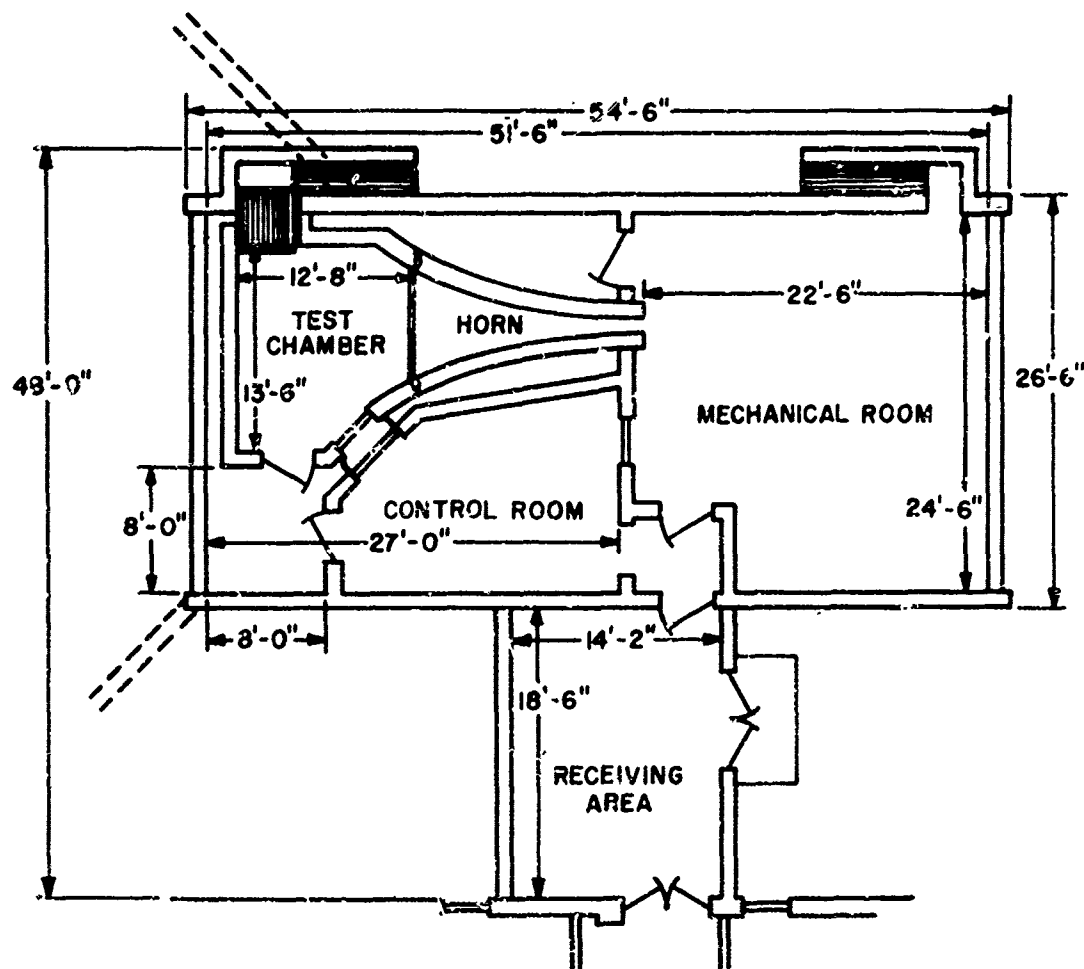
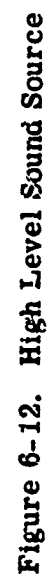


Figure 6-11. High Level Sound Facility

reverberant chamber which is energized by four loudspeakers mounted on exponential connectors. The maximum power-handling capacity of the four units is sufficient to produce a sound pressure level of 157 db. The shape of the enclosure, as is indicated in figure 6-13, is unsymmetrical.

6-60. Reverberant chambers are irregularly shaped in order to minimize the distortion of the original noise spectrum. The walls of an irregularly shaped enclosure have more complex modes of vibration and more natural frequencies. In addition, each wall, being dimensionally different, has frequencies different from those of the other walls. Since the amount of energy absorbed by a wall depends on the relation of its natural frequency to the frequency of the sound being reflected, the multiplicity of wall natural frequencies lessens the possibility of absorption occurring at preferred frequencies. Unsymmetrical aiming of the drivers gives more irregular reflection angles and hence gives a more uniform pressure level throughout the chamber.

6-61. **RESONANT CHAMBERS (STANDING-WAVE TUBES).** Resonant chambers, also referred to as standing-wave tubes, are the most efficient enclosures for producing high sound pressures. Sound pressure levels as high as 185 db have been



obtained. However, resonant chambers operate only at single frequencies or in narrow frequency bands and are not suitable for wideband testing. Also, from a physical damage standpoint, standing waves are not as severe as progressive waves. These two facts rule against the use of standing-wave tubes for testing electronic components. They are useful primarily in testing microphones.

6-62. MISCELLANEOUS NOISE SIMULATORS. RCA obtains 145 db overall intensity in their test chamber over a band of frequencies from 30 to 10,000 cps. The enclosure, shown in figure 6-14, is 7 feet high, 3 feet wide, and 1-1/2 feet deep. The excitation is partially reverberant; but, because of the number of speakers (48) on the walls and ceiling, the reflecting surfaces necessary for a reverberant chamber do not exist. There are 30 low-frequency speakers in the sides and 18 high-frequency speakers in the ceiling.

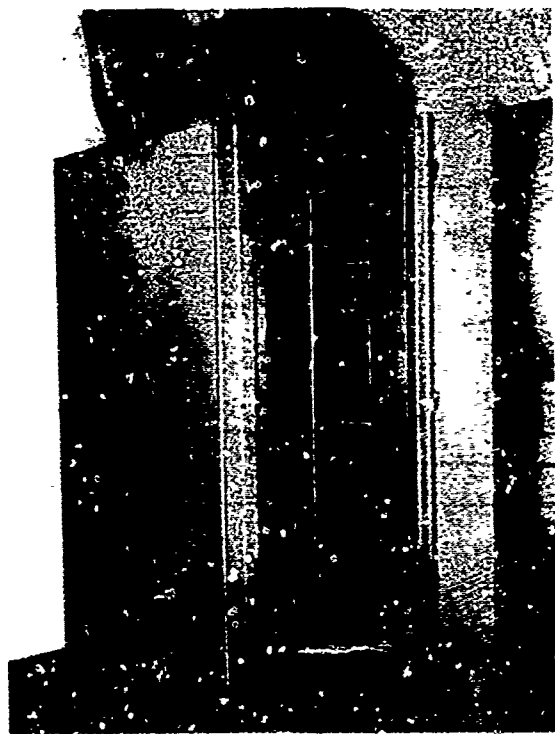


Figure 6-14. RCA Sound Chamber

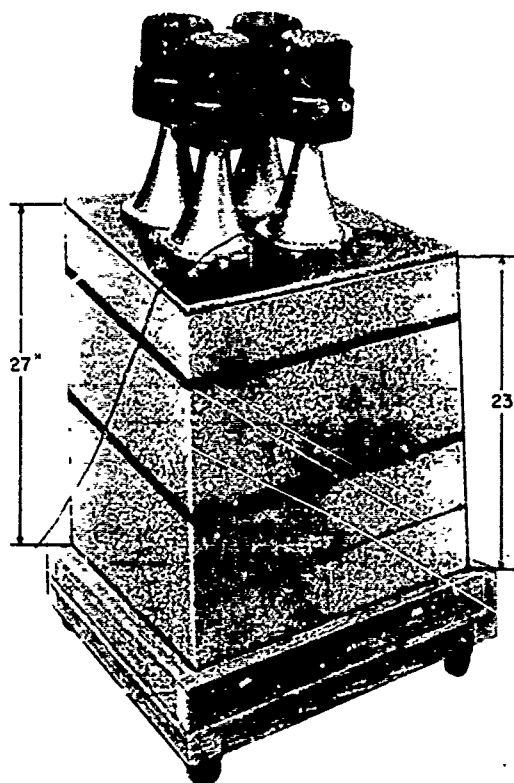


Figure 6-13. Reverberant Chamber with Four Drivers

Forty-eight 70-watt power amplifiers are used for the speakers.

6-63. As discussed previously, using jet engines specifically as noise sources is impractical, but this test setup is sometimes used during routine jet engine testing. The electronic equipment is simply mounted in the test cell and exposed to the random noise. The test cell can be instrumented to determine sound pressure levels and the noise can be recorded for wave analysis.

6-64. RCA is currently investigating the possibilities of the air-modulated speaker (or more correctly, moving-coil-modulated, compressed-air loud-speaker). This speaker is capable of producing high sound levels with a relatively small electrical power input. The main source of input energy is compressed air (as in a siren); however,

since the modulation is produced by a moving-coil rather than by a constant speed chopper, the air-modulated speaker is capable of reproducing complex sound. As with electrodynamic loudspeakers, two air-modulated speakers, for high and low frequency ranges, are required for wide-band sound.

#### 6-65. MICROPHONES FOR MEASURING HIGH-INTENSITY SOUND.

6-66. Several types of microphones may be used for measuring high-intensity sound, and each has advantages for particular circumstances. Basically, the principles of operation are the same as for conventional microphones; however, they are specially designed to have linear response at high sound levels.

6-67. The two most commonly used types are the condenser and piezoelectric types. The condenser microphone indicates sound pressure levels by the variations in separation between a stretched diaphragm and a parallel rigid plate. The separation variations, caused by the sound waves impinging on the diaphragm, change the electrical capacitance between the two plates. A voltage, either ac or dc, applied to the capacitor plates is modulated by the sound wave.

6-68. The piezoelectric type of microphone, either crystal or ceramic, operates in the same manner as the shock and vibration pickup in that the deformation of the crystal (or ceramic unit) generates a voltage. Sound waves excite a diaphragm which causes the crystal deformation. The materials used are barium titanate, zirconate, and ammonium dihydrogen phosphate (ADP).

6-69. Other sound pressure transducers used are electrokinetic and strain gage types. The electrokinetic transducer contains a homopolar fluid which is forced through a porous membrane to generate a voltage. Again the force is supplied by a diaphragm excited by sound waves. The strain gage type operates in the same manner as the shock and vibration pickups (see paragraph 6-103) except for the exciting force.

#### 6-70. SHOCK AND VIBRATION PICKUPS.

6-71. Instrumentation is necessary to determine the shock and vibration environments that must be simulated, and also to indicate whether the simulating equipment is performing the task. Shock and vibration pickups are the first link in the instrumentation since they detect the motion of a vibrating or shocked equipment.

6-72. Shock and vibration pickups are of three general types: optical, electrical, and mechanical. The optical types are relatively delicate and not suitable for field measurements. They are, however, useful as references because they require no calibration and are frequently used to calibrate electrical pickups. Electrical pickups are the most versatile type, useful both in the laboratory and in the field. Mechanical pickups are large and comparatively heavy and thus are not adaptable to testing smaller components. Their weight would affect the natural frequencies of all but the large equipments.

6-73. There is little distinction between pickups for recording shock and those for recording vibration, except that the former must be designed to handle greater accelerations and displacements. In general, only accelerometers are adaptable

to measuring shock.

#### 6-74. OPTICAL PICKUPS.

6-75. Optical pickups are magnifying systems which focus on a point arbitrarily set on a shaker table or on an equipment. A vibratory motion along one axis causes the moving point to appear as a straight line in the lens of the pickup. The vibratory excursion can be measured by (1) observing the length of the line on the instrument's reticle, (2) moving the optical instrument so as to sight at each extreme of the line and recording the amount of movement, and (3) recording the angle through which a prism or mirror, located inside the optical instrument, must be turned in order to sight at each extreme of the point's excursion. Figure 6-15 shows a collimator which is one type of optical pickup.

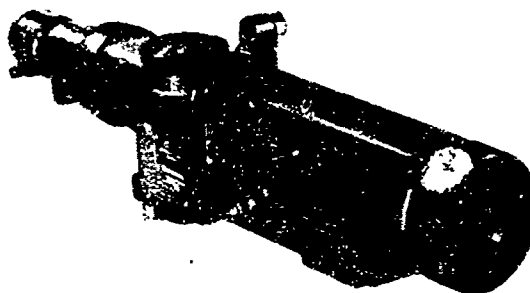


Figure 6-15. A Collimator

6-76. A stroboscopic light source can be used in conjunction with an optical pickup to flash its intermittent light on the table and equipment so that they appear to be barely moving. An optical pickup can then sight on any equipment component for each extreme of the vibration excursion. The strobe light can also be used for determining the frequency of vibration. With the frequency and displacement known, the other parameters of velocity and acceleration can be calculated, provided the waveform is known.

6-77. The major advantages of the optical pickup are: (1) it is unnecessary to calibrate this instrument and thus it has no calibration to lose; (2) mounting techniques are not a factor as it is with pickups mounted on the vibrating object; and (3) because nothing need be mounted on the test item, there is no added weight to affect the vibration characteristics of the test item.

6-78. The disadvantages of the optical pickup are: (1) it is not a versatile instrument, being limited to laboratory usage; and (2) it cannot measure small amplitudes that occur at high frequencies.

#### 6-79. ELECTRICAL PICKUPS.

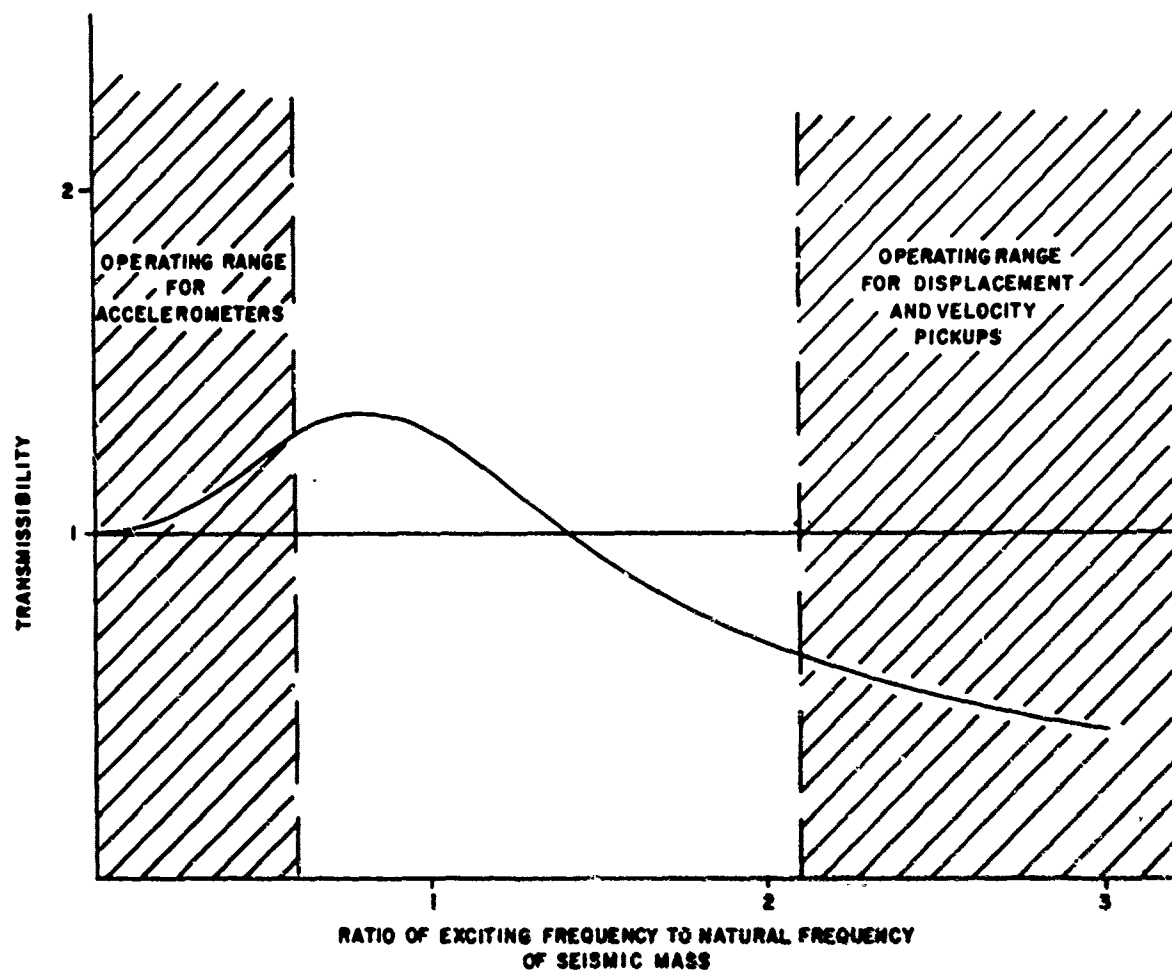
6-80. The most commonly used commercial vibration pickup is one which transmits the vibration information in the form of an electrical signal. The electrical signal is usually dependent upon the motion of a mass suspended elastically in a case. A description of the relative motion between mass and case is made in terms of displacement, velocity, or acceleration. However, any one term can be derived from either of the other two terms by one or two differentiations or integrations, provided the frequency and waveform are known. For simple harmonic motions, the maximum acceleration in g's is approximately  $1/10f^2x$ , where  $f$  is the frequency in cps and  $x$  is the single amplitude in inches.

6-81. Electrical pickups are divided into three general categories: the displacement pickup, the velocity pickup, and the accelerometer. The displacement pickup

measures vibrations occurring at frequencies above its own natural frequency where the relative displacement of the mass is almost equal to the applied displacement. It usually requires an electrical input for operation. The velocity pickup is similar to a displacement instrument except that it requires no electrical input, but instead operates as an electromagnetic generator. The displacement is electrically differentiated so that the output is proportional to the relative velocity of the mass. The accelerometer measures vibrations occurring at frequencies below its own natural frequency. Thus, there is no isolation of the suspended mass as occurs in the displacement and velocity pickups, and the excursion of the mass is essentially the same as that of the case.

**6-82. DISPLACEMENT PICKUPS.** A displacement pickup measures variations in the position (displacement) of a test item in any one axis. Its operation is dependent upon the displacement of the instrument's case relative to a reference. The reference may be a mass, isolation mounted within the pickup, or an object which is fixed by a connection to the earth.

**6-83.** Displacement pickups with internal reference masses mount these masses on soft springs. The natural frequency of the elastically mounted mass upon its



**Figure 6-18. Transmissibility Curve Showing Operating Ranges for Displacement Pickups, Velocity Pickups, and Accelerometers**

mounting thus is quite low, and the ratio of exciting frequency to natural frequency should be high. The system's operation, therefore, is described in the right-hand portion of the transmissibility curve (figure 6-16) called the "isolation region." Here, the mass stands still while the case moves around it, and the relative motion between mass and case is proportional to the absolute motion of the case.

6-84. The relative displacement between the mass and the pickup case can be determined through the use of a differential transformer and an electrical input. The transformer core serves as the mass and as the core moves, it changes the flux linkage between the primary and secondary windings of the transformer (figure 6-17) and, therefore, varies the transformer output. Another sensing method uses a coil as the elastically suspended mass. The coil remains effectively stationary in a moving a-c field. The voltage induced in the coil is a function of its position in the a-c field, and the output of the coil will be the a-c carrier modulated by the vibration signal.

6-85. The displacement meter must be large enough to allow for the relative motion between the mass and the case. If this is inadequate, the mass will strike the stops or snubbers in the case, causing incorrect readings or even damaging the pickup. The pickups, therefore, tend to be comparatively large and heavy and are not used where the size or weight of the pickup would affect the characteristics of the system being monitored. This type of displacement pickup is also limited in usable frequency range by the natural frequency of the mass on its mountings. The frequency being measured must be well above this natural frequency if a flat response is to be obtained.

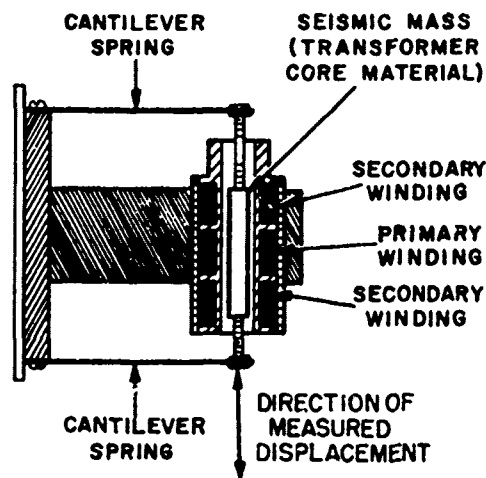


Figure 6-17. Cross Section of Differential-Transformer-Type Displacement Pickup

6-86. One type of fixed-reference displacement pickup uses a crystal element to generate a voltage proportional to the displacement. The crystal element is actuated by a drive pin which presses against the vibrating body. The pickup is mounted, or it can be held by hand, so that the drive pin maintains contact with the vibrating object through the vibration cycle. A fixed stop is usually provided to protect the crystal from damage. This type of displacement pickup is not limited in its lower usable frequency by the natural frequency of a suspended mass. The only low-frequency limitations are the leakage characteristics of the crystal and attending circuitry. Also, it does not appreciably affect the vibrating characteristics of the test item. A disadvantage of this type pickup is its limited versatility. For example, it cannot be used in aircraft because it needs an earth reference.

6-87. Other fixed-reference displacement pickups use an electric or magnetic field as the earth-connected reference. The three major types are the mutual inductance pickup, the microwave system pickup, and the capacitance pickup.

6-88. The mutual inductance displacement pickup consists of two coils held stationary close to a conductive surface on the vibrating object. (The mutual inductance pickup cannot be used if the conductive surface is ferromagnetic.) As the conducting surface moves toward and away from the two coils, their mutual inductance varies. With one coil energized, the voltage output from the other will vary with the vibratory motion of the conductive surface.

6-89. The microwave system uses a transmitter and receiver operating in the microwave frequency band. When placed close to a vibrating surface, the reflected transmitter power output will vary as the reflecting surfaces vibrate, and these variations are detected by the receiver.

6-90. The capacitance pickup uses a stationary probe surface placed close to the vibrating surface and a means of detecting changes in capacitance as the distance between the surfaces varies during vibration. The chief disadvantage of capacitance pickups is in achieving linearity and sensitivity with small probe size. The capacitance of the sensing element is often no greater than the leads connecting to it and since the two capacitances are in parallel, the effective change is not great if the probe is small and the leads are fairly long.

6-91. The advantages of using pickups employing an electrical or magnetic field as the stationary reference are: (1) there are no moving pickup parts, thereby eliminating troublesome pickup resonances or modes of vibration; (2) the pickup does not mechanically load the test item and thus does not affect its vibrating characteristics; and (3) minimum pickup lateral sensitivity provides an output relatively free of distortion from lateral vibration components.

6-92. Disadvantages are: (1) there is a lack of pickup versatility because of the need of laboratory facilities during testing; and (2) the vibrating surface detected by the probe must be a reasonable geometry, highly conductive, and, for mutual inductance pickups, not ferromagnetic.

6-93. **VELOCITY PICKUPS.** Velocity pickups measure the instantaneous velocity of a vibrating object. They do so by acting as tiny electromagnetic generators whose electrical output is proportional to the instantaneous velocity of the vibrating object.

6-94. The velocity pickup, like the internal-reference type of displacement pickup, contains a mass mounted elastically on soft springs so that the system (springs and mass) has a low natural frequency. The transmissibility of the vibratory motions from case to mass, therefore, is described in the "isolation region" of the transmissibility curve (see figure 6-16), and the mass effectively remains motionless while the case vibrates around it. Damping, viscous or magnetic, is used in velocity pickups to lower the usable frequency range and to prevent pickup damage due to overexcursion.

6-95. There are various designs for using the relative motion of case and mass to generate a signal. One model pickup, for example, uses a hollow cylinder magnet as the elastically mounted mass. The coil, fixed to the case, moves within the magnet (figure 6-18) to generate a signal. Another model uses a coil as the mass and the magnets are fixed to the case (figure 6-19). The coil is elastically mounted on the end of a pivoted shaft.

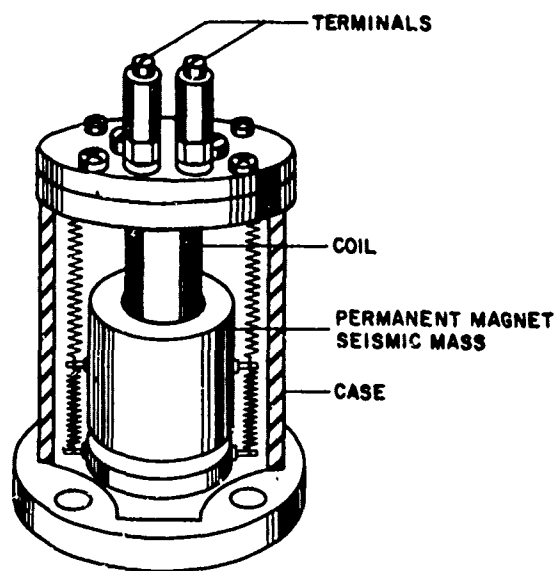


Figure 6-18. Component Parts of Velocity Pickup

6-96. Velocity pickups are versatile instruments because they require no electrical input and are not sensitive to temperature or humidity. They also have low impedance and high output. Their limitations are the same as those for seismic mass displacement pickups. They must necessarily be large to allow for movement between mass and case. They are also comparatively heavy, and thus cannot be used where the natural characteristics of the test item would be altered by the additional weight.

6-97. **ACCELERATION PICKUPS.** Acceleration pickups, also called accelerometers, measure the accelerations of a vibrating system. Like the displacement and velocity type pickups, the accelerometer employs an elastically suspended mass as part of its sensing system. Here, however, the similarity

ceases. The accelerometer mounts the mass with stiff springs, and the system (springs and mass) has a high natural frequency. This natural frequency must be well above the frequency of the vibrating object. Referring again to figure 6-16, the left portion of the transmissibility curve is now applicable, and the motion of the mass is almost equal to the motion of the case.

6-98. The accelerometer is mounted on the vibrating shaker table or equipment. A sensing element within the pickup detects the force exerted on the suspended mass and converts it into an electrical signal. Since transmissibility between the suspended mass and case is, for practical purposes, one, and force is proportional to acceleration, the electrical signal (when properly calibrated) is a measure of case (and vibrating object) acceleration.

6-99. Damping is usually necessary in accelerometers. It is used to prevent damage to the pickup and to extend the usable frequency range. Fluid is the usual damping medium. Some types are magnetically damped.

6-100. There are various types of sensing elements used to detect the forces on the suspended mass. The most common are (1) variable inductance, (2) wire strain gage, and (3) piezoelectric.

6-101. **VARIABLE-INDUCTANCE ACCELEROMETER.** The variable-inductance accelerometer consists of a transformer with the suspended mass as part

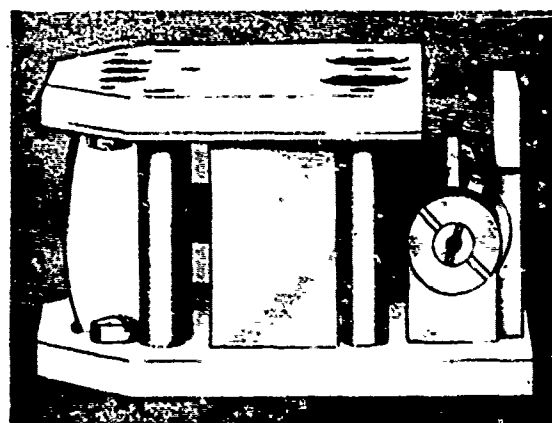


Figure 6-19. Velocity Pickup

of the magnetic circuit. This type is not self-generating but requires an electrical input with a carrier frequency of a few thousand cycles. A slight movement of the suspended mass due to pickup acceleration changes the flux linkage between the primary and secondary windings. This change is detectable and convertible to a measure of acceleration.

6-102. This type of accelerometer is useful only to approximately 100 cps. If the natural frequency of the suspended mass and springs is raised too high, the sensitivity (output per unit of acceleration) is decreased. Allowance for static acceleration is also necessary. Thus, if the test item will be subjected to 10g normal (static) acceleration and 10g peak vibration acceleration, it will be necessary to use a 20g accelerometer with only one-half the range available for testing.

6-103. **STRAIN-GAGE ACCELEROMETERS.** Strain-gage accelerometers use resistance-wire strain gages as the sensing elements. A mass, suspended by either the resistance-wire gages directly or by cantilevers to which the resistance-wire gages are cemented, provides the force which changes the wire resistance. The resistance change, proportional to the inertial force of the mass, which in turn is proportional to the acceleration of the accelerometer (and test item), is usually measured by a Wheatstone bridge circuit.

6-104. The frequency range limitation of the variable-inductance accelerometer is also applicable to the strain-gage accelerometer. Natural frequencies range from 40 cps to 200 cps. This type of accelerometer is also subject to static accelerations, and, in choosing the proper instrument, the required range is determined by the sum of the static and dynamic accelerations.

6-105. **CRYSTAL (PIEZOELECTRIC) ACCELEROMETERS.** Piezoelectric crystals generate electric charges on their surfaces when placed under stress. In the crystal accelerometer, one or more crystals are mounted so that acceleration forces produce the stress. The quantity of the charge induced is proportional to the applied stress, which, in turn, is proportional to the acceleration.

6-106. Structurally, there are two types of crystal accelerometers: the compression and the bimorph (bender) types. The compression type consists essentially of a cylindrical assembly in which a mass, under acceleration, compresses one or more disc-shaped crystals. The crystal faces are coated with a conducting material and are connected either in series or in parallel. These accelerometers (figure 6-20) generally have relatively high impedances, high natural frequencies, and high ranges.

6-107. The bimorph or bender accelerometer contains two thin crystal slabs each fastened to either side of a metallic shim. Both sides of both crystals are coated with a conducting material. The crystals are oriented so as to be connected in series or in parallel. The crystal slabs may be center-supported or end-supported (figure 6-21). Another type consists of square slabs fastened at three corners, with the fourth

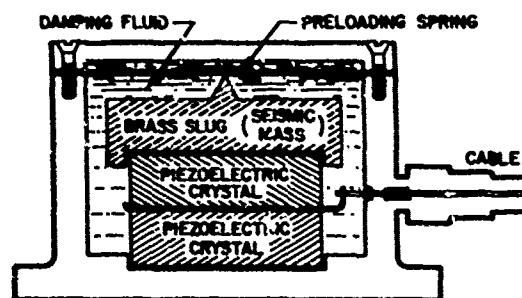


Figure 6-20. Cross Section of Compression-Type Crystal Accelerometer

corner free to move under acceleration forces. In general, bimorph crystal accelerometers have a lower impedance, a lower natural frequency, and a smaller g range than the compression type.

6-108. The crystal accelerometer uses many types of crystals, the most important being quartz, Rochelle salt, ammonium dihydrogen phosphate (ADP), and barium titanate (one of a group of electrically polarized ceramics). Quartz has a relatively low sensitivity. For example, one model quartz accelerometer has an output of 0.07 millivolt (rms) for each g of acceleration with a range of 600 g and a natural frequency of about 4000 cps. Despite its low sensitivity, quartz has advantages of low leakage and relative insensitivity to temperature changes and shock. The low leakage characteristic, which maintains electric charges on each surface of the crystal longer than the synthetic crystals (such as Rochelle salt), permits detection of lower frequencies of vibration than is possible with crystal accelerometers using other types of crystals.

6-109. Rochelle salt (sodium potassium tartrate), a synthetic crystal, when properly cut is one of the most sensitive of all piezoelectric materials. For example, some models of Rochelle salt accelerometers produce an output of about 150 millivolts (rms) for each g of acceleration at 500 cps. However, this type of crystal has limitations. Because of its sensitivity to humidity, it must be enclosed in a hermetically sealed container. It is also sensitive to temperature changes; starting at 70°F, the output of a Rochelle salt accelerometer was found to drop about 2 percent for each degree of temperature change, either increasing or decreasing, and permanent damage to the crystal can result from heat as low as 120°F. Rochelle salt is also likely to fracture with high mechanical strain.

6-110. Ammonium dihydrogen phosphate (ADP) is one of the more recently developed crystals. It is less sensitive than Rochelle salt; for example, at 500 cps it has an output of about 30 millivolts (rms) for each g of acceleration. It is much less sensitive to temperature than Rochelle salt. It has a high leakage characteristic, however, which limits its use at low frequencies. In addition, the ADP pick-up is more expensive than accelerometers using other crystals.

6-111. Barium titanate, unlike the previously discussed crystals, is a synthetic ceramic. This material has some advantages over crystals when used as accelerometer sensing elements. It can be readily produced in various shapes, thus

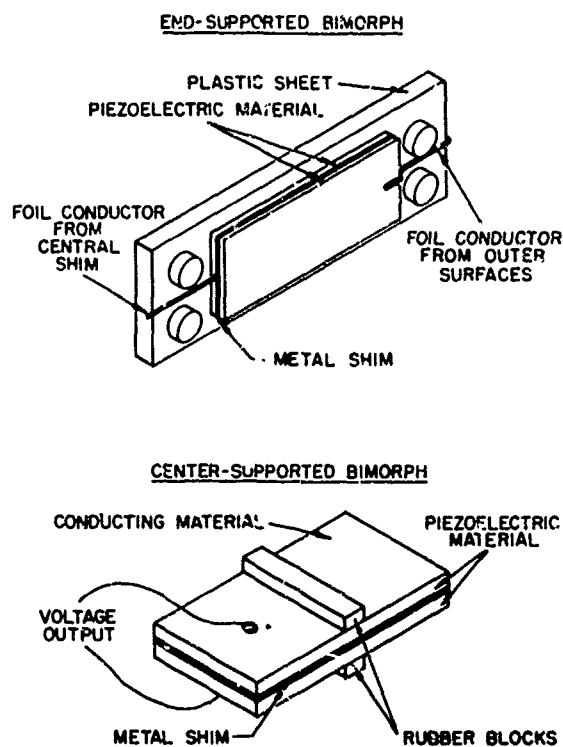


Figure 6-21. End-Supported and Center-Supported Bimorph Crystal Accelerometer Elements

making it adaptable to many different pickup designs. It has relatively good high-frequency response and is less subject to changes in sensitivity due to aging. Barium titanate is also nonhygroscopic, less temperature-sensitive than Rochelle salt, and higher in resistivity than ADP. A typical barium titanate compression-type accelerometer has an open-circuit output voltage of 10 millivolts (rms) for each g of acceleration. Starting at 80°F, the output typically drops only 0.1 percent for each degree of temperature change, either increasing or decreasing.

6-112. Accelerometers of the piezoelectric type are all high-impedance devices and, therefore, are usually coupled to an amplifier by a cathode follower for impedance matching. Also, the high impedance makes any slight impedance variation a critical item. For example, noise voltages, generated by the flexing of the coaxial cable connecting pickup and amplifier, can affect the output signal (see paragraph 3-17).

#### 6-113. MECHANICAL PICKUPS.

6-114. Mechanical pickups are, generally, not applicable to airborne environments or electronic equipment. They are bulky and heavy and suitable only for shock and vibration testing of large components. The upper frequency is limited to about 100 cps because of inertial forces on mechanical linkages. Mostly they are mechanical or combined mechanical and optical devices. The two major types of mechanical vibration pickups are vibrometers and vibrographs. The only distinction between the vibrometer and the vibrograph is that the vibrograph has a permanent recording mechanism.

6-115. A simple and useful mechanical pickup for indicating vibration accelerations of 1g is the chatter accelerometer. It consists of a small ball which rests on a platform. As vibration accelerates the platform downward at less than 1g, the ball will remain in contact with the platform. When the acceleration exceeds 1g, however, the ball will lose contact with the platform momentarily, striking the platform later in the cycle. The series of separations and contacts causes a chattering noise which indicates vibrations with accelerations above 1g. The chatter accelerometer can be used to calibrate vibration machines and accelerometers at the 1g level.

6-116. Another type of mechanical pickup is the reed instrument. The reed tachometer, for example, consists of a number of reeds, with different natural frequencies, mounted on a common housing. When the instrument is exposed to vibration, the response of each reed depends upon the relationship of its natural frequency to the frequency of the vibration. The reed showing the greatest response indicates, by its own natural frequency, the predominant frequency of the environment.

6-117. The reed gage is a simple and inexpensive mechanical sensing device which records shock spectra. The gage consists of a number of single-degree-of-freedom cantilever structures of graduated natural frequencies. The peak motions at the ends of the cantilever reeds, when the gage is subjected to a shock, portray the shock spectrum. The motion of each reed is recorded by a stylus, attached to its end, on either a detachable piece of stainless steel or, if more than one shock is involved, on a strip of waxed paper which is slowly moved under the styli. The reed gage is a relatively large and heavy instrument. Because of its weight and

size, the number of its reeds is usually limited, and hence the amount of information it provides is usually restricted to a few frequencies. Also, the scratches made by high-frequency reeds are small and difficult to read.

#### 6-118. RECORDING VIBRATION AND SHOCK DATA.

6-119. There are various means of recording vibration and shock data. The method selected depends upon the needs of the particular test situation. For example, to record transient phenomena, such as shock data, an oscilloscope and/or a camera can be used. A vibration meter may be most suitable in a field application. For a permanent record of steady-state vibration, an oscillograph could be chosen. Magnetic tape may be regarded as the best medium for eventual analysis of random vibration.

6-120. OSCILLOSCOPES. The oscilloscope can be used in conjunction with displacement, velocity, or acceleration pickups. An amplifier is used between the pickup and oscilloscope, and if the pickup is an accelerometer a cathode follower is used for impedance matching.

6-121. For steady-state vibration, both frequency and amplitude\* are indicated on the oscilloscope. When transient phenomena such as shock are being observed, the trace signal can be photographed. A calibration signal is usually fed into the oscilloscope to supply a reference height.

6-122. OSCILLOGRAPHS. Oscillographs provide the same shock and vibration measurements as an oscilloscope; however, instead of the signal being observed on a cathode-ray tube, it is recorded on moving paper to provide a permanent record of the wave shape, frequency, and amplitude. Also, whereas an oscilloscope is usually restricted to one or two channels, an oscillograph can have as many as 50 channels recording simultaneously. There are two types of oscillographs: one mechanically writes on a moving paper; the other uses a light source which reacts on sensitized paper. The direct-writing type of instrument is limited in its frequency range to approximately 100 cps because the inertia of the linkage system is a limiting factor.

6-123. The photographic-line type of recorder will record at frequencies up to 3000 cps. It uses an optical system in which a light source is used in conjunction with mirrors linked to galvanometers. When the voltage input varies, the reflecting angle of the mirror changes, and the changes in direction of the beam of light result in a signal indication on the moving sensitized paper. Figure 6-22 shows an oscillograph equipped to handle 18 channels simultaneously. Figure 6-23 shows a smaller and lighter instrument designed to handle 6 channels.

6-124. VIBRATION METERS. Vibration meters are used in conjunction with velocity pickups to provide meter readings proportional to the velocity of the vibratory motion. Integrating and differentiating networks give direct meter readings of displacement or acceleration if desired. Frequency is not indicated on these meters. A jack is provided for connection to an oscilloscope if waveform or frequency information is desired. Figure 6-24 shows a typical vibration meter.

\* Amplitude refers to displacement, velocity, or acceleration, depending upon the type of pickup that is used.

6-125. The major advantage of this type of meter is its mobility. It is battery-operated and portable, and it is adaptable to most test situations.

6-126. **MAGNETIC TAPE RECORDING.** Magnetic tape provides a convenient method of storage and lends itself to rapid and comprehensive data processing. The electrical impulses stored on the tape can be fed into an oscilloscope and viewed as described previously in the discussion of oscilloscopes. The magnetic tape can also be processed by a wave analyzer which will indicate frequencies and amplitudes or power versus frequency (mean square acceleration density spectrum). Finally, the tape can be used to excite an electrodynamic shaker and so reproduce the vibration environment without necessitating intermediate environment analysis. Figure 6-25 shows a portable tape recording system.

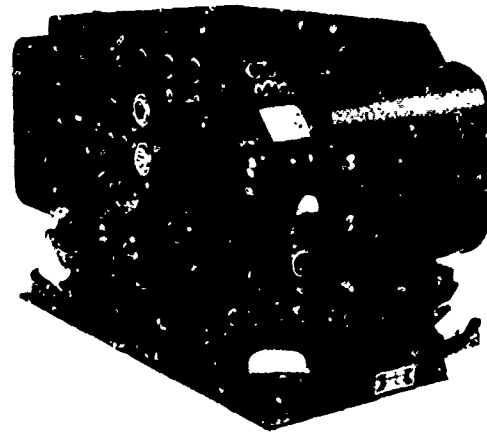


Figure 6-22. Mirror-Galvanometer Type Oscillograph (18-Channel)

6-127. **HIGH-SPEED PHOTOGRAPHY.** The high-speed camera, with speeds up to 8000 frames per second, photographically records shock and vibration phenomena. The film is then projected in slow motion allowing study of the surfaces of equipment being tested. This method of study is used mainly for investigative and not for routine testing because it is expensive and not suited to quantitative analysis except by lengthy and laborious calculations.

6-128. **DATA REDUCTION.**

6-129. Data for periodic vibrations can be reduced mathematically, mechanically, or electrically. Reducing data for random vibrations is limited to the electrical method.

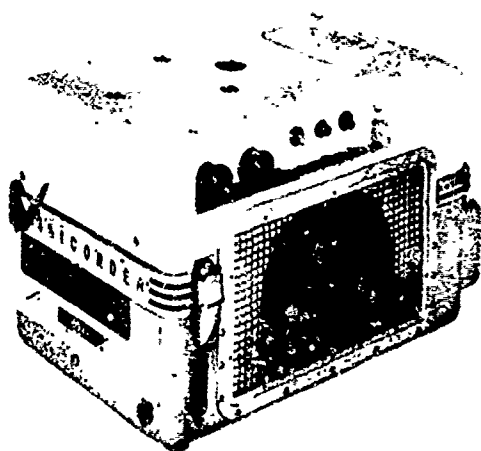


Figure 6-23. Mirror-Galvanometer Type Oscillograph (6-Channel)

6-130. **MATHEMATICAL.** Reducing periodic vibration data mathematically is feasible but tedious and time-consuming. It consists of performing a Fourier analysis on a complex waveform by using a combination of graphical and mathematical methods to determine the fundamental frequency and its harmonics and their amplitudes.

6-131. **MECHANICAL.** Mechanical reduction of periodic vibration data consists of performing a Fourier analysis on a complex waveform by using a Mader analyzer. This method, essentially, is similar to the mathematical method except that mechanical aids are

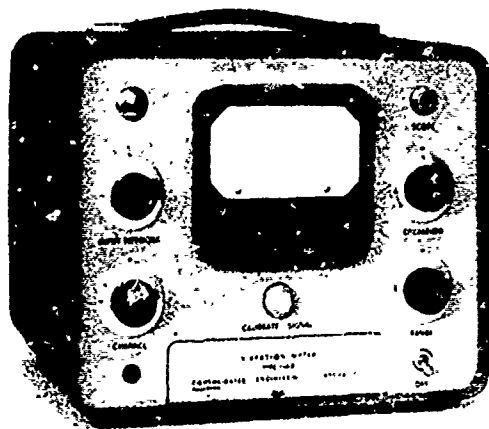


Figure 6-24. Vibration Meter

used in determining the Fourier integrals. The Mader analyzer consists of a gear, rack, and slider system, which, when used in conjunction with a planimeter (a mechanical integrator), can give readings analogous to a Fourier analysis, but with much less effort and time.

6-132. ELECTRICAL. The electrical method for reducing data can be used for both periodic and random vibrations. It is a much faster and simpler method but it does require an initial investment for a wave analyzer and the data must be recorded on magnetic tape.

6-133. A wave analyzer, shown in figure 6-26, is an electrical device which is responsive to selective, varied frequencies. The selectivity and variance can be achieved in two ways. One method employs a narrow band-pass filter, the center frequency of which is varied for scanning the input signal. The second method employs a fixed band-pass filter which selects the beat frequencies resulting from the heterodyning of the incoming signal with the output of a variable oscillator. Various commercially available analyzers use different methods for presenting the signal analysis. One type uses a high-persistence oscilloscope; others use a recording potentiometer.

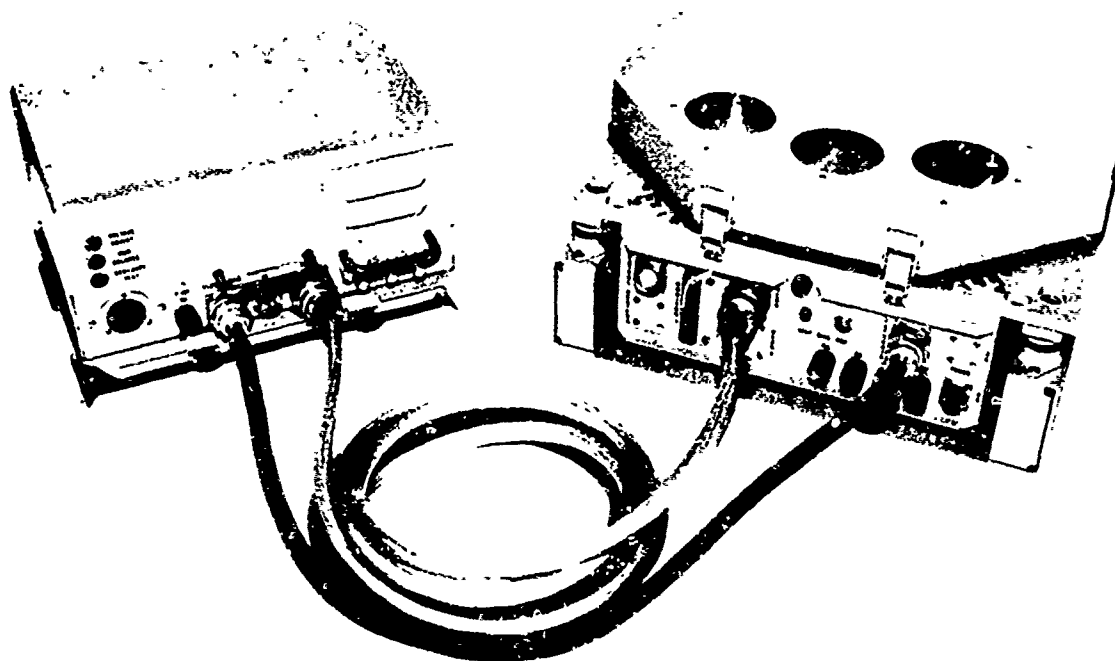


Figure 6-25. Portable Tape Recorder

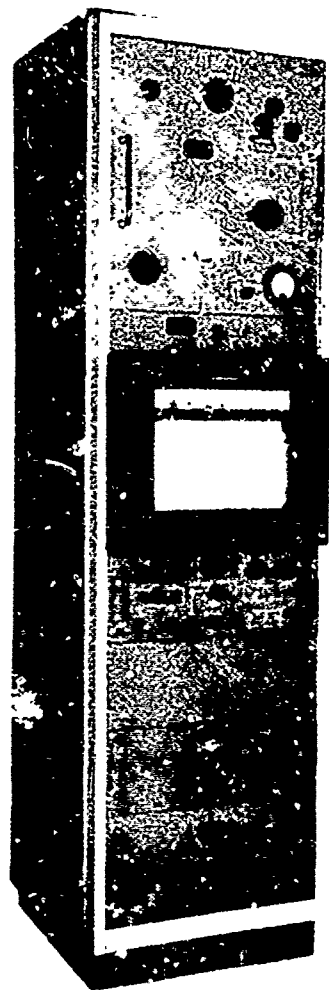


Figure 6-26. Wave Analyzer

6-134. For periodic vibration, an analyzer can simply record voltage versus frequency. With random vibration, however, the output of the filter at any frequency is a randomly varying voltage and this output must be averaged by a slow acting type of meter which indicates the rms (or mean square) value of the varying voltage. The recording of this value versus frequency is an rms (or mean square) acceleration density spectrum. In addition to the mean square acceleration density spectrum, information on the statistical distribution of acceleration peaks in a random vibration is sometimes desirable. The signal is fed into a counting circuit which indicates the number of times the acceleration exceeds a predetermined level. The counting must be done over a statistically significant length of time.

6-135. In analyzing a recording of a shock motion, one of the most promising methods which has been used is to play a recording of the shock motion into an electrical analog of a damped single-degree-of-freedom system. Varying the natural frequency of the analog circuit and noting the response to the shock motion leads to a shock spectrum as discussed in paragraph 2-9.

## SECTION VII

### SELECTION OF LABORATORY SHOCK AND VIBRATION TEST PROCEDURES

#### 7-1 GENERAL.

7-2. The final test of an equipment's ability to perform satisfactorily in a service environment occurs when a statistically significant number of the equipments are put into use under actual service conditions. However, it is impossible to wait until this stage of an equipment's life before getting a reliable measure of its anticipated performance. Not only would it be extremely expensive to modify an equipment at this time, but human life would be endangered if dependent upon the equipment's operation. It is necessary, therefore, to simulate the expected service environment in some way in order that the future in-service behavior of equipment can be predicted. Any attempt to simulate service conditions in laboratory testing (or to provide a test which does not simulate but does have correlative results) must be predicated on the assumption that the true service environmental conditions are known. In cases where the true service conditions are not known and an estimate is made of these conditions, any test based upon this estimate can be no better than the estimate.

7-3. The best test is one which gives the desired degree of reliability assurance for the smallest expenditure of time, money, and facilities. The decision as to the degree of assurance and, to some extent, the testing required to attain a given degree of assurance is arbitrary within certain limits, depending upon the ultimate objective of the equipment. Therefore, the type of test, test level, and test duration can vary for the same equipment, according to the philosophy of the person who establishes the test.

#### 7-4. THE ROLE OF STANDARD TESTS.

7-5. Standard shock and vibration tests, the results of an evolutionary process, are covered in various specifications, both governmental and industrial. These tests, which are based on past experience and accumulated flight data, represent good engineering effort and, in many cases, are adequate. No test, however, can be adopted without a thorough study of the anticipated environment. Tests affect equipment design, and, if poorly selected, can result in overdesign, or, what is more serious, underdesign. The orientation of a component or component part is important. Unless the part is positioned in the same way during a test as during its use in an equipment, and unless the test conditions reflect the environment existing in the equipment, the behavior of the component or part under test usually cannot be correlated to its eventual behavior in the equipment.

#### 7-6. SINUSOIDAL VIBRATION TESTING.

7-7. When a service vibration environment consists of periodic complex vibration, it is customary to separate, by analysis, this complex vibration into its com-

ponent frequencies and amplitudes. The information is then presented in the form of a plot of amplitude versus frequency. (Data in this form is presented in Section II.) If the service vibration environment were simple and did not change with time, only a few component frequencies would be present. It would be relatively easy to reproduce each of these single-frequency vibrations on a laboratory vibration exciter and thus determine the ability of the test item to withstand each of these component vibrations. While this test method would not test the exact response of the item to the complex vibration (i.e., all frequencies applied simultaneously), a small safety factor could compensate for this.

7-8. If the nature of the service vibration environment changes, as it does with changing flight conditions in aircraft, the number of frequencies likely to be encountered increases and the frequency-amplitude plot begins to look more like a band spectrum. When an equipment is intended for use in several aircraft or when the environment for a new aircraft must be estimated, the number of anticipated frequencies is increased. Resonant frequency testing and sweep frequency testing are the two basic methods used to test the resistance of an equipment to vibration within a harmonic spectrum.

7-9. **RESONANT FREQUENCY TESTING.** The maximum response amplitudes of a mechanical system to vibration excitation are obtained when the system is excited at its natural frequency. Thus, maximum stresses will occur at resonance, producing damage more quickly than vibration at other frequencies, and hence permitting shorter test periods. Also, many investigators believe that practically all failures of electronic equipment due to vibration are caused by resonances within the equipment. Given an envelope of environmental vibration, then, it might seem logical that for test purposes it would suffice to examine only those frequencies which coincide with natural frequencies within the equipment.

7-10. A prerequisite to resonant frequency testing is the determination of the natural frequencies of the test item which fall within the frequency range of interest. Since these natural frequencies cannot be accurately predicted, they must be experimentally determined by slowly sweeping the frequency range while observing the equipment for maximum response. Once the natural frequencies have been determined, resonant frequency testing consists of vibrating the equipment, at the appropriate input amplitudes, at each resonant frequency within the environmental frequency range, for a predetermined length of time.

7-11. In practice, it is difficult to perform a resonant frequency test. In a complex piece of electronic equipment, there are so many resonances that it becomes impossible to locate all the resonant points which lie within the frequency range. Also, it is difficult to locate resonant frequencies accurately and the response of a system falls off rapidly if the exciting frequency varies from the resonant frequency.

7-12. **SWEEP FREQUENCY TESTING.** Sweep frequency testing consists of vibrating the test item with a continuously varying input frequency. The frequency of vibration is cycled over the test frequency range while the amplitude is controlled as indicated by the test envelope. As discussed previously, a complex piece of equipment has many resonances, and locating each resonant point is difficult. Cycling over a frequency range successively excites all resonances within that range.

7-13. The time allowed for resonance build-up varies with the cycling rate. For wide frequency bands, such as 10 cps to 500 cps or more, a logarithmic time-versus-frequency relation is customarily used to give the same effective time at each frequency. The difference in the effective time at each frequency for linear and logarithmic sweep rates may be seen from the following considerations.

7-14. During a sweep frequency test, a component part may be considered to be in resonance when the exciting frequency is within a certain narrow frequency band which is centered at the resonant frequency of the part. For example, referring to figure 5-5, any exciting frequency between approximately  $0.96 f_n$  and  $1.03 f_n$  for a curve with a maximum transmissibility of 10, will produce a transmissibility of 8 or more, and the number of cycles which occur between these two frequencies could be considered as occurring at  $f_n$ . For a constant sweep rate (linear curve), as shown in figure 7-1, the number of cycles at  $f_n$  (between  $f_1$  and  $f_2$ ) is:

$$N = \int_{f_1}^{f_2} f dt = \int_{f_1}^{f_2} f \frac{df}{k} = \frac{1}{2k} (f_2^2 - f_1^2).$$

Since  $f_2 = 1.03 f_n$  and  $f_1 = 0.96 f_n$ ,

$$N = \frac{1}{2k} (1.03^2 - 0.96^2) f_n^2.$$

The effective number of cycles at any frequency is proportional to the square of that frequency for a linear sweep curve. With a logarithmic sweep curve, however, as shown in figure 7-2, the number of cycles at  $f_n$  (between  $f_1$  and  $f_2$ ) is:

$$N = \int_{f_1}^{f_2} f dt = \frac{1}{k} \int_{f_1}^{f_2} df = \frac{1}{k} (f_2 - f_1).$$

Since  $f_2 = 1.03 f_n$  and  $f_1 = 0.96 f_n$ ,

$$N = \frac{(1.03 - 0.96)}{k} f_n.$$

Thus, with a logarithmic sweep curve, the effective number of cycles at any frequency is proportional to that frequency. Another way of expressing this is that the effective time at each frequency is a constant.

7-15. The sweep rate used in a sweep frequency test should be calculated using the lowest frequency of the sweep range. A system excited at resonance does not reach maximum response instantaneously; the response amplitude increases with each successive cycle until maximum response is reached. If the cycling rate is so high that the system never reaches maximum response, the test is inadequate. The number of cycles required to reach maximum response is independent of frequency; even with the same effective time at each frequency, the least number of cycles will occur at the lowest frequency.

7-16. Multiple octave sweep testing, a variation of sweep frequency testing, has been proposed to shorten the total testing time. It consists simply of a sweep frequency test in which the overall sweep frequency range is divided into several shorter sweep frequency ranges of equal sweep times, and these shorter sweep

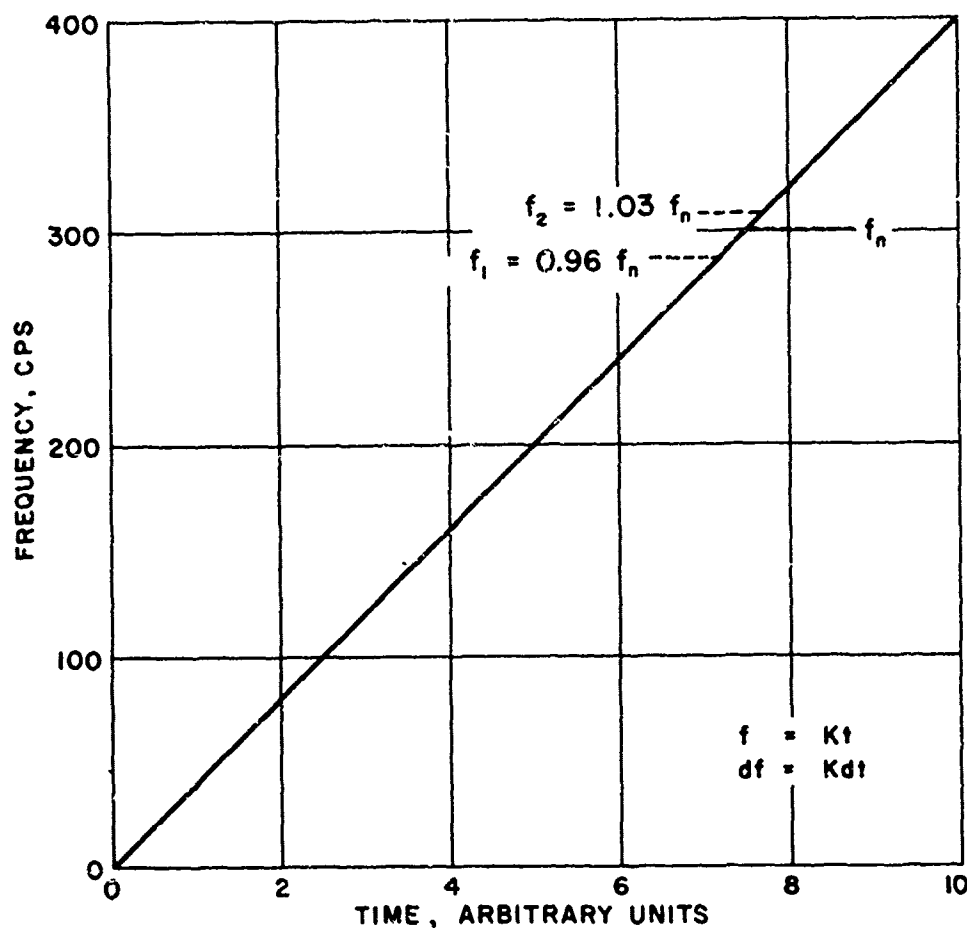


Figure 7-1. Linear Sweep Curve

frequency tests are run concurrently. The waveform at any time, then, is the result of superimposition of the vibrations from the individual sweep frequency tests. Under this type of test it would be possible to have several portions of the test item, which have different natural frequencies, in resonance at the same time. This type of test would more nearly simulate actual service conditions.

**7-17. COMBINATION RESONANT AND SWEEP FREQUENCY TESTING.** Many test specifications require both resonant frequency and sweep frequency testing. Resonant frequency testing is usually required at the major resonant frequency within a specified range; every resonant point does not necessarily have to be identified. The required range for sweep frequency testing is usually the same as that given for resonant frequency testing.

**7-18. RANDOM VIBRATION TESTING.**

**7-19.** The method of simulating random vibration environments in a laboratory is still controversial. To date, there has been no general adoption of uniform means of data analysis or of test methods. Generally, the major question has been whether a random vibration environment can be simulated by sinusoidal excitation or whether a random excitation is the only or preferable method.

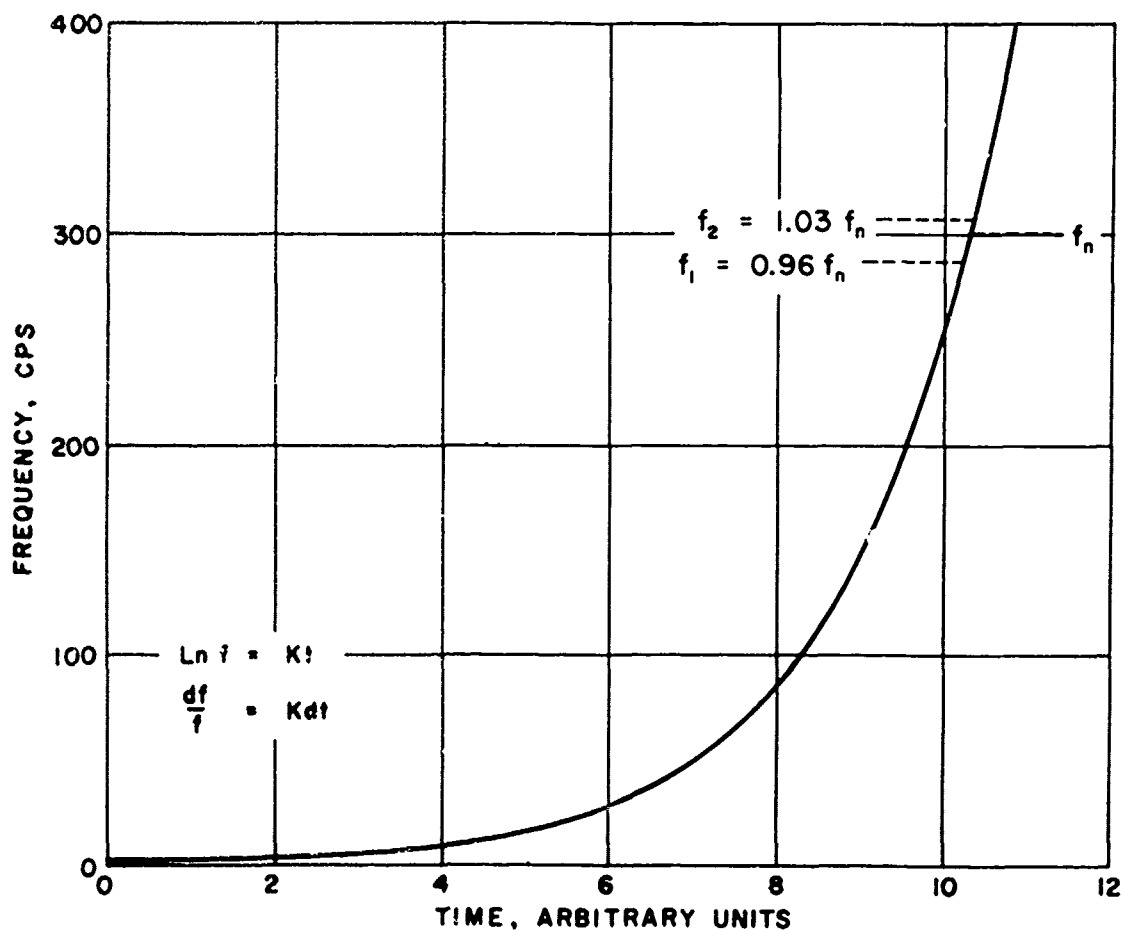


Figure 7-2. Logarithmic Sweep Curve

7-20. Proponents of random excitation testing consider that sinusoidal testing of equipments intended for use in a random environment is completely unrealistic. For example, a crude method of establishing sinusoidal excitation tests is to process the random vibration data by studying the acceleration-time curve, finding cycles which approximate sinusoids, and then measuring their apparent frequencies and amplitudes. Using these frequencies and amplitudes in steady-state single-frequency tests, even if it were possible to measure each frequency and amplitude accurately, is unduly severe, particularly in a system with little damping. In a random environment, resonant response cannot build up to a maximum in one cycle, and by the time another cycle of the same frequency and amplitude occurs, the response due to the first may be damped out.

7-21. Theoretically, it would appear feasible to statistically analyze the response of single-degree-of-freedom systems to the random environment and to use this statistical analysis to establish sinusoidal tests. As described in Section I, when a recording of a continuously random vibration\* is fed through a narrow bandpass

\* As used here, continuously random vibration is vibration distributed over a broad frequency band, as opposed to being concentrated within a few narrow frequency bands.

filter, as in a wave analyzer, the output is a randomly varying signal at the center frequency of the filter band. This filter output is analogous to the response of a single-degree-of-freedom mechanical system to random vibration, and the statistical distribution of the response amplitudes of the single-degree-of-freedom system can be determined. Then, it should be possible to use sinusoidal vibrations at resonance to test a simple system whose natural frequency was that of the filter. The input amplitudes would have to be chosen to produce the desired response amplitudes, and the time at each input amplitude would have to be based on a theory of cumulative fatigue damage. However, choosing the correct response amplitudes would be difficult since the response depends on damping within the structure.

7-22. The difficulties encountered in establishing correlation between random environments and sinusoidal testing suggest that random excitation, being characteristic of the environment, should be used for testing. The test level for simulation of a random vibration environment by continuously random excitation is a relatively straightforward matter. Having a mean square (or rms) acceleration density spectrum, the test excitation is chosen to have the same rms values over the frequency range. Factors of safety can be introduced by choosing higher rms values for the test excitation.

7-23. The advocates of sinusoidal excitation state that almost every argument in favor of random vibration testing is founded on the assumption that the environment consists of continuously random vibration with energy existing at each frequency within a frequency range. The validity of this assumption depends upon the closeness of the vibration source to the airframe location. Several structures or substructures, having different resonant frequencies and degrees of damping, normally exist between a power plant and an electronic equipment. Although the power plant may produce continuously random vibration over a broad frequency band, the vibration energy, in passing through the intermediate resonant structures, would be expected to concentrate in discrete frequencies.

7-24. That continuously random vibration is filtered by resonant structures is supported by a recent study of missile environmental data. The final report on this program states: "The initial excitation resulting from rocket power and aerodynamic forces, possibly random in nature, apparently is filtered by structural members and emerges as vibration confined to a narrow frequency band with an apparent fluctuating amplitude. This type of vibration is better simulated by a single frequency test having a prescribed acceleration amplitude. Such a test is valid for structures having any degree of damping. On the other hand, the simulation of this structural vibration by a random excitation is unconservative for structures with relatively little damping, generally the type of structure most vulnerable to damage by vibration. The single frequency test, therefore, not only appears to be more representative of the environment encountered in guided missiles but also avoids many practical testing problems."

7-25. As discussed in paragraphs A-46 through A-49, the rms response of a lightly damped, single-degree-of-freedom system to white-noise excitation can be calculated and a sinusoidal excitation chosen which will produce the same rms response. The excitation level at frequencies well removed from the system natural frequency has less effect upon the system response than does the excitation level at frequencies close to the natural frequency. Equation A-50, therefore, although

based on an excitation for which the mean square acceleration density is constant over the frequency range of zero to infinity, will give a close approximation if the mean square acceleration density is constant only in the region of the system natural frequency.

7-26. Although sinusoidal excitation may be chosen so as to produce the same rms response as would be produced by white-noise excitation, this sinusoidal excitation would not necessarily be expected to have the same damage potential. Methods are being developed for establishing sine wave tests which have the same damage potential as random vibration environments. Typical of these is the derivation of factors by which the rms accelerations are multiplied to obtain "equivalent acceleration amplitudes" for sine wave testing. The factor for an excitation having a Rayleigh distribution of peak accelerations was calculated to be 2.71. This value was based on equations for (1) an idealized S-N curve, (2) the response of an undamped, single-degree-of-freedom system, and (3) damage accumulation under cyclic loading at various stress levels. Studies of the correlation between sinusoidal and random excitations are being continued.

7-27. The choice of random versus sinusoidal testing for simulation of missile (or any other) vibration environment apparently depends primarily on the true nature of the environment. If the environment is continuously random, correlation between laboratory test results and in-service behavior would be expected only if random excitation were used in the laboratory. If, however, the environments are characterized by narrow frequency bands within which most of the energy lies, the advisability of using random excitation for laboratory testing can be seriously questioned. As stated previously, there has been no general adoption of uniform testing. This can be attributed to the general lack of definite knowledge as to the true nature of the environments being encountered. Investigations in this field are continuing.

#### 7-28. PROOF-OF-DESIGN AND PROOF-OF-WORKMANSHIP TESTS.

7-29. In theory, proof-of-design testing establishes the acceptability of a design and, once a design is finalized, proof-of-workmanship testing controls the quality of production items. A proof-of-design test is an attempt to predict whether or not an equipment will be able to withstand a service vibration environment. A proof-of-workmanship test is a quality-control test to uncover faulty workmanship without aging the item, and makes no attempt to simulate a service environment.

7-30. For vibration testing, a proof-of-workmanship test, consisting of simple, short-duration shaking to uncover faulty electrical connections, loose threaded fasteners, dirt, etc., is an inadequate acceptance test for production equipment. Construction faults such as surface cracking at bends of chassis or brackets due to the use of a wrong alloy or temper are not necessarily uncovered. Therefore, in practice, the same tests are often used for both design and production.

7-31. Considering the complexity of the problem of establishing tests, the expediency of using one test cannot be questioned. It is conceivable, however, that an electronic equipment may pass the proof-of-design test by the "skin of its teeth" and that, as a result of progressive deterioration, the equipment is approaching failure at the conclusion of the test. If this same test is used for proof-of-workmanship, equipments may be aged by the test to the point that failure will rapidly

occur in service. To reduce the chances of this occurring, a more sensible procedure would be to use, for production items, a short-duration proof-of-workmanship test on all equipments and proof-of-design testing on a low-percentage sampling basis. The few equipments which receive the proof-of-design tests would be considered expendable.

7-32. One approach to proof-of-design testing is to separate it into ultimate-strength testing and fatigue testing. The ultimate-strength tests consist of testing at maximum service conditions for a short period of time. The fatigue test subjects the equipment to long-duration testing at a level approximating the average steady-state service condition, and for a time equal to the expected equipment life.

7-33. To shorten the duration of a vibration fatigue test, equipments are often tested under conditions which are more severe than the average service conditions, based on the philosophy that there is a correlation between service stress and equipment life. Undoubtedly some correlation exists, although it has never been exactly defined and is not likely to be, since correlation would vary between equipments. In any event, accelerated tests are aging tests only and, if the test conditions are more severe than the most severe service conditions, the equipment should not be expected to be operable during the test. At the completion of the accelerated test, however, the equipment should be operable under maximum service conditions.

7-34. One method of establishing accelerated vibration life test requirements is to draw an envelope enclosing all of the data points on an amplitude-versus-frequency plot, to increase the amplitudes of this envelope by 50%, and to use this as the test envelope for one million cycles. This method was chosen because observation of the envelope of several S-N (stress-versus-number-of-cycles-to-failure) curves indicated that, if an item can withstand one million cycles at a stress 40% above the service stress, it can withstand the service stress indefinitely. One million cycles was chosen as a reasonable number and a 50% increase in amplitudes was used (instead of a 40%) to allow for slight errors in recording the original environmental data. This test would seem to be overly conservative since the first envelope encloses all of the data points, some of which undoubtedly represent conditions which have a low probability of occurrence.

7-35. It is customary, when choosing an envelope of environmental data points, to choose an envelope shape which lends itself readily to laboratory reproduction. This simply means that the parameters used to describe the vibration level are kept constant over a wide range of frequencies.

7-36. In addition to design and workmanship tests, individual component parts and chassis subassemblies are tested for developmental purposes. In selecting component parts, available information (either advertising literature or previous experience with a component part's performance) about the fragility of a part is often inadequate. Laboratory environmental testing is the best method of choosing between electrically similar component parts (although frequently neglected by designers). These tests are not fixed but are chosen to produce particular information. A test may consist simply of determining resonant frequencies, or determining vibration levels necessary to cause malfunction or complete failure. At the other extreme, a series of tests can determine the time-to-failure distribution for several different vibration levels. Chassis subassemblies are tested during

development to determine their individual transmissibilities, which directly affect the environments of the component mounted upon them.

#### 7-37. SHOCK TESTING.

7-38. Basically, there are three methods of specifying a shock motion: (1) by describing the method of producing the motion, (2) by giving the time-history of a parameter such as acceleration, or (3) by giving the damage potential of the shock motion. By specifying the type of shock simulator and the test methods (e.g., drop height and arresting mechanisms), the shock pulse, although not known exactly, is theoretically standardized. However, different mechanisms of the same design, used with identical test values, do not always produce the same shock motion.

7-39. Definitions of shock motion by giving the peak-acceleration level, the duration of the shock pulse, and a general statement about pulse shape are usually inadequate. In measuring pulse duration, the end point of a shock motion is arbitrary and the pulse shape does not lend itself to verbal description, particularly as to the smoothness (amount of hash) of the acceleration-time curve. The pulse shape has often been chosen for ease of reproduction or ease of mathematical description rather than for its similarity to pulse shapes of shocks experienced in service.

7-40. Throughout most of the evolutionary stages of shock testing, the objective has been to reproduce the shock motions which exist in service, and this philosophy still guides the selection of most shock test requirements. Concurrent with this growth of shock testing, investigators attempted to answer basic questions about the response of equipments to shock, the importance of pulse shape and duration, and the definition of a shock motion in terms of severity or damage potential. From the standpoint of test selection, a means of defining shock in terms of damage potential is important. No two service shocks are exactly alike and, with no measure of damage potential, there is no basis for choosing a single type of shock motion. Consequently, the shock spectrum method of defining the severity of a shock pulse is increasing in popularity. If several shock spectra are plotted together, an envelope can be drawn and compared with the shock spectra produced by various shock simulators. In this way, a shock simulator and test conditions can be chosen to simulate the severest aspects of several individual service shock motions. Typical shock spectra for testing machines are shown in figure 7-3.

7-41. The shock spectrum method of defining shock severity was the primary basis for the design of the flat-spectrum drop-type shock simulator. In establishing a shock test for components or component parts, the severity of several individual service shocks must be considered. Although each service shock gives a jagged response spectrum, the envelope of several response spectra is relatively flat, and therefore a shock pulse with a flat response spectrum is desirable for testing. Theoretical considerations of a terminal-peak sawtooth acceleration pulse show that it tends to give a flat response spectrum. Figure 7-4 shows a theoretical terminal-peak sawtooth pulse and an actual pulse obtained by using a conical-top lead pellet as the arresting medium for a drop-type simulator. Figure 7-5 gives the response spectra for the pulses of figure 7-4. The simulator described in paragraph 6-14 produces this type of pulse.

7-42. Many specifications require that an equipment withstand three shocks in each of three mutually perpendicular directions, giving a total of nine shocks in all.

This type of testing does not consider that an equipment, which responds with damped oscillations to the shock pulse, may be able to withstand a few shocks but may fail through fatigue after repeated shocks.

7-43. Although not generally practiced, shock tests, similar to vibration tests, should be divided into two types: ultimate-strength and fatigue tests. A shock ultimate-strength test would consist of subjecting an equipment to a limited number of shocks at the maximum severity it would encounter in service.

7-44. There are two general types of shock motion. If the velocity of an item is the same after a shock as before, the shock motion must consist of both positive and negative acceleration\* pulses with the area under each being equal. This type of shock is experienced by a depth-charged submarine, a tank hit by shell fire, or a plane close to an aerial explosion. The hi-impact shock simulator reproduces this type of motion. If an item experiences a change in velocity as a result of the shock, the shock consists primarily of an acceleration in a single direction. Although this type of shock motion may contain other accelerations, they are negligible. Unidirectional accelerations are experienced by a landing plane, a missile at launch, or an item dropped during handling. The drop-type shock simulators produce this shock motion.

7-45. In selecting the service shock environment that is to be simulated, handling and transportation environments must also be considered. This is especially important for shock testing, since the most severe shocks can occur at a loading platform. Also, the number of shocks to which an equipment is subjected during handling and transportation may far exceed the number encountered in service. This is especially true for a short-lived equipment such as a short-range missile. An unpackaged equipment, of course, should not be required to withstand the shipping environment unless it is to be shipped unpackaged. A packaged item test is a test of the combination of shipping container and equipment; what the equipment lacks in ruggedness, the container must provide in protection and vice versa. The customary (and usually adequate) packaged item test consists of a number of bumps or drops, the height of drop depending on the weight and size of the equipment.

7-46. Boost-phase shock loads caused by ignition of a solid propellant booster are often the greatest environmental accelerations experienced by missile components. The intensity of the load experienced by a component varies depending upon the component's position relative to the booster and the characteristics of the missile airframe, which may either amplify or attenuate the shock. In certain missiles, the boost-phase shock exhibits a rapid acceleration build-up followed by a steady-state acceleration which persists until the booster burns out. Although the missile frame may respond to the rapid build-up of acceleration with high-frequency transient vibrations, the thrust output of the booster is relatively smooth. The shocks encountered may vary from 20g for 50 milliseconds to 60g for 20 milli-

\* The only difference between positive and negative acceleration is direction and the positive direction is an arbitrary one. From an equipment's point of view, the difference between so-called positive and negative accelerations is a 180-degree difference in direction of acceleration. With proper orientation, therefore, a unidirectional acceleration pulse can be reproduced either by accelerating the equipment from zero velocity to a new velocity or by decelerating an equipment from a finite velocity to zero velocity.

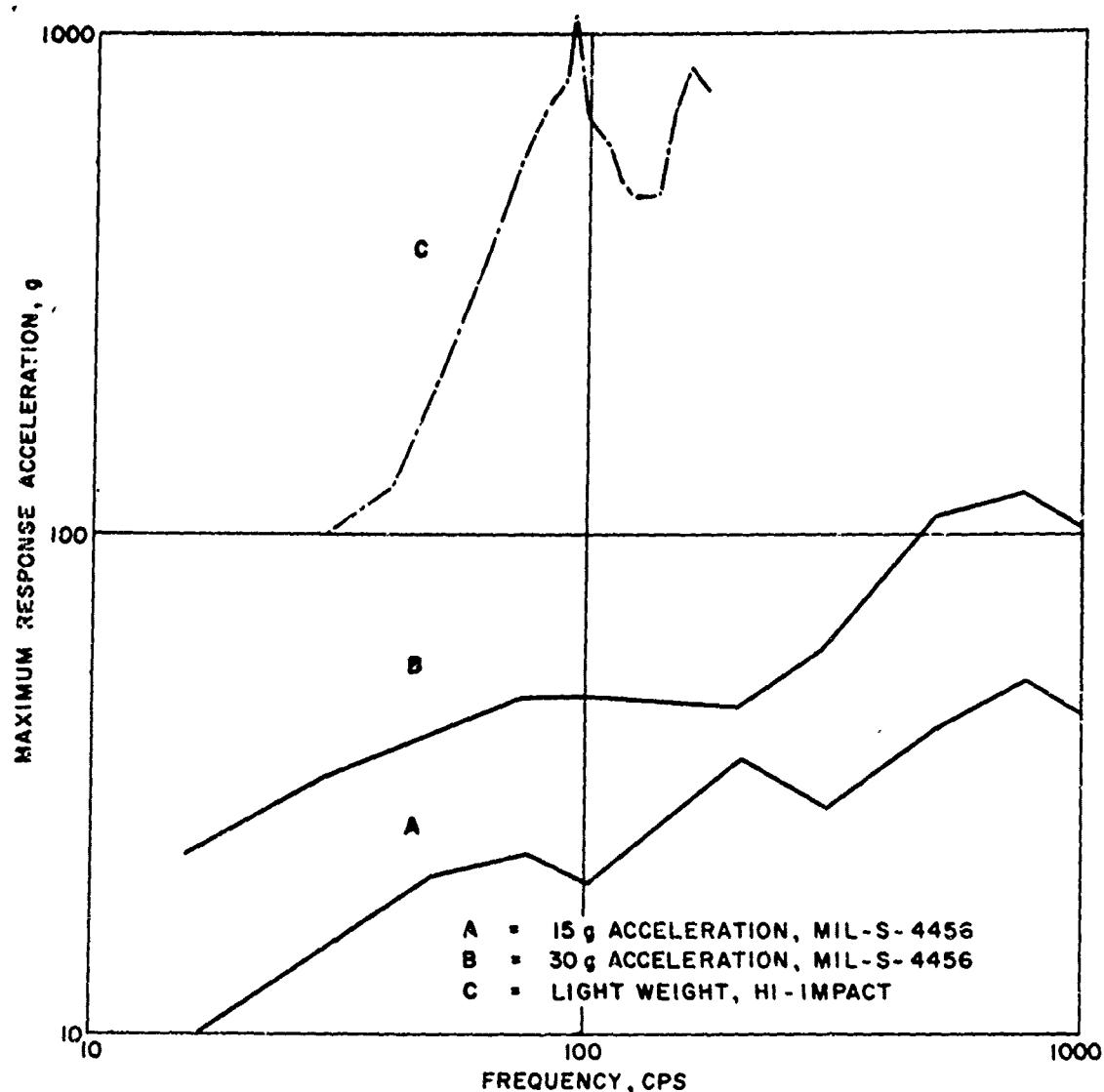


Figure 7-3. Typical Shock Test Spectra

seconds with rise times of approximately 10-30 milliseconds.

7-47. Rapidly applied long-duration accelerations, such as experienced in missile boost, rocket setback, or water penetration, can be simulated by rocket sled, actual weapon firings, or by a rotary accelerator. A conventional centrifuge is not satisfactory for rapidly applied accelerations since the acceleration of the specimen changes direction (from tangential to radial) as the centrifuge speed is rapidly changed.

7-48. The drop-type shock machines which employ sand or lead as an arresting medium do not provide sufficient travel during arrest to reproduce the long-duration shocks needed to simulate boost-phase shock and acceleration. Special arresting mechanisms are used, such as a hydraulic-pneumatic shock strut which provides a long arresting stroke, by means of hydraulic oil acting on an orifice plate through which the oil is forced at a rate predetermined by a contoured metering pin.

#### 7-49. COMBINED ENVIRONMENTS.

7-50. An equipment able to withstand two or more environments when subjected to them separately, may fail when subjected to the same environments simultaneously. Constant acceleration, temperature, and barometric pressure are environments which may combine with vibration or shock to produce short-term failure. Although the probability of environmental interactions has long been recognized by design and test engineers, the lack of adequate test facilities has often prevented testing for these interactions. The major emphasis has been on the most common combination of environments, vibration and temperature, and testing under this combination is performed frequently. Shock testing at extreme temperatures is more difficult to perform because of the types of shock machines in common use.

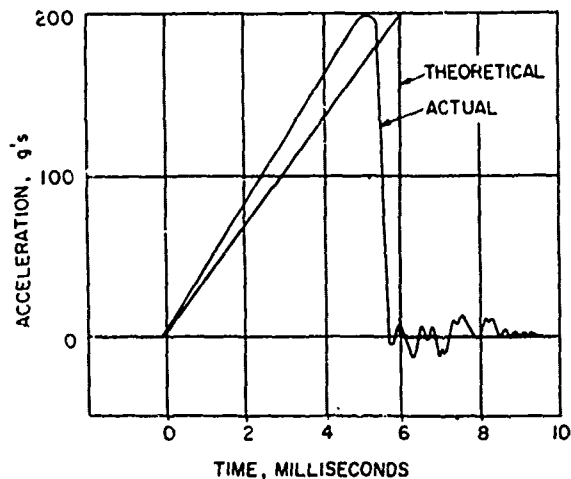


Figure 7-4. Terminal-Peak Sawtooth Acceleration Pulses

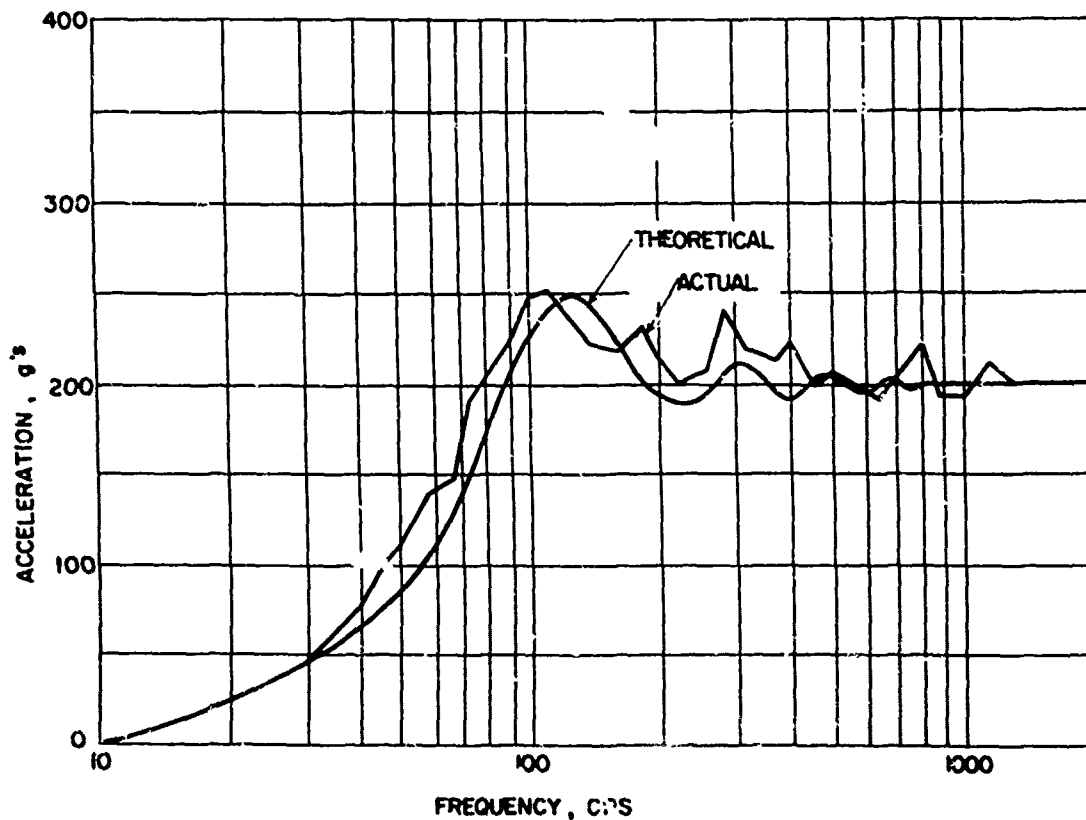


Figure 7-5. Response Spectra for Pulses of Figure 7-4

7-51. Vibration combined with constant acceleration such as might occur during dive pullout in fighter aircraft or during the boost phase of missile flight is usually difficult to simulate. In the vibration testing of isolation systems, superimposed static acceleration can be simulated by using a long spring (such as bungee cord) to produce a force on the equipment. The spring force should act through the equipment's center of gravity and be equal to the equipment weight multiplied by the static acceleration. With a long spring, the spring force is practically constant over the travel of the vibrating equipment and the spring adds little effective mass to the equipment. At high frequencies, however, the distributed mass of the spring could cause spring surge. Although useful for testing isolation systems, this method cannot be used to simulate the effect of static acceleration on the component parts of a vibrating equipment. To accomplish this necessitates mounting a vibration simulator on a centrifuge; the expense of such a test setup, particularly for testing large components, has discouraged its use. Convair, however, is presently planning this type of facility, as was discussed in Section VI.

## SECTION VIII

### VIBRATION AND SHOCK PROTECTIVE DEVICES

#### 8-1. GENERAL.

8-2. Vibration and shock protective devices are used to prevent the vibratory and shock motions of a supporting structure from reaching the supported equipment. The devices or isolators are resilient materials placed between the equipment and the supporting structure. Vibrations or shock motions of the support flex the resilient isolators leaving the equipment comparatively motionless.

8-3. An example of a highly refined isolation system is an automobile suspension. In the automobile, soft metal springs and viscous dampers (shock absorbers) isolate the vibration and shock motion of the wheels from the body. Space and weight are not important considerations in this application.

8-4. In aircraft, however, there are exacting limitations on the size and weight of electronic equipment. For example, if the space were available and the added weight permissible, an electronic equipment could be encapsulated in a large block of soft sponge rubber. This treatment would, in all likelihood, afford excellent protection to the equipment. But clearance around equipment is allotted in precious fractions of an inch, and light weight is an ever-present requirement. And there are other problems.

8-5. The presence in some aircraft and missiles of steady vibration with intermittent shocks causes a difficult isolator design problem. The requirements for a shock isolator usually conflict with those for a vibration isolator in airborne environment. The lower the natural frequency relative to the exciting frequency, the more effective the vibration isolation. However, the low spring constant (soft springs) of the isolator, necessary to achieve a low natural frequency, can cause pounding against the snubbers during shock. A high spring constant, giving good shock protection, can result in an equipment natural frequency that will coincide with environmental vibration frequencies and result in destructive resonance.

8-6. The types of isolators, their performance characteristics, and criteria for selecting them are discussed in the following paragraphs. The discussion of shock isolators, although discussed separately, is concerned mainly with the shock protective features of devices which are primarily vibration isolators.

#### 8-7. VIBRATION ISOLATORS.

8-8. Isolators used for airborne electronic equipment are mainly of two types; the cup type which has the resilient element enclosed in a metal container, and the open type which has an exposed resilient element. These two basic types are subdivided according to the construction and material used in the resilient element. The resilient element can be a coil spring, woven metal mesh, a combination of

coil spring and woven metal mesh, or rubber. The construction of the isolator will result in a certain spring rate which will largely determine its performance.

8-9. **SPRING RATE.** Spring rate (or spring constant) is the rate at which the spring deflects under an additional load. Spring rate may be linear or nonlinear. If increasing the load by equal increments produces equal changes in deflection, the spring is linear. Otherwise it is nonlinear. Rubber performs as a nonlinear spring as does woven metal mesh. Figure 8-1 shows examples of linear and nonlinear coil springs.

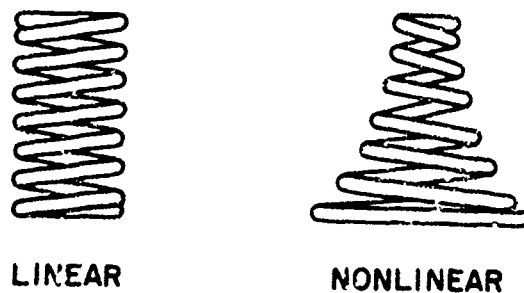


Figure 8-1. Linear and Nonlinear Coil Springs

8-10. Graphically, spring rate is the force-deflection curve of the spring. Figures 8-2 and 8-3 show force-deflection curves for both linear and nonlinear springs. In figure 8-2, the linear spring rate is 100 pounds per inch. The spring rate for the nonlinear spring (figure 8-3) changes as the spring is displaced: ten pounds displaces the spring three inches; thirty pounds, five inches; and 50 pounds, six inches.

8-11. Spring rate is important because it determines the natural frequency of the mounting system. Figure 8-4 shows typical relationships of natural frequency and loading for both linear and nonlinear springs. For both springs, zero load gives an infinitely high natural frequency, but this is the only way in which they are similar. The greater the load on linear isolators, the lower is the system natural frequency. The greater the load on nonlinear isolators, the lower is the natural frequency until a minimum point is reached; further loading then will increase the natural frequency. Commercially available nonlinear isolators are designed to

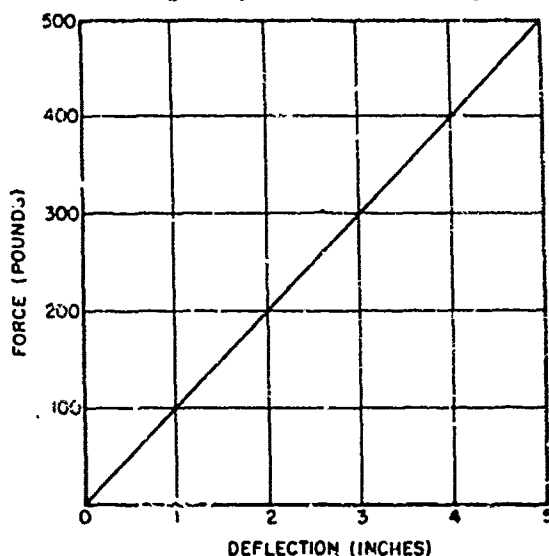


Figure 8-2. Force-Deflection Curve for a Linear Spring

operate at the portion of the curve where added load lowers the natural frequency. The advantages or disadvantages of linear or nonlinear springs are summarized in table 8-1.

8-12. **HORIZONTAL-TO-VERTICAL STIFFNESS RATIO.** The ratio of horizontal to vertical stiffness of an isolator affects the characteristics of the intended mounting system. Underneath mounting systems are more effective with mounts having a greater vertical than horizontal stiffness; while center-of-gravity mounting systems are planned with equal horizontal and vertical stiffnesses for each isolator. Section IX discusses in detail the preferred horizontal-to-vertical stiffness ratios of the various mounting systems.

TABLE 8-1. COMPARISON OF THE PERFORMANCES OF  
LINEAR AND NONLINEAR SPRINGS

Linear	Nonlinear
Narrow load range.	Wide load range.
Absorbs less energy in shock before bottoming.	Absorbs more energy in shock before bottoming.
Natural frequency lowers with added load.	Change in natural frequency with added load is less than for linear.
Natural frequency is independent of vibration amplitude.	Natural frequency varies with changes in vibration amplitude.
Natural frequency is independent of static acceleration.	Natural frequency is dependent on static acceleration.
Greater danger of bottoming in static acceleration.	Less danger of bottoming in static acceleration.

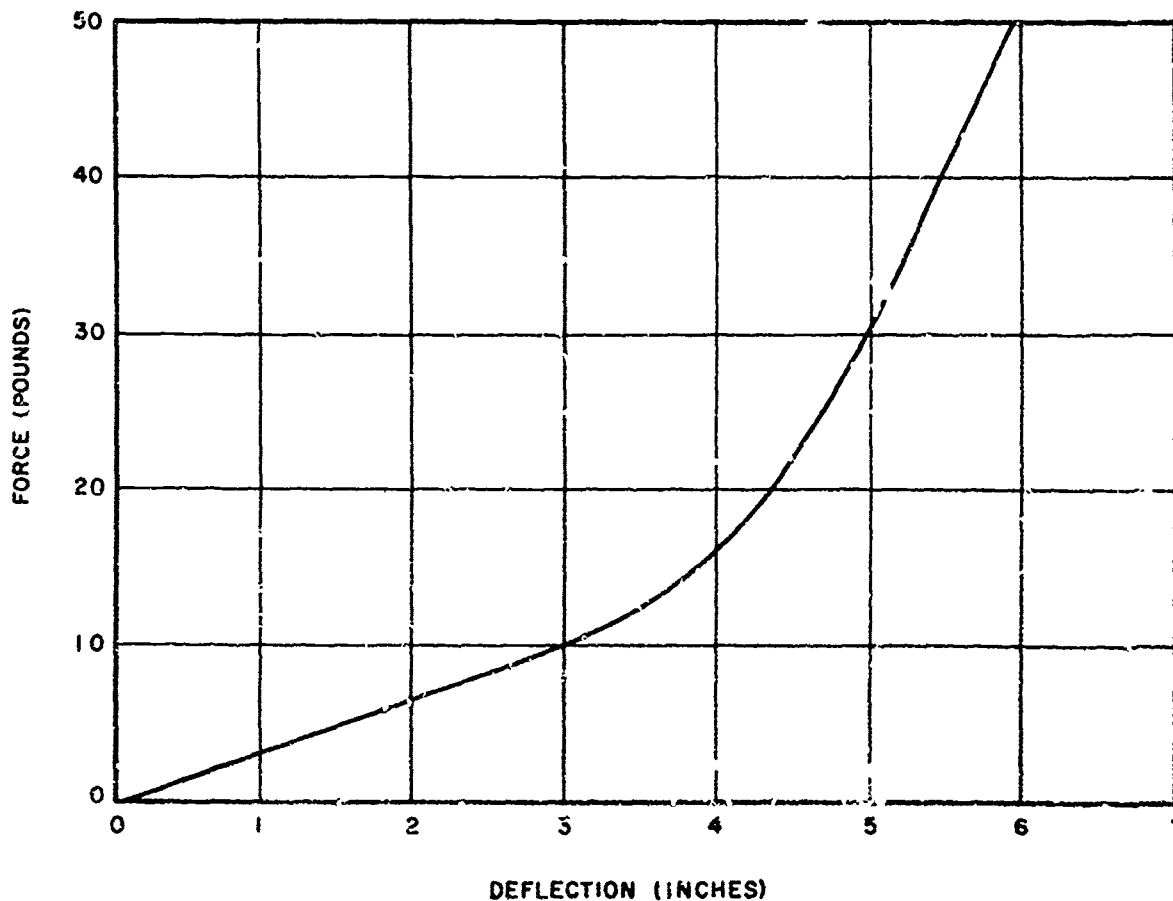


Figure 8-3. Force-Deflection Curve for a Nonlinear Spring

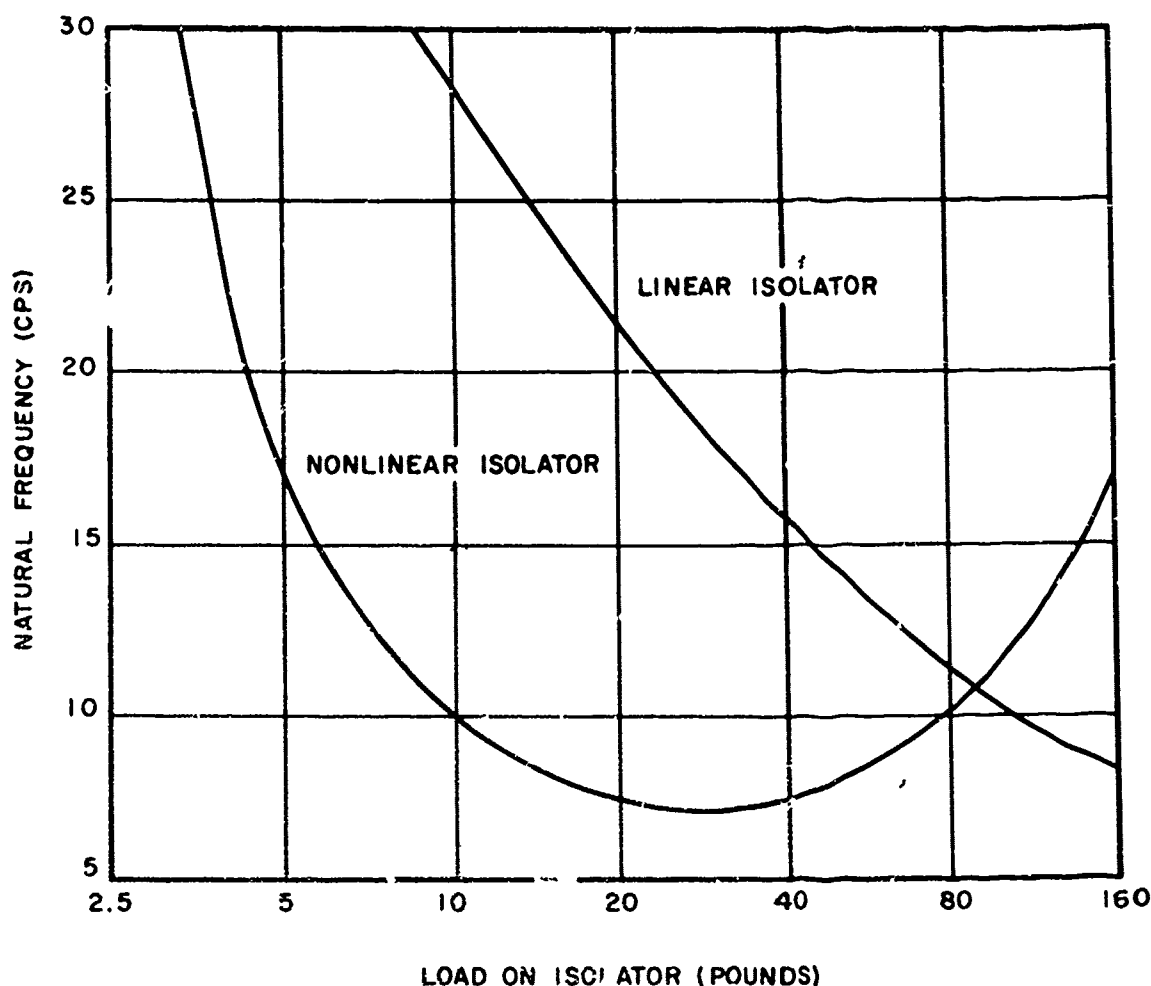


Figure 8-4. Effect of Load on Natural Frequency for Linear and Nonlinear Isolators

### 8-13. RUBBER ISOLATORS.

8-14. Rubber has long been used in vibration and shock isolators, and, until World War II, practically all vibration isolators were made of natural rubber. During the war, when natural rubber became scarce, synthetics were substituted. These synthetics exhibited qualities in special applications that were preferable to natural rubber. For example, Buna N and Neoprene maintain their elasticity and tensile strength at higher temperatures than does natural rubber, and are less apt to deteriorate when exposed to oil. Natural rubber, however, has the best all-around characteristics and still is used in the majority of the rubber isolators.

8-15. USE OF RUBBER AT TEMPERATURE EXTREMES. Most natural and synthetic rubber compounds do not give satisfactory performance at temperature extremes. At temperatures greater than 150° F, most rubber compounds develop surface cracks and lose strength. At low temperatures they become stiff. Neoprene can be compounded to resist high temperatures, but is the first to become

rigid at low temperatures. Natural rubber performs well at low temperatures, but has undesirable high-temperature characteristics.

8-16. Silicone rubber is suitable for use at temperature extremes. It retains its flexibility at  $-70^{\circ}\text{F}$  and physical properties at temperatures as high as  $400^{\circ}\text{F}$ . However, its physical properties, such as tensile strength and tear resistance, generally are inferior to other compounds. Resilient elements made of silicone rubber or other low-temperature elastomers are available in several styles of vibration isolators for airborne use. The performance of one such isolator is illustrated in figure 8-5. Three transmissibility curves are shown: one for operation at room temperature, another at  $300^{\circ}\text{F}$ , and a third at  $-67^{\circ}\text{F}$ .

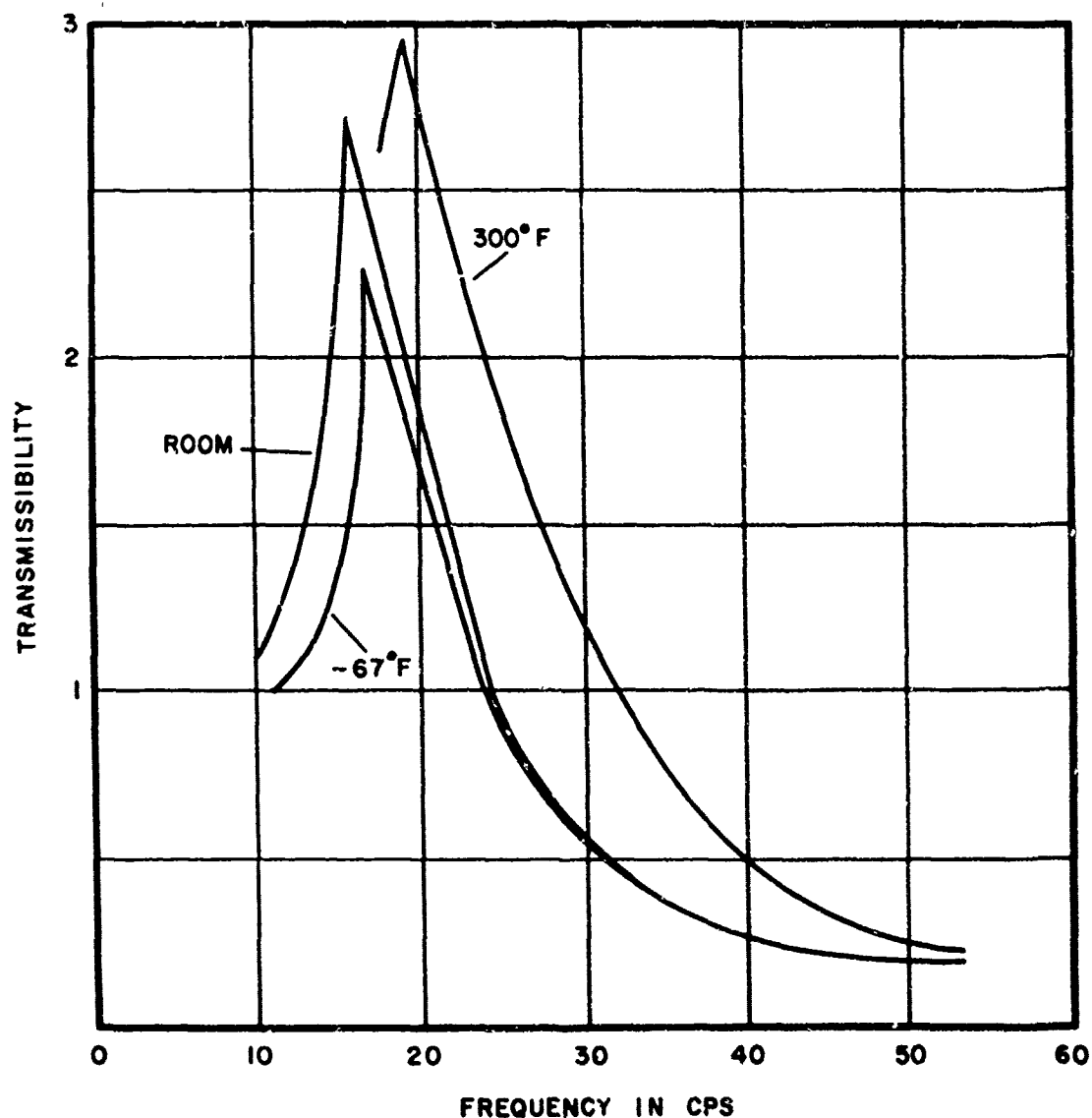


Figure 8-5. Transmissibility Curves for an Isolator Using a Special Extreme-Temperature-Resistant Rubber Compound as the Resilient Element

8-17. **DRIFT OR CREEP.** The deflection of a rubber isolator will increase with time under continuous loading, especially at high temperatures. This slow distortion is known as drift or creep. In an airborne environment where excursion space is limited, drifting can eventually result in bottoming of the isolators during high amplitude vibration. In a well designed rubber mount, the recommended loading will be conservative enough to eliminate this danger.

8-18. **DAMPING IN RUBBER.** Rubber has inherent damping qualities. Auxiliary damping devices often are not needed with a properly compounded rubber resilient element, as they are with coil spring isolators. The damping ratio  $c/c_c$  increases as the rubber hardness increases (see figure 8-6). If, however, rubber isolators are exposed to resonance for an extended period (when damping is needed most), they tend to drift, nullifying the usefulness of their characteristic damping. Rubber isolators are best suited for applications where the resonant frequency of the system occurs infrequently in the environment.

8-19. **CONSTRUCTION OF RUBBER VIBRATION ISOLATORS.** There are two types of constructions for rubber isolators: open and cup types. An open-type isolator is illustrated in figure 8-7. It consists of a molded rubber form bonded to a metal mounting flange and to a core. The core, a cylinder in the center of the isolator, also is metal and attaches to the equipment.

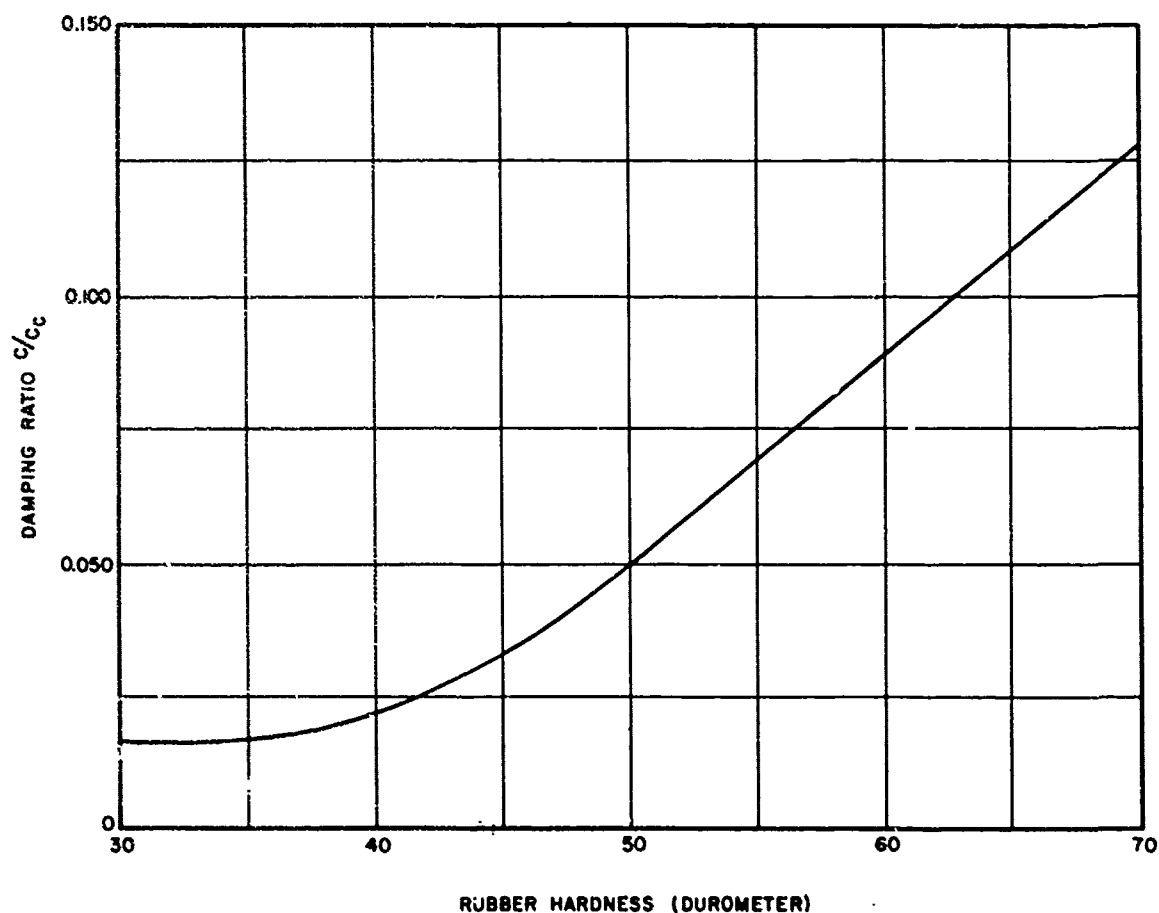


Figure 8-6. Damping Characteristics of Rubber Related to Rubber Hardness

8-20. Figure 8-8 is a photograph of cup-type isolators. The resilient element for each isolator is rubber and is enclosed in, and bonded to, the cup (housing). The core, shown protruding from the cup, also is bonded to the rubber. This isolator has an advantage over the open type in that, even with complete failure of the resilient element, the cup and core will hold the equipment captive. Both the open- and cup-type isolators are available with different rubber compounds, spring rates, and horizontal-to-vertical stiffness ratios.



Figure 8-7. Open-Type Rubber Vibration Isolator

#### 8-21. METAL-SPRING ISOLATORS.

8-22. Metal-spring isolators have advantages and disadvantages when compared to rubber isolators. Metal springs do not drift, are not affected by temperatures found in the airborne environment, and their service life usually is unlimited. Their inherent damping characteristics, however, are inadequate (see paragraph 5-47). Without auxiliary damping devices, which are costly and often unreliable, metal-spring isolators would allow intolerable transmissibilities at resonance and provide poor shock protection.

8-23. AUXILIARY DAMPING. Auxiliary damping devices may consist of vented air sacks, friction elements, or metal mesh. The principles of air damping and friction damping are detailed in Section V. Metal mesh is, essentially, a nonlinear spring of knitted wire damped by the friction between the interlocking wire loops. In some applications, mesh is used simply as a friction element and bears no load. In others, it acts as the main resilient element, bearing the major portion of the load in addition to providing damping.

8-24. CONSTRUCTION OF METAL-SPRING VIBRATION ISOLATORS. Metal-spring vibration isolators also are available in cup and open types. The cup types employ either coil springs or metal mesh, or a combination of the two, as the resilient element. Construction of the coil-spring, cup-type isolators varies according to the damping method employed in the isolator and the type of spring (linear or nonlinear) used. The metal-mesh, cup-type isolators need no additional damping; the mesh doubles as the damping agent. The metal mesh is always nonlinear.

8-25. The open-type metal-spring vibration isolators use metal mesh as the resilient element in order to include damping action in the isolator performance. Since there is no cup (housing), it would be difficult to provide damping

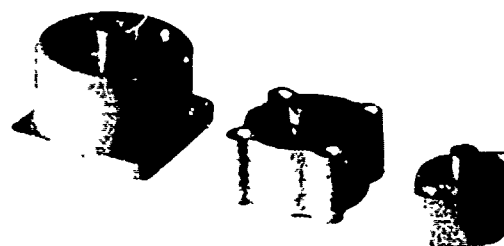


Figure 8-8. Cup-Type Rubber Vibration Isolator

by other methods. Either the cup or open types can be single acting or double acting. A single-acting isolator will provide isolation only when the equipment is oriented so that its weight is acting to compress the isolator (unidirectional static acceleration). A double-acting isolator will provide isolation no matter how the equipment is oriented.

8-26. Figures 8-9 through 8-12 illustrate different types of metal-spring vibration isolators. Figure 8-9 shows a cup-type isolator with auxiliary friction damping. The friction damping is provided by a composition damping element rubbing against the inside surface of the cup. The damper-and-auxiliary-load spring bears a portion of the load, and the topmost coil presses the damper tight against the cup. A synthetic rubber grommet provides snubbing at the top limit of travel and the damping element provides snubbing at the bottom of travel.

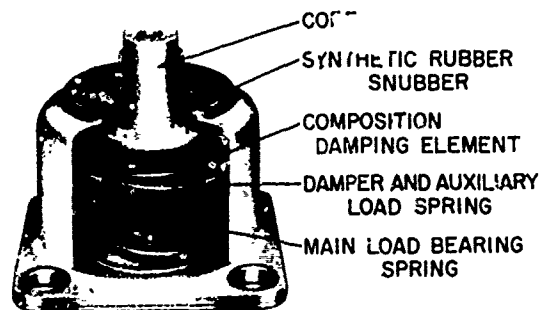


Figure 8-9. Cup-Type Metal Spring Vibration Isolator with Friction Damping

8-27. Figure 8-10 shows an example of an all-metal, cup-type isolator that uses metal-mesh as the friction damping element. A heavy, finger-shaped metal mesh rubs against the inside surface of the core and against the coils of the load-bearing spring during movement of the core. The metal mesh bears no load and functions strictly as a friction element. Top and bottom snubbing is provided by a metal-mesh pad fixed on the flange of the core.

8-28. Figure 8-11 illustrates a cup-type, air-damped isolator. A synthetic rubber bellows damps the system by the flow of air through an orifice. A nonlinear load-bearing spring is pictured inside the bellows. A rubber grommet in the opening of the cup provides top snubbing and limits side to side motion. This type of damping loses some of its effectiveness at higher altitudes.

8-29. Metal mesh is used as a load-bearing, nonlinear spring in the isolator shown in figure 8-12. The coil spring around the outside of the mesh functions

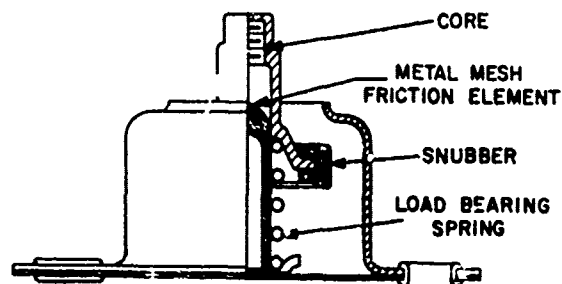


Figure 8-10. All-Metal, Cup-Type Vibration Isolator with Metal Mesh as the Friction Element

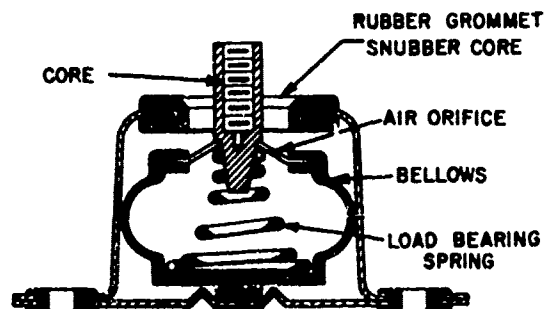


Figure 8-11. Cup-Type, Metal-Spring Vibration Isolator with Air Damping

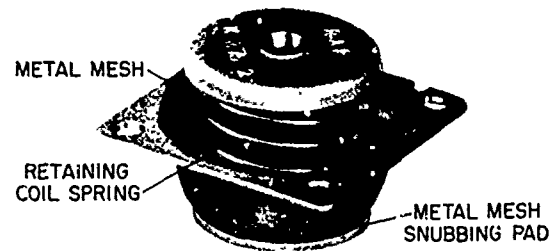
mainly as a retainer for the mesh, and bears little of the load. Damping comes from the mechanical interaction of the loops in the mesh. The snubbing pad on the bottom of the isolator limits travel in an upward direction. In the downward direction the isolator is snubbed by solid compression of the mesh spring.

**8-30. DOUBLE-ACTING, METAL-SPRING VIBRATION ISOLATORS.** A double-acting, metal-spring isolator, when mounted under or aside an equipment, will perform equally well whether the equipment is in a normal or an inverted position. At attitudes between these two positions, performance varies according to individual designs. Figure 8-13 shows an example of a double-acting isolator that uses nonlinear coil springs and friction damping. The core is fastened to the two steel washers centered between the coil springs which provide the isolation. The washers grip the phenolic damping element to provide damping in side-to-side motions of the core. Damping in the vertical direction is provided by the phenolic damper rubbing against the inside surface of the cup. The damping spring maintains a constant pressure between the damper and the cup. Top and bottom snubbing occurs when the damper contacts either of the molded nylon spring seats.

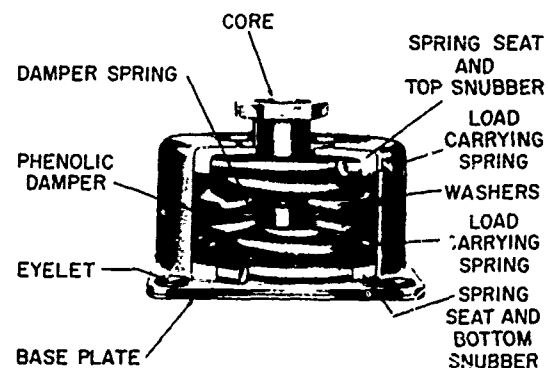
**8-31.** Pictured in figure 8-14 is another style of double-acting mount. It consists, essentially, of two of the resilient elements of the isolator in figure 8-12 joined back to back. The equipment may be fastened either to the axial bolt, or to the center flange.

#### **8-32. SHOCK ISOLATORS.**

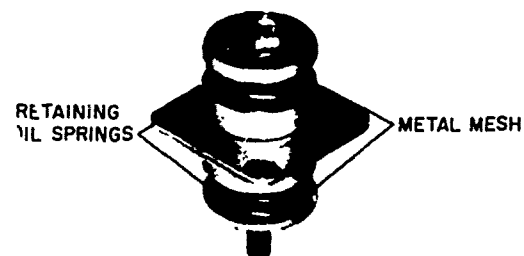
**8-33.** Shock isolators have stiffer springs than do vibration isolators and, consequently, have a higher natural frequency. Their resilient elements are always nonlinear, whereas some



**Figure 8-12. Vibration Isolator with Metal Mesh Resilient Element**



**Figure 8-13. Double-Acting, Metal-Spring Vibration Isolator with Friction Damping**



**Figure 8-14. Double-Acting Vibration Isolator with Metal Mesh Resilient Element**

vibration isolators use linear springs. Table 8-2 compares the characteristics of shock isolators and vibration isolators.

**TABLE 8-2. CHARACTERISTICS OF SHOCK ISOLATORS  
COMPARED TO VIBRATION ISOLATORS**

Shock Isolators	Vibration Isolators
20-40 cps natural frequency.	7-25 cps natural frequency.
Resilient elements are highly nonlinear.	Resilient elements are linear or nonlinear.
Natural frequency changes with high amplitude vibration.	Natural frequency changes little or not at all with high amplitude vibration.
Very little provision for excursion.	Provision for excursion.

8-34. Shock isolators are used to mount electronic equipment when the anticipated environment is such that vibration fatigue is less a hazard to the equipment than is shock. This environment is most likely to occur in missiles. Missiles have a short operational life reducing the hazard of vibration fatigue. Shocks occur due to the acceleration of takeoff, the separation of engines, and the acceleration of a new stage.

8-35. Shock isolators, as defined in table 8-2, are not used in protecting electronic equipment in manned aircraft. Under high-amplitude, low-frequency vibration, the use of such shock isolators could be more detrimental to the equipment than rigid mounting. Vibration, not shock is the important consideration in protecting equipment in an airplane. Where severe shock is expected in planes, devices which are primarily vibration isolators are modified for protection against shock. Some shock protective features of vibration isolators are stiffer linear springs, nonlinear springs, and damping.

#### 8-36. SELECTION OF ISOLATORS.

8-37. Isolators are selected on the basis of the weight they will be required to support, the type of mounting system in which they will be used, the critical frequencies of the equipment, and the environment they must be able to change and withstand.

8-38. **WEIGHT OF EQUIPMENT.** The weight of the equipment and the desired natural frequency determine the spring rate of the isolator. Figure 8-15 is a chart from which the spring rate for a linear isolator can be selected that will give a system natural frequency for a given equipment weight.

8-39. The spring constant of each isolator in a mounting system can be equal or not, depending upon the configuration of the equipment and whether decoupling of vibratory modes is desired. If the center of gravity is such that the load on each isolator is equal, then all spring constants will be equal. When the center of gravity

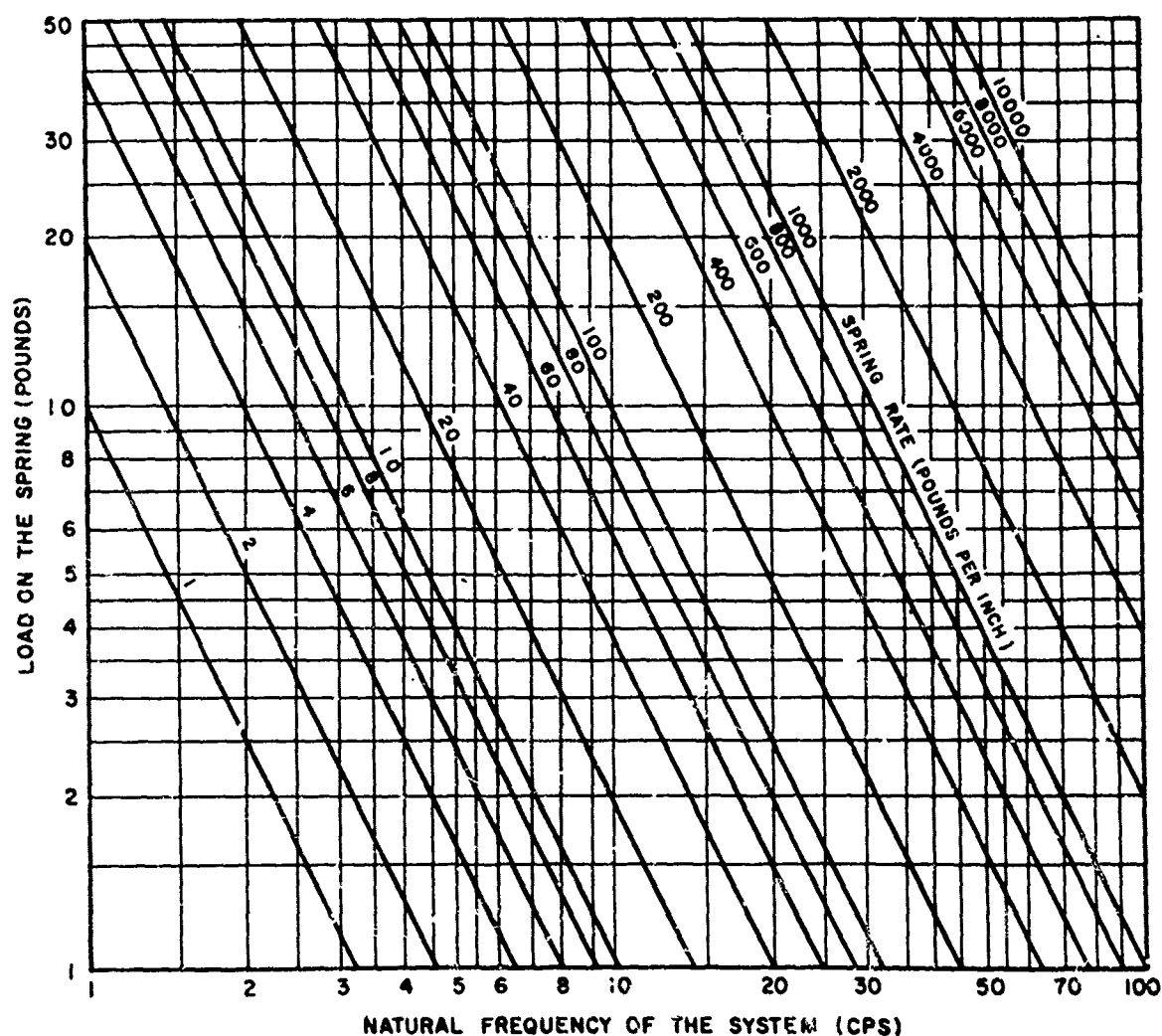


Figure 8-15. Natural Frequency as a Function of Spring Rate for a Mass Supported by Linear Springs

results in an unequal distribution of weight to the isolators, however, adjustments can be made in the spring constants so that decoupling can still be achieved within a mounting system. (Methods of varying the spring constants to achieve decoupling are discussed in detail in Section IX.)

**8-40. MOUNTING SYSTEM.** The type of mounting system used will influence the isolator horizontal-to-vertical-stiffness ratio. For example, in underneath and inclined-isolator mounting systems, the horizontal and vertical stiffnesses are different for maximum system effectiveness. In the center-of-gravity and double-side mounting systems, equal stiffnesses are used in the horizontal and vertical directions.

**8-41. FRAGILITY LEVEL OR CRITICAL FREQUENCIES OF AN EQUIPMENT.** Vibration isolators reduce the vibration environment to a level that can be reliably tolerated by the equipment. The fragility level of an equipment (the maximum levels of vibration, at discrete frequencies, the equipment can withstand) must be

kept sufficiently above the level of the vibration environment. Figure 8-16 illustrates the manner in which vibration isolators can accomplish this objective. The three levels shown are the fragility level, the level of vibration environment, and the level of the environment reaching the equipment via the isolators.

8-42. The equipment fragility level is high (i. e., less apt to fail) at frequencies below approximately 40 cps. Vibration environments are normally at a low level below 50 cps. The two curves intersect at approximately 50 cps, and at this point failure could be expected. Failure also would be expected at all the higher frequencies. Mounting the system on isolators gives the system a natural frequency of approximately 20 cps. This raises considerably the level of vibration transmitted to the equipment in the area below 30 cps; however, this is where the fragility level of the equipment is highest. At all points above 30 cps the transmitted environment is below the natural environment and, also, below the fragility level of the equipment. Thus, it can be expected that the equipment will perform satisfactorily.

8-43. **THE ENVIRONMENT.** As discussed previously in Section II, electronic equipment is subjected to two types of environment: (1) transportation and handling prior to installation in the aircraft and (2) operational environments. The environment of transportation and handling is not influenced by the type of equipment nor by the type of aircraft in which the equipment will be eventually installed. The levels of shock and vibration that can be expected during transportation and handling are indicated in Section II. The operational environments, however, do differ depending upon the type of aircraft.

8-44. The different classifications of aircraft are fighter, bomber, cargo, trainer, helicopter and missile. There are significant differences in the design

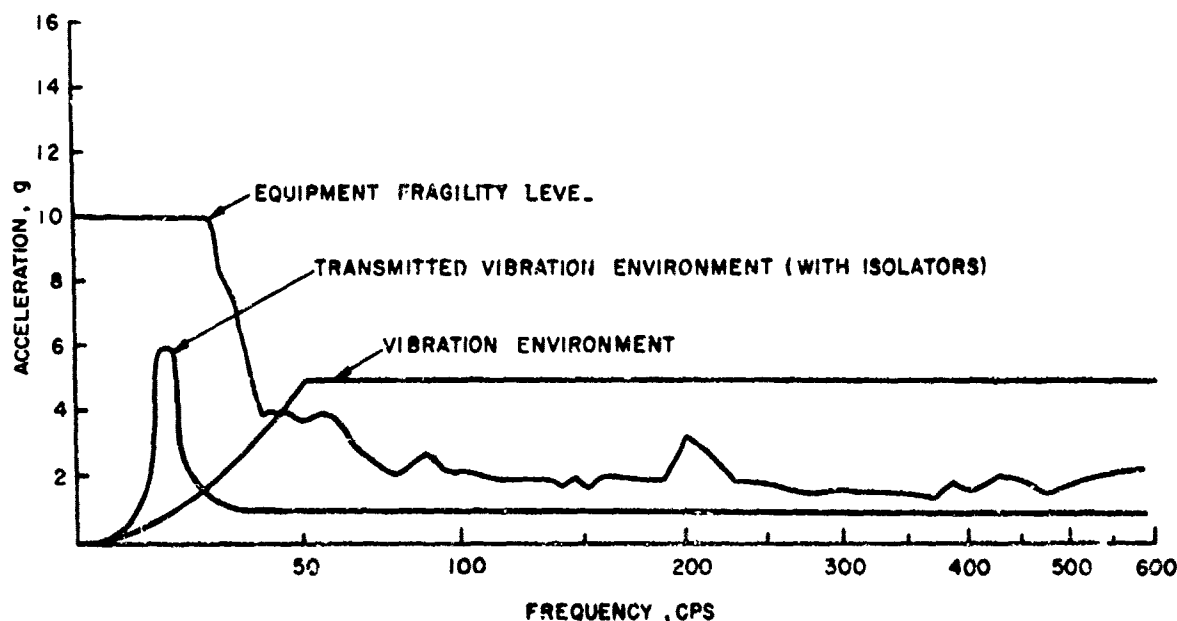


Figure 8-16. Using an Equipment Fragility Curve as a Means for Selecting Isolators to Modify an Environment

strength of these different aircraft and, consequently, the flexibility of wings, tail surfaces, and fuselage varies. This results in different aircraft response to excitations such as the power plant, gunfire, maneuvers, wind buffeting, and landing shock. The type of power plant (reciprocating engine, turbo jet, turbo prop, ram jet, or rocket), also provides different excitations, further complicating the situation.

8-45. Although isolators are selected mainly for the protection they offer against vibration, shock environments must be given careful consideration. The most severe shock environments occur during landing and booster-assisted or catapult-assisted takeoffs. The direction of the shock is as important as the intensity. Shock in a downward direction will not compress the load-bearing spring of a single-acting isolator, and the shock will be transmitted directly through the top snubbers. Shock in a lateral direction will also cause snubber contact if horizontal stiffness of the isolators is low, as it can be for bottom-mounted equipment (see Section IX).

8-46. Interceptor and fighter aircraft, and guided missiles spend considerable time in maneuvers such as sustained near-vertical climbing, and uncoordinated slow rolls and slow loops, and, as a result, the attitudes a craft will assume with respect to earth should be considered when selecting isolators. Figure 8-17 shows

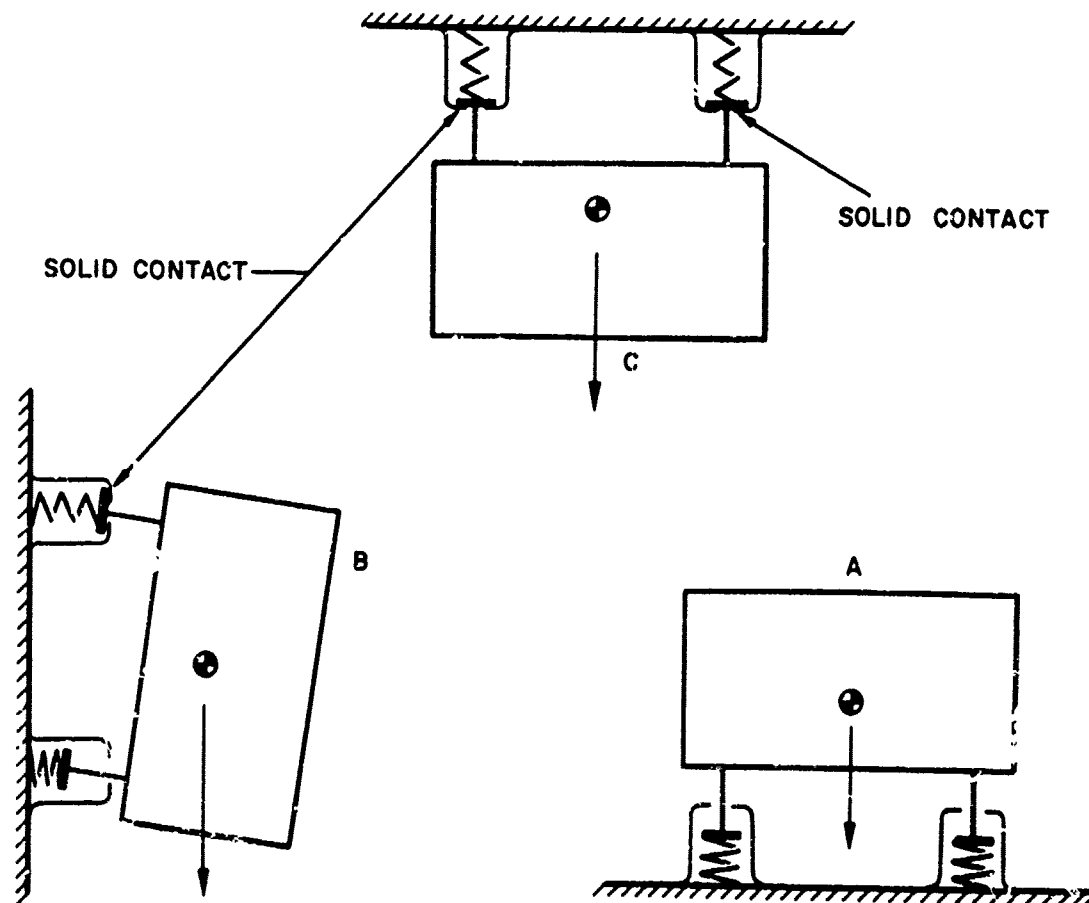


Figure 8-17. The Effect of Aircraft Attitude on Single-Acting Vibration Isolators

the difficulties that might occur if single-acting isolators were used in this type of environment. In level flight (position A) the single-acting isolators work perfectly. In position B, as might be encountered in a near-vertical climb or dive, gravity acts to unload the equipment from the top isolator and load the bottom isolator. At the top of a slow loop or slow roll, the equipment would be hanging from the mounts as shown by position C. In positions B and C, where there is solid contact at the top of the snubbers, the isolators would not function.

8-47. Figure 8-18 shows the effects on vibration isolators of static acceleration due to maneuvering. When nosing down, static acceleration acts to drive the equipment away from the points of connection. In a dive pullout, the equipment is forced toward the points of connection. Obviously, double-acting isolators would be necessary if effective isolation is to be maintained, regardless of the attitude of the aircraft.

8-48. In addition to the different shock and vibration environments resulting from the various classifications of aircraft, the environments within a particular aircraft will vary depending upon the location of the electronic equipment relative to the source of excitation. For example, "g" loadings will vary considerably between

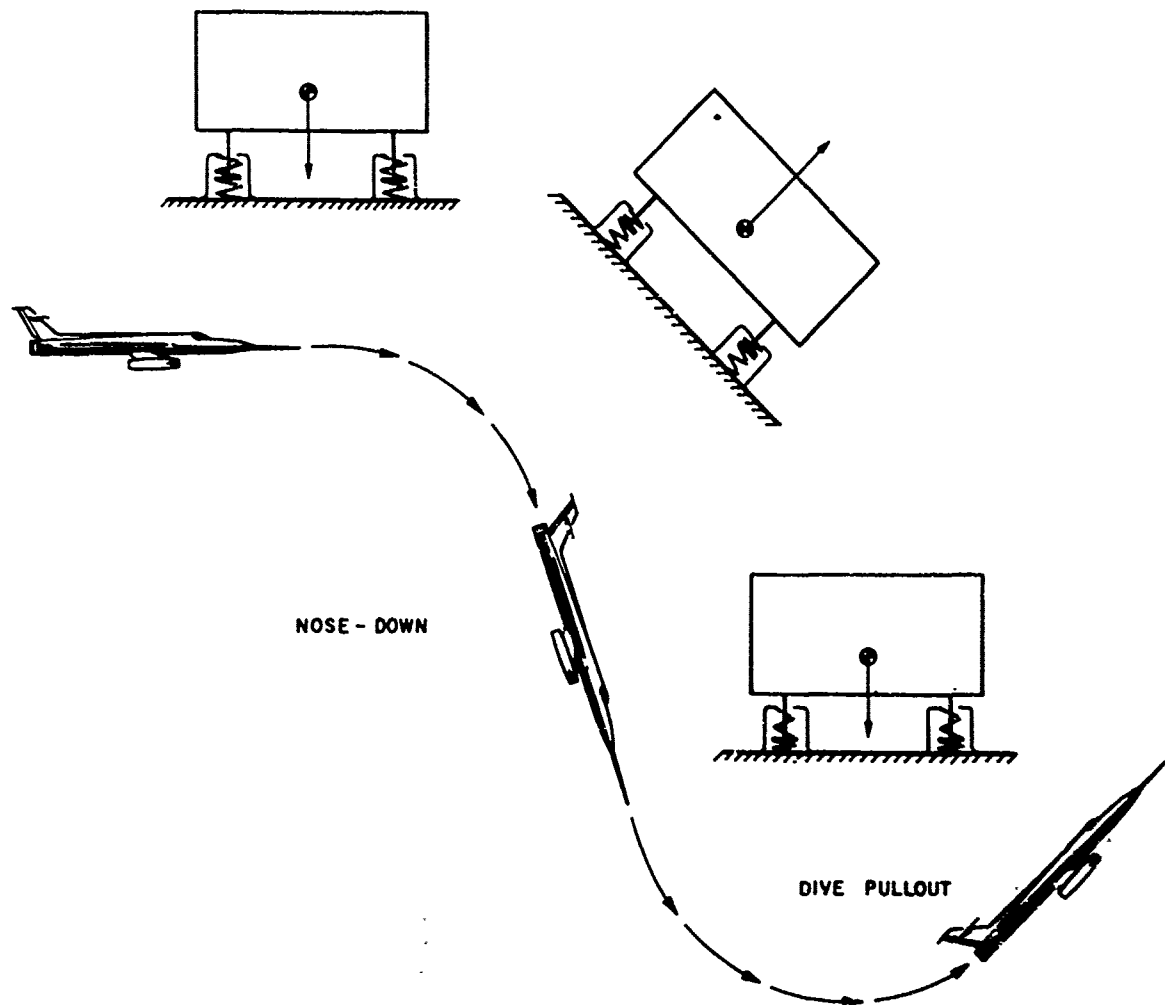


Figure 8-18. Effect of Static Acceleration on Isolators

the fuselage, tail structure, and wings of an aircraft. Also, if the electronic equipment is to be mounted in the vicinity of the guns which provide severe transient excitation, this factor should be considered.

8-49. Exciting vibrations in various aircraft environments are graphically summarized in Section II. Data is given for jet aircraft, helicopters, and various missiles. The data is far from comprehensive, but a few generalizations can be made about vibration environments which would be of value in selecting isolators.

8-50. Isolators for use in jet aircraft are chosen to give a vertical natural frequency of from 12 to 20 cps. The practice of planning the natural frequency of the mounting system and equipment to be substantially lower than the exciting frequency is impossible for equipment mounted in helicopters, since the exciting frequencies are as low as 3 cps. Thus, a natural frequency for the mounting system and equipment is chosen about half way between the critical exciting frequencies, and the system must be heavily damped to limit transmissibility at and near resonance.

8-51. Vibration and shock environments in missiles vary drastically between types, prohibiting any valid generalizations. In fact, because of the severe vibration and shock forces involved, the decision even to use isolators cannot be made without study. Each missile is a problem in itself. In some missiles, the environment is such that the electronic equipments should be mounted on vibration isolators; in others, shock isolators should be used; in still others, the equipment should be bolted down solidly.

#### 8-52. STABILIZERS.

8-53. Tall electronic equipment which is bottom mounted may sway excessively under resonant or shock conditions. The top of the equipment may swing through a large enough arc to strike adjacent bulkheads or equipment. Devices designed to prevent such occurrences are termed stabilizers. Stabilizers have very little axial stiffness and a nonlinear spring rate in the lateral direction. The construction of a stabilizer is shown in figure 9-10. More detailed information on the application of stabilizers is given in Section IX.

#### 8-54. CRASH-SHOCK-ABSORBER LINKS.

8-55. Electronic equipment installed in aircraft can, during a minor plane crash, fly loose from its supports and become a hazard to personnel who would otherwise be uninjured. One solution to the problem is to strengthen the isolators and mounting bases. Stainless steel can be used instead of aluminum, and the parts of the mounting base can be interlocked and overlapped in such a way that separation becomes unlikely. This is an expensive method, however, and adds weight to the installation.

8-56. Crash-shock-absorber links are available which prevent dislodgement of equipment during a plane crash. They consist of two short lengths of wire rope joined with a tough, expandable shock-absorbing link. One end of the device is attached securely to the airframe, and the other is attached to the chassis of the equipment (not the dust cover). If the chassis is not accessible, the equipment may be bound in web strapping and the retaining link attached to the strapping.

## SECTION IX

### EQUIPMENT MOUNTING AND INSTALLATION TECHNIQUES

#### 9-1. GENERAL.

9-2. Isolators are selected on the basis of the dimensions, weight, and weight distribution of an equipment, the anticipated environment, and the type of mounting system that is planned. Thus, the choice usually occurs during the late design stages of an equipment. The type of mounting system, however, should be considered much earlier because it influences the design of the primary chassis sections and the placement of subassemblies that will attach to them.

9-3. The selection of an isolation mounting system is dependent mainly upon the space allowed for both the isolation system and the expected sway of the equipment. For example, if the slot necessitates an equipment be higher than it is wide, resulting in a high center of gravity, a bottom-mounting system should be avoided, unless stabilizers are used. The anticipated environment, including the exciting frequencies, and the stiffness of the supporting structure, and the required stability of the equipment are other mounting-system determinants in the early design stages. Later, it is necessary to know the weight, mass distribution, location of the center of gravity, and the radii of gyration (three) of the equipment.

9-4. Theoretical discussions of single-degree-of-freedom systems serve to provide a basic understanding of vibration isolation. In airborne electronic installations, however, single-degree-of-freedom systems rarely occur. If an equipment were constrained to move in a vertical direction only, the horizontal vibrations or shocks would be transmitted directly through the rigid guides to the equipment, without the benefit of isolation.

9-5. An equipment mounted in an aircraft will be subjected to vibration and shock in all directions; consequently, it must be free to move in all directions for total isolation. This results in a six-degree-of-freedom system. The equipment is free to move translationally in vertical, longitudinal, and lateral directions, and rotationally about the vertical, longitudinal, and lateral axes (figure 9-1). The system has a natural frequency in each of these natural modes and all must be considered when planning the isolation system.

9-6. When resilient supports of equal stiffness are located unsymmetrically about the center of gravity of an equipment, certain of the rotational and translational modes will couple. When coupled, vibration cannot exist in one mode without existing in its coupled mode or modes. Thus, a horizontal force through the center of gravity will not only displace the equipment horizontally, but also will cause it to rotate. Each coupled mode has its own frequency and, again, all must be considered when planning the isolation system.

9-7. As discussed for a single-degree-of-freedom system in paragraph 1-25, satisfactory isolation will occur if the natural frequency is well below the forcing frequency. In a six-degree-of-freedom system with all frequencies coupled, all coupled frequencies must be below the forcing frequency. However, predicting the frequencies for coupled modes of vibration means solving six simultaneous equations, which is a difficult and time-consuming task. The practice is to decouple some or all of the modes by arranging the isolators into mounting systems. This simplifies the frequency analysis considerably. In addition, it generally is true that the natural frequencies can be made equal only if the modes are not coupled.

9-8. Obviously, it is advantageous to attempt to determine the natural frequencies of an isolator system in the preliminary design stages of the equipment. This aids in avoiding the possibility of a resonant condition developing and also assists in anticipating the effectiveness of the intended isolator system. Vibration isolators may be arranged in many ways, but, generally, all arrangements are variations of three basic systems: (1) the underneath mounting system, (2) the center-of-gravity mounting system, and (3) the double-side mounting system. Each system has its advantages and limitations. The following discussion assumes a rigid supporting structure for the mounting systems. In general, to prevent the supporting structure from disturbing the performance of the system, the natural frequency of the supporting structure should be at least three times the natural frequency of the equipment on its isolators.

#### 9-9. UNDERNEATH MOUNTING SYSTEMS.

9-10. The underneath mounting system is the most frequently used for isolating equipment against vibration. There are various reasons for its popularity. Most supports for equipments, such as shelves and decks, accommodate bottom-mounted equipments. A bottom-mounted equipment will fit just about anywhere making it a versatile installation. If a mounting system is not planned in the early design stages of an equipment, in all probability, the chassis will best accommodate isolators mounted underneath it. And, finally, some designers unfortunately are unfamiliar with the advantages of other mounting systems.

9-11. COUPLING OF MODES. Isolators mounted underneath the equipment are, of course, unsymmetrically placed about the horizontal plane through the center of gravity; thus, rotational and horizontal translational modes will couple. If identical isolators\* also are placed unsymmetrically about both vertical planes through the center of gravity, all of the rotational and translational modes will couple. In all cases, the location of the center of gravity must be considered with cables and connectors attached to the equipment.

9-12. Figure 9-2 shows the three possible placings of isolators with respect to the vertical planes through the center of gravity. In figure 9-2a, the isolators are

\* Since the practice is to use identical isolators in a mounting system, the discussion throughout this section refers to identical isolators as the general case. Therefore, all discussions about degrees of coupling and the symmetrical placing of isolators about a plane through the center of gravity assume that identical isolators are used.

placed unsymmetrically about both the XY and the YZ planes. Thus, as is indicated, all modes are coupled to each other, and vibration in any one predominant mode has elements of vibration in all other modes. Those modes adjacent to the predominant mode have the major effect on the vibratory motion. With total coupling, the vibratory motions are extremely complex and difficult to analyze.

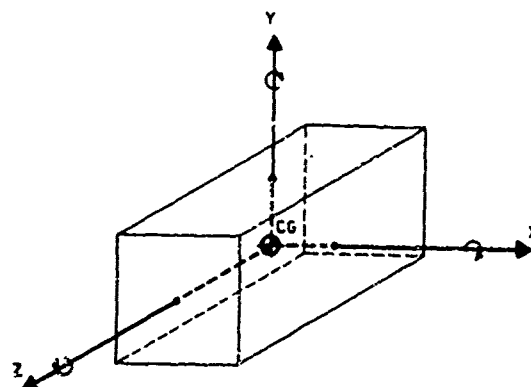


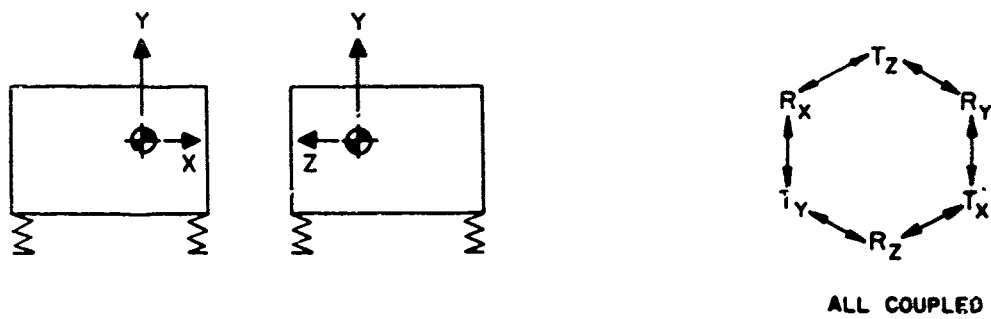
Figure 9-1. Six Degrees of Freedom:  
Three in the Translation Modes and  
Three in the Rotational

9-13. In figure 9-2b, the isolators are symmetrically placed about the YZ plane but unsymmetrically placed about the XY plane. No mode is completely decoupled, but all modes are not coupled to each other. As is indicated in the figure, translation in the Z and Y directions will couple with rotation about the X axis, and rotations about the Y and Z axes will couple with translation in the X direction. However, the two groupings are decoupled from each other because of the symmetrical placing of the isolators with respect to the YZ plane. Although less complex, the motion in any mode is still difficult to analyze.

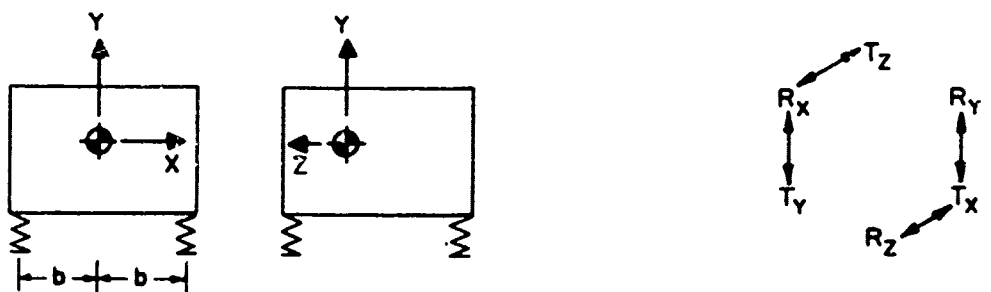
9-14. In figure 9-2c, the isolators are symmetrically placed about both the XY and YZ planes. Translation in the Y direction is decoupled from all other modes as is rotation about the Y axis. Translation in the Z direction is coupled with rotation about the X axis and translation in the X direction is coupled with rotation about the Z axis, but each couple is decoupled from the other. This is as uncomplicated a motion as is possible using the underneath mounting system. The six different motions can be described as: (1) vertical, (2) rotational about the Y axis, (3) low rocking in which the box rotates about an axis which is parallel to and below the Z axis, (4) high rocking in which the box rotates about an axis parallel to and above the Z axis, (5) low rocking in which the box rotates about an axis parallel to and below the X axis, and (6) high rocking in which the box rotates about an axis parallel to and above the X axis.

9-15. **DECOUPLING OF MODES FOR AN UNSYMMETRICAL BOX.** When identical mounts are located equidistant from the center of gravity in the X and Z directions (figure 9-2c), decoupling is accomplished, and the resulting vibratory motions for an underneath mounting system are simplified to the extent that the frequencies can be approximately predicted. Determining the load requirements for each isolator mount is a simple matter of dividing the total weight of the equipment by four. When it is impossible to locate the four mounts symmetrically about the XY and YZ planes, decoupling of modes still can be accomplished by using isolators of different spring constants. Figure 9-3 gives four formulas for determining the load on each isolator. The four isolators then are selected to give the same static deflection and have the same natural frequency when carrying their share of the load.

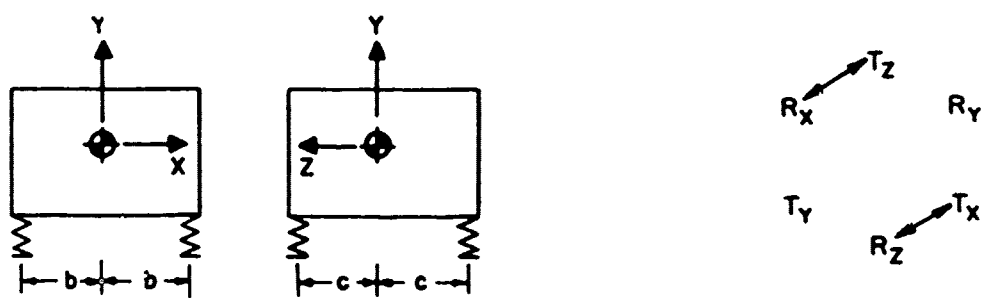
9-16. **PREDICTING NATURAL FREQUENCIES.** Predicting the natural frequencies for an underneath mounting system with a decoupled vertical mode is relatively



(A)



(B)



(C)

Figure 9-2. Degrees of Coupling of Rotational and Translational Modes in an Underneath Mounting System

simple compared to what it would be if it were not decoupled. The natural frequency of the vertical vibration mode is calculated and then is used as a reference for determining the coupled natural frequencies. In general, underneath mounting systems have at least one natural frequency greater than the vertical natural frequency. Consequently, the vertical natural frequency must be lower than it would be normally to maintain the desired separation between the forcing frequency and any natural frequency, if the coupled modes were not considered.

9-17. The uncoupled vertical natural frequency is calculated by using the formula

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

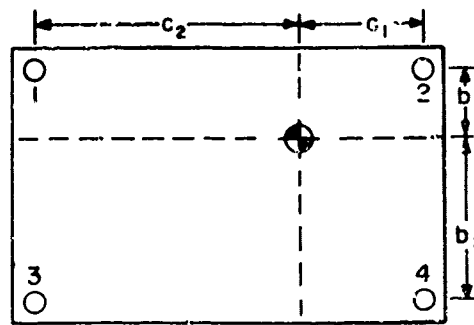
As described in paragraph 1-24, this is the natural frequency of a single-degree-of-freedom mass-spring system. The mass "m" can be the total mass of the equipment or the mass supported by any one of the isolators. If the total mass is used, then the total of the four spring constants must be used. If the mass considered is that supported by one of the isolators, then "k" is the spring constant of the one isolator. The mass on one isolator is either 1/4 of the total mass if identical mounts are located equidistant from the center of gravity in the X and Z directions, or, if mounted unsymmetrically, it is the mass supported by one of the isolators as calculated by the formulas given in figure 9-3.

9-18. The static deflection also can be used as a parameter in calculating the vertical natural frequency. In paragraph 4-13, it was shown that the natural frequency of a single-degree-of-freedom system is proportional to the static deflection. Thus, the relationship

$$f_n = 3.13 \sqrt{\frac{1}{\delta}}$$

can be used where  $\delta$  is equal to the static deflection in inches.

9-19. With the vertical natural frequency known, the coupled frequencies can be determined by reference to a chart. Figure 9-4 permits the determination of the four coupled frequencies: the low and high rocking modes about axes parallel to the Z and X axes (see paragraph 9-14). To use this chart, however, it is necessary to calculate the radii of gyration of the equipment.



$$\text{Load on Isolator 1} = W \left( \frac{c_1}{c_1 + c_2} \right) \left( \frac{b_2}{b_1 + b_2} \right)$$

$$\text{Load on Isolator 2} = W \left( \frac{c_2}{c_1 + c_2} \right) \left( \frac{b_2}{b_1 + b_2} \right)$$

$$\text{Load on Isolator 3} = W \left( \frac{c_1}{c_1 + c_2} \right) \left( \frac{b_1}{b_1 + b_2} \right)$$

$$\text{Load on Isolator 4} = W \left( \frac{c_2}{c_1 + c_2} \right) \left( \frac{b_1}{b_1 + b_2} \right)$$

Note: W is the weight of the equipment with connectors and cables attached

Figure 9-3. Determining the Load on Each Isolator when the Center of Gravity is Located Unsymmetrically in the Horizontal Direction

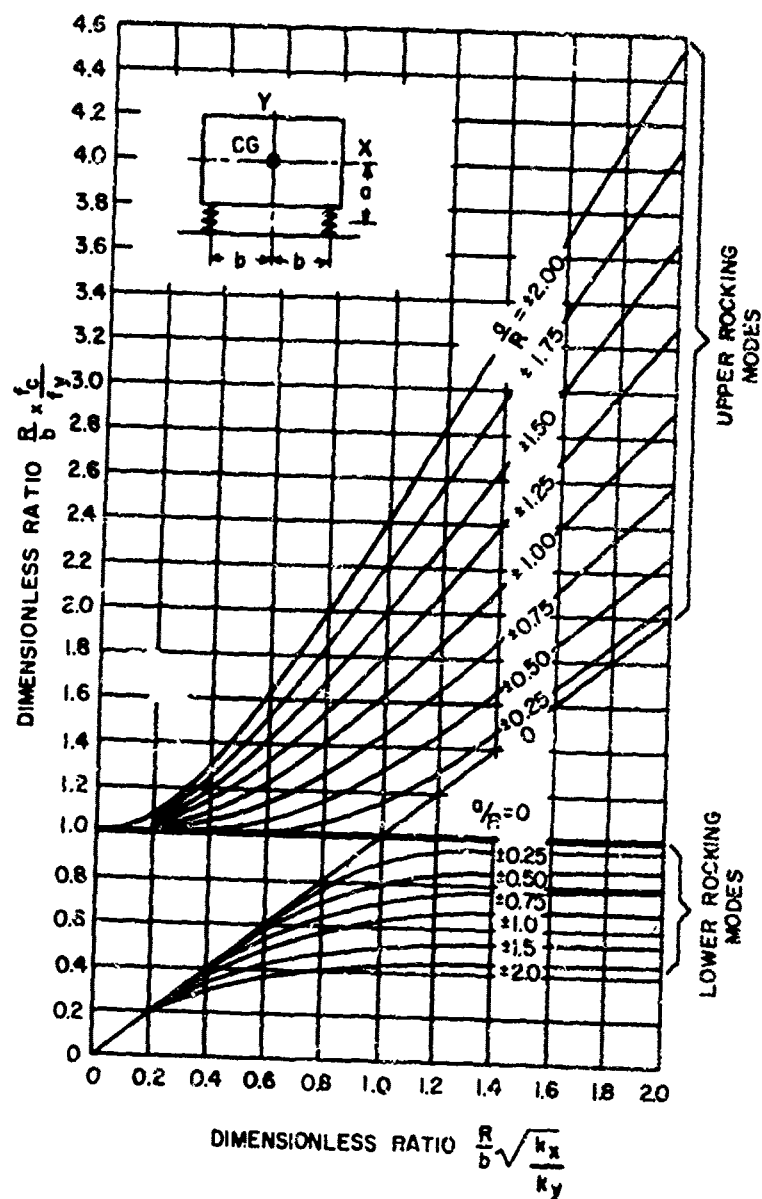


Figure 9-4. Determination of Coupled Natural Frequencies for an Underneath Mounting System

9-20. Each box has three principal radii of gyration: one each about the X, Y, and Z axes. Since the coupled modes exist about the X and Z axes only, it is unnecessary to calculate the third radius of gyration, unless the natural frequency of the rotational mode about the Y axis also is desired. If the body is a homogeneous\* cuboid (rectangular parallelepiped) or cylindrically shaped object, the radii of gyration can be accurately calculated by referring to figure 9-5.

\* In which case the center of gravity will be in the geometric center of the equipment.

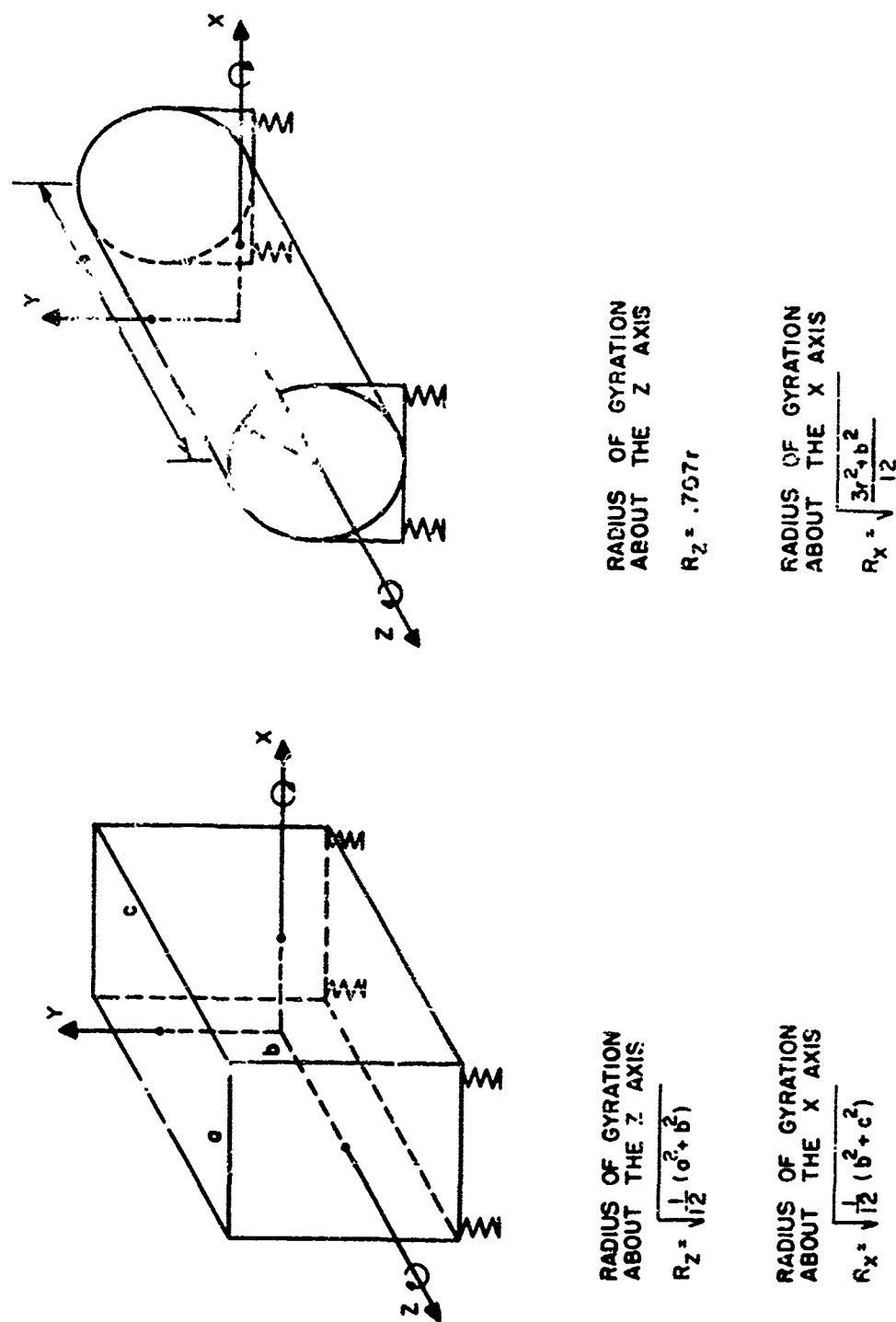
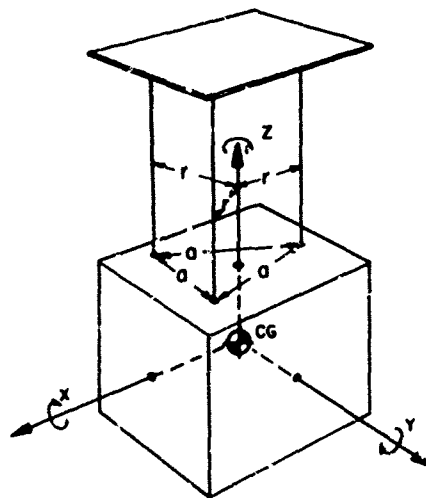


Figure 9-5. Determining the Radii of Gyration About the X and Z Axes for Homogeneous Equipments

9-21. If the equipment is heterogeneous, the formulas given in figure 9-5 still can be used for calculating the radii of gyration, but they will give only approximations. Fortunately, a rough figure usually is sufficient for estimating natural frequencies, since resonant frequencies usually are avoided by a good margin.

9-22. If the equipment is available, an experimental method gives precise values for the radii of gyration regardless of the homogeneity of the equipment. The equipment is hung from three wires of equal length equidistant from the center of gravity and from each other, as shown in figure 9-6. (See paragraph 9-42 for methods of determining the center of gravity.) To find the radius of gyration about the Z axis, the equipment is oriented so that the Z axis is parallel to the suspension wires. The period of oscillation as a torsional pendulum rotating in a horizontal plane is determined by counting and timing with a stop watch. The values then are substituted in the formula given in figure 9-6. The equipment is reoriented and the procedure repeated for finding the radii of gyration about the other axes.



- 1. RADIUS FROM THE CENTER OF GRAVITY TO THE SUSPENSION WIRE IN INCHES
- 2. LENGTH OF SUSPENSION WIRE IN INCHES
- 3. PERIOD OF TORSIONAL OSCILLATION IN SECONDS

$$R_z^2 = \frac{386 \cdot l^2}{4 \cdot \pi^2}$$

Figure 9-6. Determining the Radius of Gyration by the Experimental Method

9-23. Returning to figure 9-4, with values for the vertical natural frequency, the radius of gyration, the dimension "a" which is the vertical distance from the center of gravity to the center of the isolator, the dimension "b" which is the horizontal distance from the center of gravity to the center of the isolator, and the vertical and horizontal stiffnesses of the isolator, all the information necessary to determine the coupled frequencies is available. The dimensions on the drawing in the upper left-hand corner of the chart are set up to give the coupled frequencies of the upper and lower rocking modes about axes parallel to the Z axis (perpendicular to the paper) for an equipment with its mounts located equidistant from the center of gravity.

9-24. Using the radius of gyration about the Z axis " $R_z$ " and the dimension "a", the first step is to select two of the curves, one for the low rocking mode, and the other for the high rocking mode. The dimensionless ratio at the bottom of the chart then is calculated, using " $R_z$ ", the dimension "b", and the horizontal and vertical stiffnesses of the isolators. The ratio of horizontal-to-vertical stiffness\* is somewhat arbitrary at this stage of design, but should be fairly well established in accordance with the general system planning (see paragraph 9-28). With the

\* It should be noted that the horizontal-to-vertical stiffnesses of the isolators and the spacing of the isolators, are the only values that can be changed conveniently in order to vary the coupled natural frequencies.

value set on the horizontal axis of the chart, and the two curves selected, projecting up to the two curves and then across to the vertical axis results in two values. Thus, with  $N_1$  and  $N_2$  designated as the two values, and using the dimensionless ratio on the vertical axis, the coupled upper and lower rocking modes about axes parallel to the Z axis are found by:

$$f_{c1} = \frac{N_1 bfy}{R_z} \quad \text{and} \quad f_{c2} = \frac{N_2 bfy}{R_z}$$

The whole procedure is repeated for finding the upper and lower rocking modes in axes parallel to the X axis, using the radius of the gyration about the X axis and the dimensions "a" and "b" resulting from the new orientation of the equipment.

9-25. If the mounts are located unsymmetrically in either or both the X and Z directions with respect to the center of gravity, but the modes have been decoupled by varying the spring constants of the isolators, figure 9-4 still can be used, but the "b" dimension must be changed to meet the unbalanced situation. Referring to figure 9-7,  $\sqrt{b_1 b_2}$  is analogous to, and is used instead of, "b." If the shape of the equipment is such that the isolators are on different levels (figure 9-7), "a" in figure 9-4 becomes  $\sqrt{a_1 a_2}$ . The use of this latter dimension is not rigidly correct and will introduce some error in the frequency calculations; however, for most applications the results will be sufficiently accurate.

9-26. Figure 9-8 is a chart that gives coupled natural frequencies somewhat more simply than figure 9-4, but, it can be used only for a special case. The chart is applicable only to underneath mounted equipments, cuboid in shape, of uniform mass distribution, and with the isolators attached precisely at the corners. As discussed previously with reference to figure 9-4, to find the coupled frequencies parallel to the Z axis, the dimensions are those taken with the equipment oriented so that the reader is looking into the Z axis, and, likewise, for the X axis.

9-27. With the determination of the vertical and the four coupled natural frequencies, the remaining mode in which the natural frequency is unknown is rotation about the Y axis. Normally, excitation in this mode of vibration is uncommon in airborne installations. However, if necessary, this natural frequency can be obtained by referring to figure 9-9.

9-28. **HORIZONTAL-TO-VERTICAL-STIFFNESS RATIO OF ISOLATORS.** As noted previously, varying the horizontal-to-vertical-stiffness ratio of the isolators will change the coupled natural frequencies in an underneath mounting system. Lowering the horizontal stiffness while keeping the same vertical stiffness will result in lower coupled natural frequencies. Conversely, raising the horizontal stiffness will raise

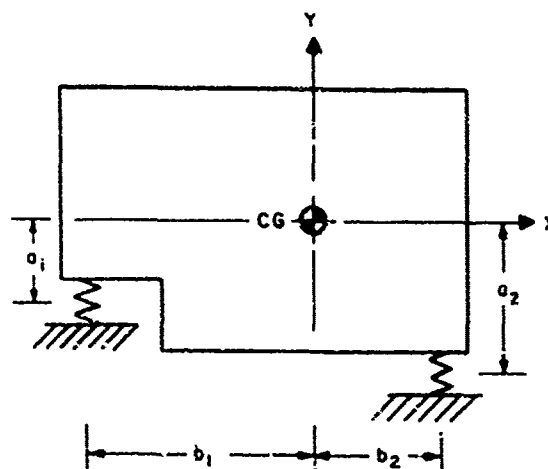


Figure 9-7. Underneath Mounting System with Isolators Mounted at Different Levels and, Also, Unsymmetrically About the YZ Plane

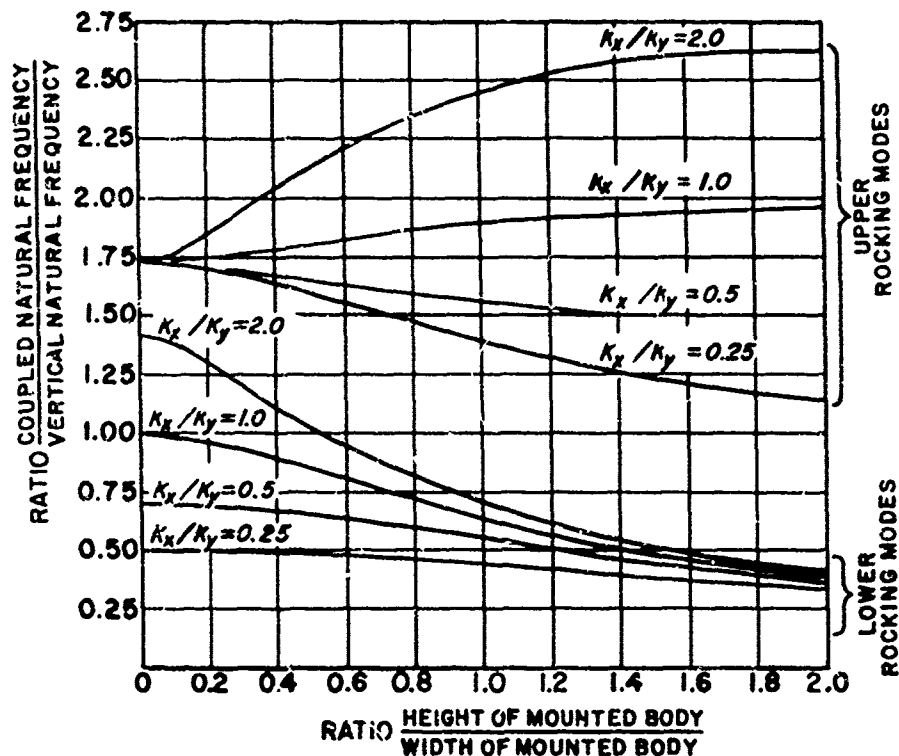


Figure 9-8. Coupled Natural Frequencies for an Underneath Mounting System as a Function of Isolator Stiffness Ratio

the coupled natural frequencies. This is readily apparent from figure 9-8, and, although less obvious, it also is evident from figure 9-4.

9-29. Since it is desirable to maintain the system natural frequency well below the forcing frequency, it would appear that the lower the horizontal stiffness, the better the installation. However, there are practical limitations to the horizontal-to-vertical stiffness ratio, because with no horizontal stiffness, the equipment could not be supported by the mounts. Generally speaking, horizontal-to-vertical stiffness ratios of from 0.25 to 0.50 have proved successful.

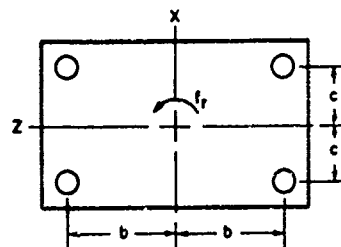
9-30. EQUIPMENT HEIGHT-TO-ISOLATOR SPACING. As the equipment height-to-isolator spacing is increased, the natural frequency of the lower rocking mode decreases. The frequency of the upper rocking mode also decreases if the horizontal-to-vertical stiffness ratio is maintained about 0.5 or less. However, when the ratio approaches 1.0, the trend reverses and the frequency of the upper rocking mode increases. These relationships are evident from figure 9-8. Although the bottom axis in the figure relates height and width, the isolators are assumed to be located at the corners; thus, this axis also can be referred to as height-to-isolator spacing.

9-31. If the horizontal-to-vertical-stiffness ratio were 0.5, or less, it would appear that the higher the equipment relative to its isolator spacing, the lower the rocking frequencies of the system, and, thus, the more desirable the installation.

Experience has indicated, however, that when the height exceeds the isolator spacing by more than 1-1/2 times, the frequency of the lower rocking mode is so low that the equipment tends to exhibit successive lateral motion under shock and resonant vibration conditions. This lateral motion can result in striking adjacent equipment or bulkheads unless sufficient clearance is allowed. Since space is at a premium in aircraft, excessive lateral motion must be avoided. This suggests that another mounting system be used better suited to accommodate tall equipments, or, if this is impractical, that stabilizers be used to limit the lateral motion.

**9-32. STABILIZERS.** Stabilizers are designed to limit the lateral motion of underneath mounted equipment and offer protection to the equipment against severe shock. While performing these functions, the stabilizer does not affect the efficiency of the mounting system. The construction and mounting of the stabilizer is illustrated in figure 9-10. The stabilizer effectively has no stiffness in the vertical direction throughout its operating range. Horizontal stiffness is provided by a resilient element, of predetermined stiffness characteristics, which buckles under a light horizontal load, buckles again under increasing horizontal loads, and then stiffens slowly in compression under severe horizontal loads. The stabilizer has a quick-disconnect method of attachment to the equipment. As indicated in figure 9-10, the stabilizer is positioned between the equipment and a rigid overhead supporting structure.

**9-33. Isolators,** intended as stabilizers, are sometimes mounted on the side of a tall equipment as a supplement to an underneath mounting system to limit lateral motion of the equipment and provide protection against severe shock. The isolator, however, with both horizontal and vertical stiffnesses, alters the frequency characteristics of



$$\frac{f_r}{f_y} = \sqrt{\frac{k_x (b^2 + c^2)}{k_y R_y^2}}$$

$f_r$  = THE ROTATIONAL FREQUENCY ABOUT THE Y AXIS

$f_y$  = THE VERTICAL NATURAL FREQUENCY PREVIOUSLY CALCULATED.

$k_x$  = THE HORIZONTAL STIFFNESS OF THE ISOLATOR. (IT IS ASSUMED THAT  $k_x$  AND  $k_y$  ARE EQUAL.)

$k_y$  = THE VERTICAL STIFFNESS OF THE ISOLATOR.

$R_y$  = THE RADIUS OF GYRATION ABOUT THE Y AXIS.

Figure 9-9. Determining the Natural Frequency for the Rotational Mode of Vibration About the Y Axis

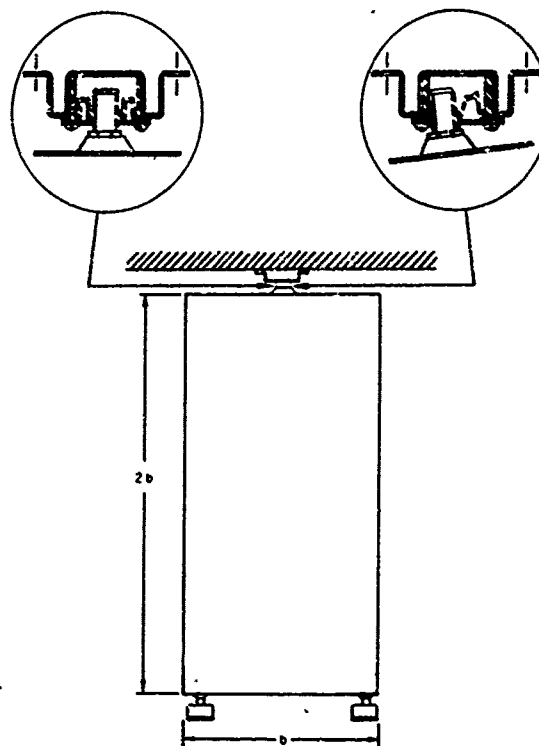


Figure 9-10. Stabilizer Under Static and Horizontal Shock Conditions

the mounting system, and, thus, is less desirable than the stabilizer. Isolators used in this way improve a poorly planned underneath mounting system but should not, during planning, be considered as part of a mounting system.

#### 9-34. CENTER-OF-GRAVITY MOUNTING SYSTEMS.

9-35. In center-of-gravity mounting systems, the isolators are located in a plane that passes through the center of gravity of the mounted equipment. It is a refinement of the underneath mounting system in that, while coupling of certain rotational and horizontal modes in the underneath system is unavoidable because of the unsymmetrical placing of the isolators relative to the horizontal plane through the center of gravity, these same modes will always be decoupled in center-of-gravity systems.

9-36. With coupled modes, the spread between natural frequencies of a system usually is greater; thus, isolation efficiency is reduced. Setting the highest natural frequency of the mounting system well below the forcing frequency may put the lowest frequency at a level that will introduce instability into the system. This necessitates compromises which reduce the overall system effectiveness. In a center-of-gravity system it is possible to decouple all translational and rotational modes, and also have the principal natural frequencies equal.

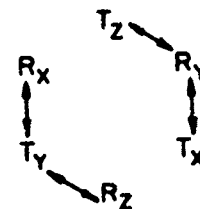
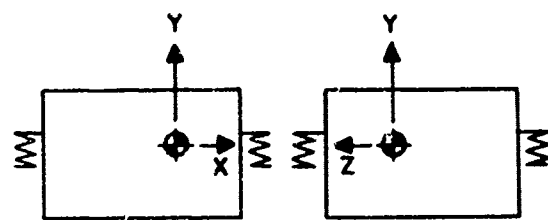
9-37. COUPLING OF MODES. By locating equal mounts in a horizontal plane through the center of gravity, partial decoupling results regardless of the location of the center of gravity relative to the XY and YZ vertical planes. Figure 9-11 shows the degrees of coupling, using a center-of-gravity mounting system, for (a) no vertical planes of symmetry\*, (b) one vertical plane of symmetry, and (c) two vertical planes of symmetry.

9-38. In figure 9-11a, with no vertical planes of symmetry, all modes are coupled, but all modes are not coupled to each other. This is the same situation that exists for a bottom-mounted equipment with one plane of symmetry, and, this again is a fairly complex motion that is difficult to analyze.

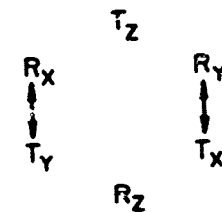
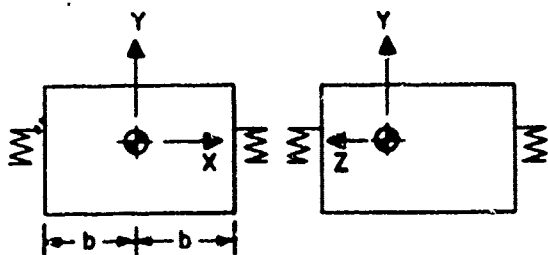
9-39. In figure 9-11b, with one vertical plane of symmetry, translation in the Z direction is decoupled from all other modes, as is rotation about the Z axis. Rotation about the X axis is coupled to translation in the Y direction and rotation about the Y axis is coupled to translation in the X direction. This is the same degree of coupling that exists when an underneath mounting system has two planes of symmetry. The vibratory motion is similar in complexity to that of an underneath mounting system with maximum decoupling, but since further decoupling is possible, and since a center-of-gravity system is used for total decoupling, this vibration and its natural frequencies will not be discussed further.

9-40. In figure 9-11c, with two vertical planes of symmetry, all modes are decoupled. Each mode of vibration exists independently of the others, and excitation in one mode will not cause motion in another. This obviously is advantageous if the predominant excitation is in one direction; it is necessary to be concerned with but one natural frequency rather than the two of coupled modes. The vibratory

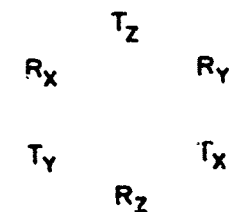
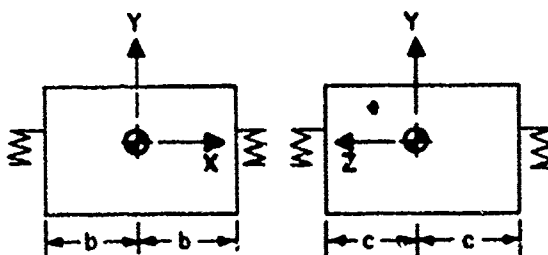
\* In this usage, a plane of symmetry exists when isolators are placed symmetrically about a plane through the center of gravity.



(A)



(B)



ALL MODES  
DECOUPLED

(C)

Figure 9-11. Degrees of Coupling of Rotational and Translational Modes in a Center-of-Gravity Mounting System

motions are easier to analyze with all modes decoupled, and it permits further refinements, such as reducing the spread of natural frequencies.

**9-41. DECOUPLING OF MODES WHEN LESS THAN TWO VERTICAL PLANES OF SYMMETRY OCCUR.** When equal mounts are located equidistant from and in a plane through the center of gravity, all modes are decoupled. Determining the load requirements for each mount is, again (assuming four mounts are used), a simple matter of dividing the total weight of the equipment by four. When less than two vertical planes of symmetry exist, total decoupling still can be achieved by using isolators of different spring constants. Figure 9-3 gives four formulas for determining the load on each isolator for an underneath mounting system. These four formulas can be used for the same purpose for a center-of-gravity system. The four isolators then are selected that will give the same static deflection and have the same natural frequency when carrying their share of the load.

**9-42. DETERMINING THE CENTER OF GRAVITY FOR AN EQUIPMENT.** There are various methods for determining the center of gravity. Two simple methods involve suspending the equipment by cord or balancing the equipment on a knife edge.

**9-43.** In the first method, the equipment is suspended by either one or two cords. When one cord is used, the center of gravity of the equipment is on the vertical line coinciding with the axis of the cord. By attaching the cord to another part of the equipment, a new vertical line can be projected through the equipment. Theoretically, these two lines should intersect at the center of gravity. Actually, it is difficult to be precise, so additional hangings are necessary before the center of gravity can be fairly accurately determined. Using two cords permits easier marking of the equipment, because if hung from the corners, the sides will be perpendicular to the ground. The center of gravity of the equipment will lie in the vertical plane formed by the two cords. At least three hangings are necessary here because the intersection of three planes is necessary to determine a point.

**9-44.** The second method requires at least three balancings before the center of gravity can be determined. When the equipment is balanced on the knife edge, the center of gravity is known to lie directly above the knife edge somewhere along its length. The equipment then is rotated about its vertical axis so that the new line formed by the equipment and the knife edge intersects the previous one. The center of gravity then is known to lie vertically above the point of intersection of the two lines. Turning the equipment on its side and repeating the process will give the elevation of the center of gravity above its base, and, thus, all the determinants necessary for fixing the center of gravity.

**9-45. PREDICTING NATURAL FREQUENCIES.** The method of predicting the natural frequencies for a center-of-gravity mounting system is similar to that used for an underneath mounting system. The vertical natural frequency is first calculated and then used as a reference for finding the natural frequencies of the other modes of vibration. Since all modes are decoupled, the procedure is somewhat simpler than it is for the underneath mounting system.

**9-46.** The vertical natural frequency is found as described previously for an underneath mounting system (paragraphs 9-17 and 9-18). With the vertical frequency known, it is possible to use figure 9-4 again for determining the other natural

frequencies. However, since the dimension "a" is now zero, only two curves are applicable. These curves are shown in figure 9-12.

9-47. All modes are decoupled in a center-of-gravity system, so by using the same vertical and horizontal stiffnesses, the three translational natural frequencies are equal. Varying the vertical-to-horizontal stiffness ratio does not benefit a center-of-gravity system (such as reducing the spread of natural frequencies) as it does an underneath mounting system. Also, having the vertical and horizontal stiffnesses equal lends stability to the system. Consequently, in all further discussions it is assumed that the individual isolator horizontal and vertical stiffnesses are equal.

9-48. In figure 9-12, two curves are shown: one giving values for the natural frequencies in translation, the other representing the natural frequencies in rotation. These vibratory modes, when coupled to each other, constitute the previously discussed rocking modes. Orientation of the equipment shown will give values for translation in the X axis and rotation about the Z axis. Turning the equipment 90 degrees about its Y axis then would show frequencies for translation in the Z axis and rotation about the X axis. Since  $k_x$  and  $k_y$  are equal in this figure, they and the radical sign do not appear in the ratio on the horizontal axis, as shown previously in figure 9-4.

9-49. It is evident from figure 9-12, that the relationship of "b", the distance from the isolator to the center of gravity, to the radius of gyration about the Z axis determines the natural frequencies. When the mounts are not located equidistant from the center of gravity, but the spring constants have been adjusted so that total decoupling results, the value  $\sqrt{b_1 b_2}$  is analogous to and is substituted for "b", where "b<sub>1</sub>" and "b<sub>2</sub>" are the distances from the center of gravity to each isolator. If "b" is equal to the radius of gyration, the dimensionless ratio is 1, and at this value, the translational and rotational curves intersect. At this point, then, the rotational and translational natural frequencies are equal. If the distance between the isolators in the Z direction is also twice the radius of gyration about the X axis, these frequencies, translation in the Z direction and rotation about the X axis, also are equal. Since all the translational frequencies are equal (the hori-

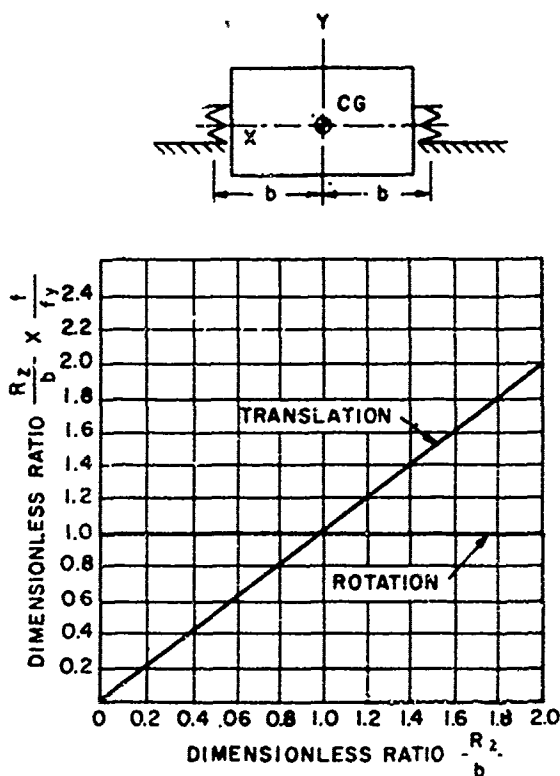


Figure 9-12. Determining the Natural Frequency of the Rotational Mode of Vibration in a Center-of-Gravity Mounting System when the Mounts are Located in a Horizontal Plane

zontal and vertical stiffnesses of the mounts are equal\*), then the natural frequencies of rotation about the X and Z axes are equal to each other, and also equal to the three translational natural frequencies. This is the ideal for which to strive. The lack of a lower natural frequency in any mode of vibration gives a general stability to the isolation system.

9-50. When the mounts are not separated by a distance equal to twice the radius of gyration, the rotational natural frequency will be greater or less depending upon the configuration of the equipment. If the equipment is tall and narrow and the mounts are located inside the radius of gyration, the rotational frequency will be less than the vertical natural frequency. For example, if  $\frac{R_z}{b} = 1.4$ , solving the dimensionless ratio would make  $f_r = \frac{f_y}{1.4}$ . If the mounts are located outside the radius of gyration, the reverse is true.

9-51. In most cases, it is unlikely that the distances from the sides of an equipment to the center of gravity will be equal to the radius of gyration. Of course, in a tall and narrow equipment it would be possible to extend mounting brackets from the sides so that the isolator spacing would be twice the radius of gyration. But space and weight considerations, as well as the necessity for taking the flexibility of the bracket into consideration, makes this solution impractical. However, it is not necessary to mount the isolators in a horizontal plane through the center of gravity for the system to be effective. The mounts can be located on an inclined plane through the center of gravity at distances equal to the radius of gyration, as shown in figure 9-13. Figure 9-14 is a photograph of a rack designed as a center-of-gravity mounting with the isolators on an inclined plane.

9-52. The chassis of an equipment can be designed to use a center-of-gravity system or the equipment can be mounted in a center-of-gravity mounting rack, such as is illustrated in figure 9-15. It is evident that this installation necessitates somewhat more space than a bottom-mounted equipment of comparable size. However, this is offset somewhat in that less sway space is necessary because of the more stable installation.

9-53. It may be impossible to locate the isolators at a distance from the center of gravity equal to the radii of gyration of an equipment, either in a horizontal or an inclined plane, in which case, the rotational natural frequencies about the X and Z axes will not be equal to the translational frequencies. If the mounts are in a horizontal plane, the natural frequencies of the rotational modes are obtained, as described previously, by referring to figure 9-12. If the mounts are located on an inclined plane, the rotational natural frequencies are found by referring to figure 9-16. The dimension "a," the distance from the isolator to XZ plane, divided by the radius of gyration "R<sub>z</sub>" will select the proper curve. The dimension "b," the distance from the YZ plane to the isolator, divided by the radius of gyration "R<sub>z</sub>"

\* The translation frequencies also are shown to be equal in figure 9-12, regardless of the relationship of R<sub>z</sub> and b. For example, if  $\frac{R_z}{b} = 0.6$ , this would also give a value for 0.6 for the vertical axis. Solving the dimensionless ratio would equate the natural frequency of translation in the X axis to translation in the Y axis. This is true for all values of R<sub>z</sub> and b.

locates a point on the curve which, when projected across to the vertical axis, will give the ratio of the rotational to the vertical natural frequency. If the dimension "a" is unequal on either side of the center of gravity  $\sqrt{a_1 a_2}$  is analogous to and is substituted for "a" and  $\sqrt{b_1 b_2}$  is analogous to and is substituted for "b."

#### 9-54. DOUBLE-SIDE MOUNTING SYSTEMS.

9-55. Double-side mounting systems normally are used on equipments whose height-to-width ratio is greater than two and, thus, may have a tendency for excessive flexing. Eight isolators are used, with four each placed symmetrically on opposing sides of the equipment so that the figure described by the isolators is a cuboid. The extra isolators provide additional support points which distribute the load more equitably to the chassis. There doesn't seem to be a definite limit to this system; satisfactory results have been obtained with equipments having a height-to-width ratio of five. Figures 4-9 and 4-12 are photographs of double-side-mounted equipments.

9-56. COUPLING OF MODES. All modes of vibration are coupled when the isolators are located unsymmetrically about the center of gravity. Similarly, as discussed previously with the underneath and center-of-gravity mounting systems, as more planes of symmetry are established, the greater is the degree of decoupling. In an underneath mounting system, it is possible to have no planes of symmetry to a maximum of two planes of symmetry. In a center-of-gravity system, a minimum of one plane of symmetry is possible because, by definition, the isolators are placed in a plane through the center of gravity. The maximum is three planes of symmetry in which all modes are decoupled. For a double-side mounting system, it is possible to have no planes of symmetry to the maximum of three planes of symmetry in which all modes of vibration are decoupled.

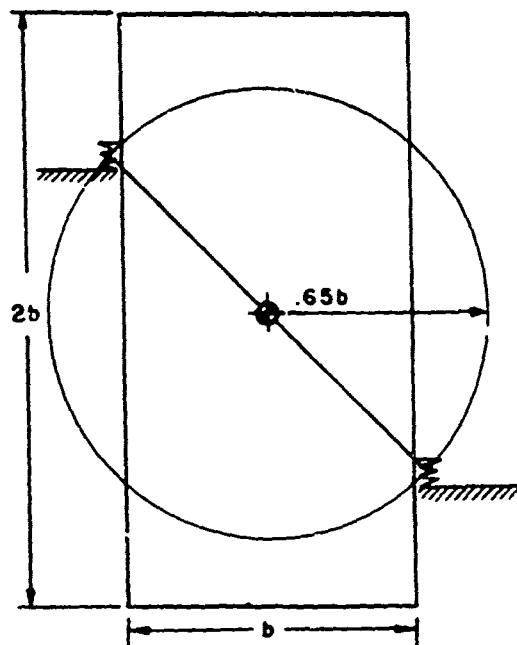


Figure 9-13. Isolators Mounted on an Inclined Plane Through the Center of Gravity at the Radius of Gyration

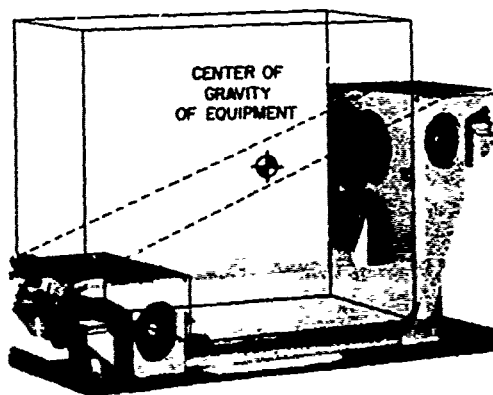


Figure 9-14. Center-of-Gravity Mounting System

9-57. Figure 9-17 shows the degrees of coupling possible in a double-side mounting system. In the first instance, there is no symmetry in the positioning of the isolators with respect to the center of gravity; therefore, all translational and rotational modes are coupled. In the second situation,  $a_1 = a_2$ , thus, the XZ plane becomes a plane of symmetry; i. e., the isolators are located symmetrically about the XZ plane. Here, all are coupled modes; but the coupled modes of rotation about the X axis, translation in the Y direction, and rotation about the Z axis are decoupled from the coupled modes of translation in the Z direction, rotation in the Y direction, and translation in the X direction. Similarly, if  $b_1 = b_2$  instead of  $a_1 = a_2$ , the plane of symmetry would be the XY plane and the degree of coupling would be the same, but the modes would couple differently.

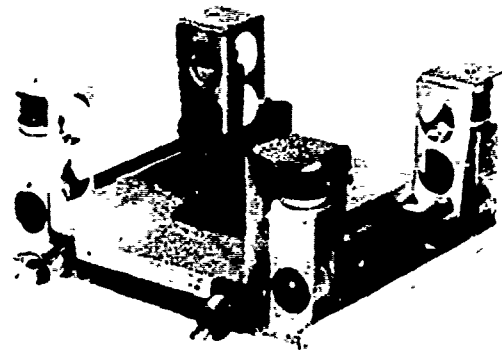


Figure 9-15. Center-of-Gravity Mounting Rack

9-58. In the third example of figure 9-17, two planes of symmetry exist. When  $a_1 = a_2$  and  $b_1 = b_2$ , the planes of symmetry are the XZ and XY planes, or if

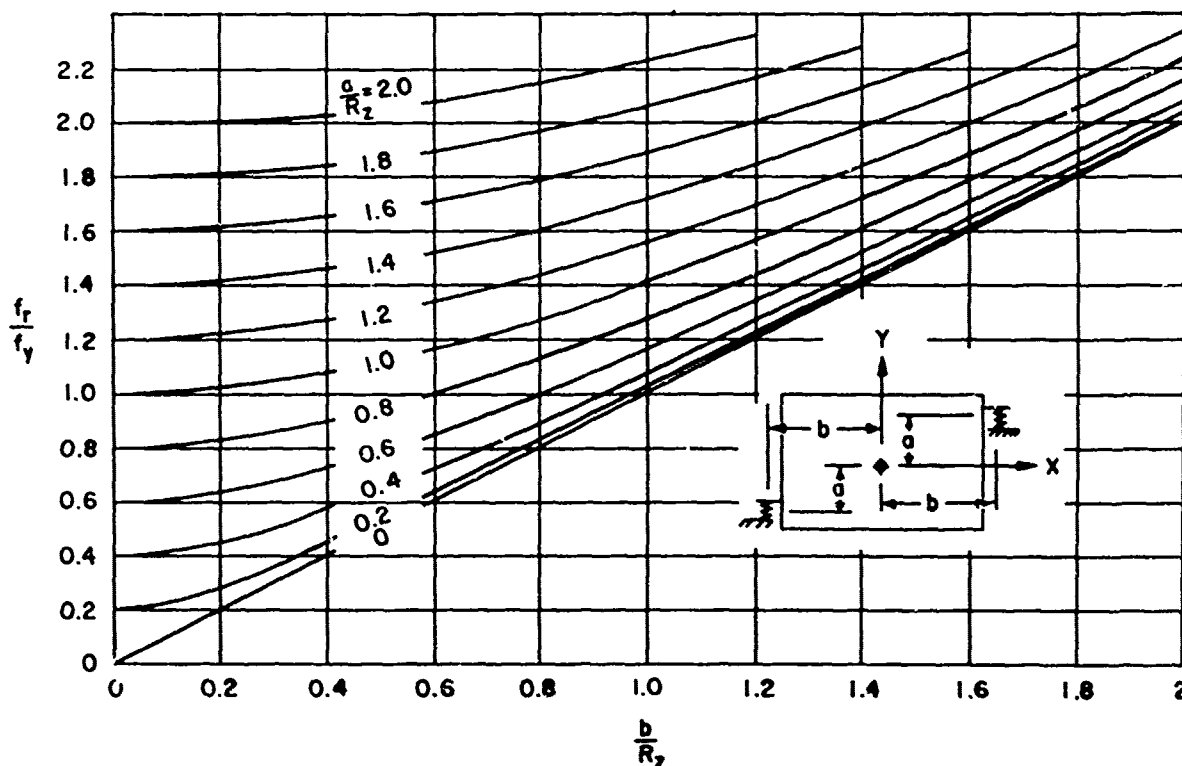
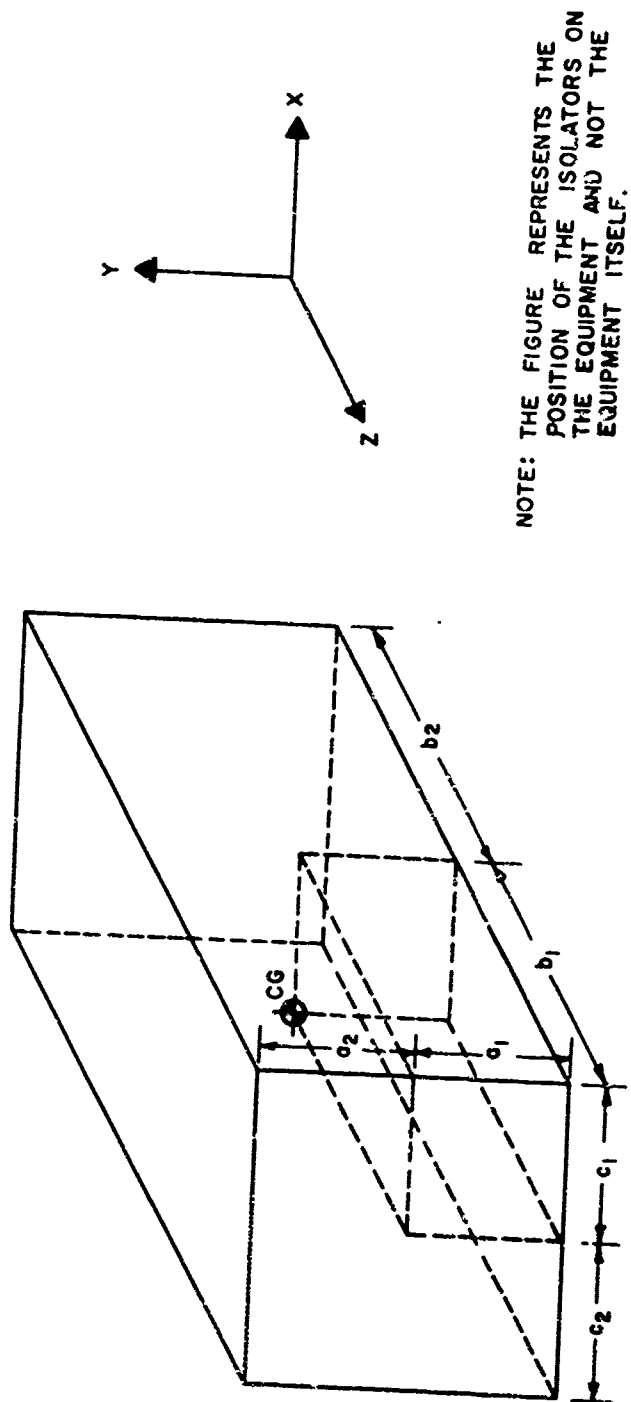


Figure 9-16. Determining the Natural Frequency of the Rotational Mode of Vibration in a Center-of-Gravity Mounting System when the Mounts are Located on an Inclined Plane



NOTE: THE FIGURE REPRESENTS THE POSITION OF THE ISOLATORS ON THE EQUIPMENT AND NOT THE EQUIPMENT ITSELF.

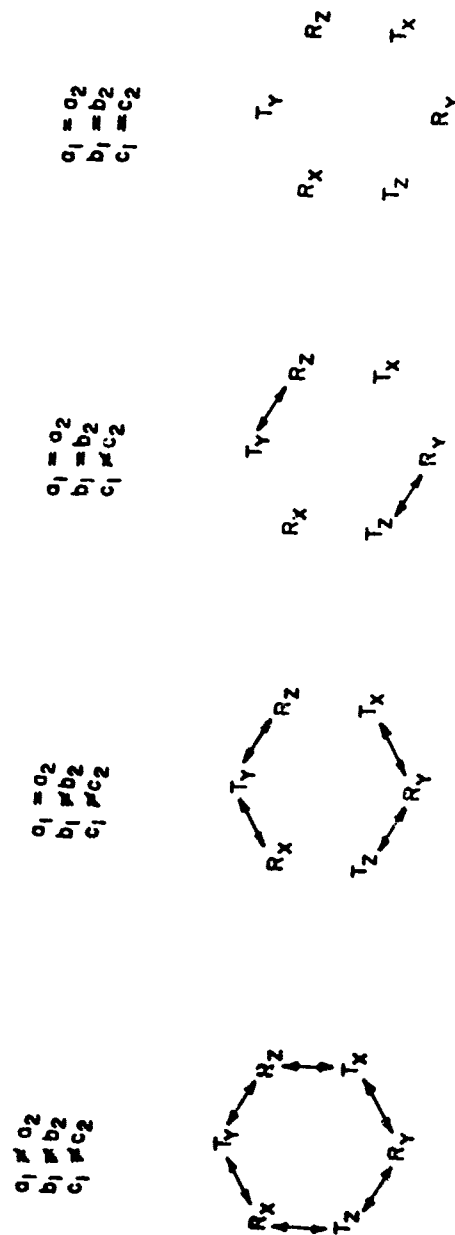


Figure 9-17. Degrees of Coupling Possible in a Double-Side Mounting System

$c_1 = c_2$  is substituted for one of the above, the YZ plane becomes one of the planes of symmetry. One translational and one rotational mode will be decoupled and each of the other two translational modes will couple with a rotational mode. In the last example,  $a_1 = a_2$ ,  $b_1 = b_2$ , and  $c_1 = c_2$ . The center of gravity, therefore, is in the geometric center of the figure described by the isolators, three planes of symmetry occur, and all modes are decoupled.

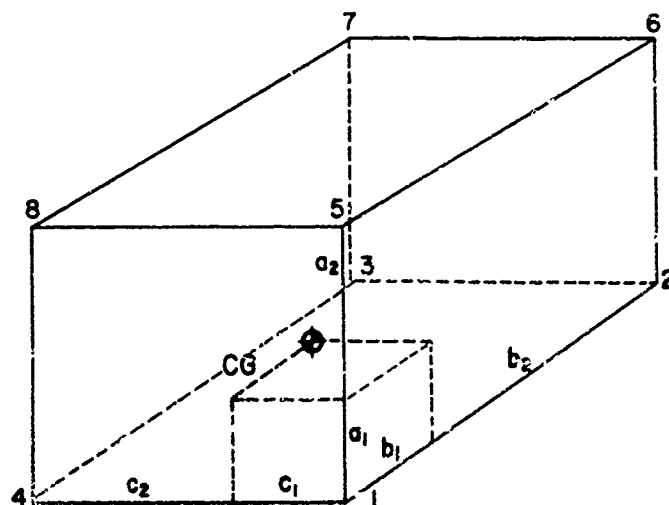
9-59. **DECOUPLING OF MODES.** When three planes of symmetry exist and all modes are decoupled, the load requirements for each isolator are determined by dividing the weight of the equipment by eight. The spring constant best suited for the installation can be chosen and all eight spring constants will be equal. If less than three planes of symmetry occur, coupling exists, but decoupling of all modes still can be achieved by using different spring constants for the eight isolators.

9-60. Figure 9-18 gives eight formulas for determining the spring constant for each of the eight isolators so that all modes will be decoupled. The three translational natural frequencies will be equal since it is assumed that the horizontal and vertical stiffnesses for each isolator are equal. The total spring constant,  $k_{yt}$ , is determined by the total weight of the equipment and the natural frequency desired for the mounting system.

9-61. **PREDICTING NATURAL FREQUENCIES.** As discussed previously in reference to the center-of-gravity mounting systems, the narrower the spread of the six natural frequencies, the more desirable the installation. Thus, if practicable, the system should be planned to have all six natural frequencies equal. It is assumed in the following discussion that all modes are decoupled.

9-62. There are two methods by which the six natural frequencies can be made equal. The first covers the situation when decoupling results from the symmetrical placing of isolators relative to the center of gravity (three planes of symmetry). The second covers the situation when decoupling results from the adjustment of the spring constants of the isolators when less than three planes of symmetry exist. When three planes of symmetry exist, the six natural frequencies will be equal if: (1) the isolators have a vertical-to-horizontal stiffness ratio of one, (2) the spring constants of all the isolators are the same, and (3) all the isolators are located on the three radii of gyration. With equal vertical and horizontal stiffnesses, the three translational frequencies are equal. The vertical natural frequency can be found by reference to the formulas given in paragraphs 9-17 and 9-18, with only the slight adjustment of considering eight isolators instead of four. Thus, the natural frequencies of the three translational modes are easily determined. The spring constants of all the isolators must be equal because, with the isolators located on the radii of gyration, the center of gravity is in the geometric center of a figure described by the isolators, and equal spring constants is one of the conditions of decoupling. Locating the mounts on the radii of gyration will equate the rotational to the translational natural frequencies; therefore, the vertical natural frequency determines all the natural frequencies. In most installations, it is sufficient to consider only the radii of gyration about the X and Z axes. The natural frequency of rotation about the Y axis will not be equal to the other five natural frequencies, but is is not a common form of excitation.

9-63. When less than three planes of symmetry exist, and decoupling results from the adjustment of the spring constants of the isolators, the natural frequencies of the six modes of vibration can be made equal by using the following three formulas:



NOTE: THE CORNERS OF THE FIGURE REPRESENT THE LOCATION OF EACH ISOLATOR.

$$\begin{aligned}
 k_{y1} &= b_2 c_2 a_2 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right] & k_{y5} &= b_2 c_2 c_1 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right] \\
 k_{y2} &= b_1 c_2 a_2 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right] & k_{y6} &= b_1 c_2 a_1 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right] \\
 k_{y3} &= b_1 c_1 a_2 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right] & k_{y7} &= b_1 c_1 a_1 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right] \\
 k_{y4} &= b_2 c_1 a_2 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right] & k_{y8} &= b_2 c_1 a_1 \left[ \frac{k_{yt}}{(b_1 + b_2)(c_1 + c_2)(a_1 + a_2)} \right]
 \end{aligned}$$

LET:

$$k_y = k_x = k_z$$

$k_{yt}$  = THE TOTAL SPRING CONSTANT IN THE Y DIRECTION

Figure 9-18. Determining the Spring Constant for Each Isolator in a Double-Side Mounting System so that All Modes Will Be Decoupled

$$a_1 a_2 = \frac{R_x^2 + R_z^2 - R_y^2}{2}$$

$$b_1 b_2 = \frac{R_x^2 + R_y^2 - R_z^2}{2}$$

$$c_1 c_2 = \frac{R_z^2 + R_y^2 - R_x^2}{2}$$

where "R" is the radius of gyration, and (referring to figure 9-18) "a<sub>1</sub>" is the dimension down from the center of gravity, "a<sub>2</sub>" is the dimension up from the center of gravity, and similarly for dimensions "b<sub>1</sub>," "b<sub>2</sub>," "c<sub>1</sub>," and "c<sub>2</sub>." The location of the isolators must satisfy the three equations for equal natural frequencies. However, because the dimension is given as a combination, some latitude is allowed in positioning the isolators. For example, if a<sub>1</sub> a<sub>2</sub> = 36, a<sub>1</sub> could be 12 and a<sub>2</sub> could be 3, or the combination could be 9 and 4 or 6 and 3, etc. Here again, with all natural frequencies equal, finding the vertical natural frequency identifies all the frequencies. In this instance, of course, all the spring constants will not be equal. After the isolators are located, figure 9-18 determines the spring constant for each isolator for total decoupling.

9-64. If the isolators are located in such a manner that the natural frequencies of the six modes of vibration are not equal (assuming that all modes are decoupled by adjusting the spring constants of the isolators), the natural frequencies of the three rotational modes can be found by the following formulas:

$$f_{rx} = f_y \sqrt{\frac{b_1 b_2 + a_1 a_2}{R_x^2}}$$

$$f_{ry} = f_y \sqrt{\frac{c_1 c_2 + b_1 b_2}{R_y^2}}$$

$$f_{rz} = f_y \sqrt{\frac{a_1 a_2 + c_1 c_2}{R_z^2}}$$

where f<sub>r</sub> is the rotational frequency about the axis denoted by the subscript and f<sub>y</sub> is the vertical natural frequency. Since the horizontal-to-vertical stiffness ratio remains one, the three translational frequencies still are equal.

#### 9-65. OVER-AND-UNDER MOUNTING SYSTEMS.

9-66. In an over-and-under mounting system, eight isolators are used, but instead of being mounted at the sides of the equipment as in a double-side mounting system, the mounts are located at the top and bottom of the equipment. This system is used when the slot designated for the equipment provides an overhead support to which isolators can be connected, and, also, when the height-to-width ratio of the equipment exceeds 1-1/2, so that a bottom-mounted installation alone would tend to be unstable (see paragraph 9-31). Most of the discussion about a double-side mounting system also is applicable here since the isolators still describe a cuboid. It is assumed in this system that the mounts located at the top of the

equipment will carry an equal share of the load with the bottom mounts. To do this, double-acting mounts are used, or, the isolators are mounted so that they support the equipment as do the bottom mounts (figure 9-19).

9-67. **COUPLING OF MODES.** The same degrees of coupling can occur in this system as in the double-side mounting system. Referring to figure 9-17, the figure described by the isolators can also be that of a top-and-bottom system. The only way that the relationship of dimensions that results in total decoupling can be satisfied, however, is by the center of gravity being located at the mid-height of the equipment and by all the mounts being equal. Since this is, in most cases, unlikely, total decoupling will not be achieved by the placement of isolators.

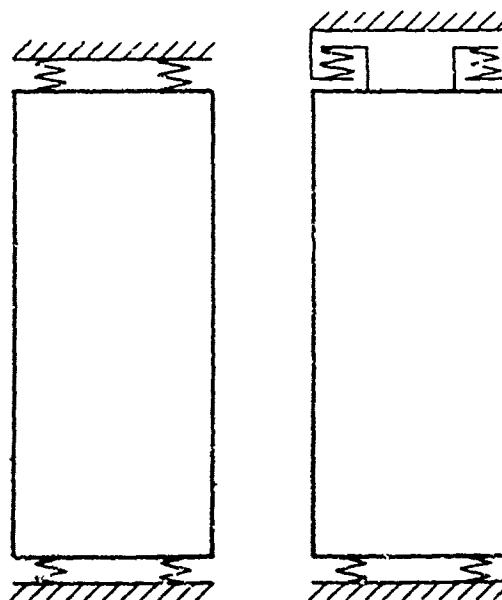


Figure 9-19. Over-and-Under Mounting System

9-68. **DECOUPLING OF MODES.** The translational and rotational modes for an over-and-under mounting system can be decoupled by varying the spring constants of the eight isolators. The formulas of figure 9-18 also are applicable to this system. Again, it is assumed that the horizontal and vertical stiffnesses for each isolator are equal.

9-69. **PREDICTING NATURAL FREQUENCIES.** In this system, as in the previously discussed center-of-gravity and double-side mounting systems, the horizontal and vertical stiffness ratios for each isolator are equal. Thus, finding the vertical natural frequency (paragraphs 9-17 and 9-18) will give three translational frequencies, since these natural frequencies are equal.

9-70. Making the three rotational natural frequencies equal to the translational frequencies is unlikely in the over-and-under mounting system. There is considerable latitude in placing the isolators on the sides of the equipment; however, the necessity of placing the isolators on top and bottom of the equipment limits possibilities prohibitively. For example, in practically all cases, the radii of gyration about the Z and X axes will not intersect the top of the equipment. This makes it impossible to locate the isolators on the radii of gyration and, through this means, to make all the natural frequencies equal. Also, it is extremely unlikely that the formulas in paragraph 9-64, by which it is possible to equate all natural frequencies, will be satisfied.

9-71. Assuming that the spring constants of the isolators have been adjusted so that all vibration modes are decoupled, it is possible to find the natural frequencies of the rotational modes by using the formulas in paragraph 9-64.

9-72. In general, the best that can be said for this system is that the vibratory modes can be decoupled, and, as discussed previously, only one natural frequency

(rather than the two of a coupled mode) must be considered when the predominant excitation is in one direction. Also, this system raises what would be the natural frequency of the lower rocking mode if the equipment were bottom-mounted alone. This will reduce any tendency the equipment may have to excessive lateral motion.

#### 9-73. INCLINED-ISOLATOR MOUNTING SYSTEM.

9-74. When four isolators are used and they cannot be located in a plane through the center of gravity, decoupling still can be accomplished by inclining the isolators, as shown in figure 9-20. As discussed previously, when an equipment is bottom mounted coupling occurs between translational and rotational modes because of the unsymmetrical position of the isolators relative to a horizontal plane through the center of gravity. This dissymmetry causes external forces, from the isolator horizontal stiffnesses, to apply a turning moment to the equipment when the equipment is displaced sideways. If, however, the isolators are inclined instead of being positioned vertically, motion along either of the principal axes results in deflection of the isolators in both the radial and axial\* directions. Thus, a translational motion results in external forces from both the radial and axial isolator stiffnesses, as shown in figure 9-20.

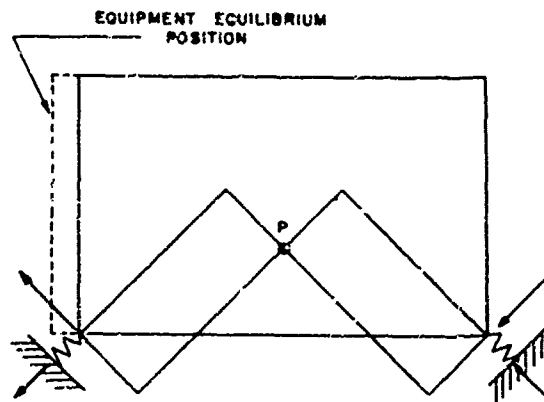


Figure 9-20. Inclined-Isolator Mounting System

9-75. Figure 9-20 also shows that the torque about an arbitrarily selected point "P" resulting from the axial isolator stiffnesses opposes the torque caused by the radial isolator stiffnesses. There is a point P where the combination of the angle of isolator inclination and the radial-to-axial stiffness ratio makes the opposing torques equal in magnitude and the resultant torque about "P" is zero. If this point "P" coincides with the center of gravity, the vibrational modes will be decoupled since a horizontal motion of the equipment will not result in a turning moment being applied to the equipment by the isolator.

9-76. The angle of isolator inclination and the isolator horizontal and vertical stiffnesses are quite critical in this mounting system. It is a special purpose application of isolators and requires detailed analysis to be successful.

\* Because the isolators are inclined, the usual reference of horizontal and vertical is changed to radial and axial.

## APPENDIX

### A-1. FREE VIBRATION OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM.

A-2. **UNDAMPED SYSTEM.** The only external force acting on the mass of the undamped single-degree-of-freedom system, shown schematically in figure A-1, is that due to the spring. Choosing the downward direction as positive for displacement and force, the displacement is positive (downward) when the spring force is negative (upward). According to Newton's law, the motion of the mass can be described by

$$m\ddot{y} = \Sigma F,$$

$$m\ddot{y} = -ky,^*$$

$$\ddot{y} = -\frac{k}{m} y. \quad \text{A-1}$$

Letting  $\omega^2 = \frac{k}{m}$  and multiplying by  $2\dot{y}$ ,

$$2\dot{y}\ddot{y} = -2\omega^2 y\dot{y}.$$

Integrating,  $\dot{y}^2 = -\omega^2 y^2 + C_1^2$

$$\dot{y} = \sqrt{C_1^2 - \omega^2 y^2} \quad \text{A-2}$$

$$\frac{dy}{\sqrt{\frac{C_1^2}{\omega^2} - y^2}} = \omega dt$$

Integrating

$$\sin^{-1} \frac{y}{C_1/\omega} = \omega t + C_2$$

$$y = \frac{C_1}{\omega} \sin(\omega t + C_2) \quad \text{A-3}$$

Assume the mass has been displaced from equilibrium a distance  $y_0$  and released with zero velocity; then, when  $t = 0$ ,

$$y = y_0 \quad \text{and} \quad \dot{y} = 0.$$

\* The total spring force acting on the mass is  $k(y + \delta)$  where  $\delta$  is the static deflection due to the weight of the mass and  $y$  is the displacement from the static equilibrium position (see paragraph 1-18). The spring force due to the static deflection is equal and opposite to the weight and these two forces cancel one another. The weight (force due to static acceleration) determines the equilibrium position but does not enter into the equation of motion if a linear spring rate is assumed.

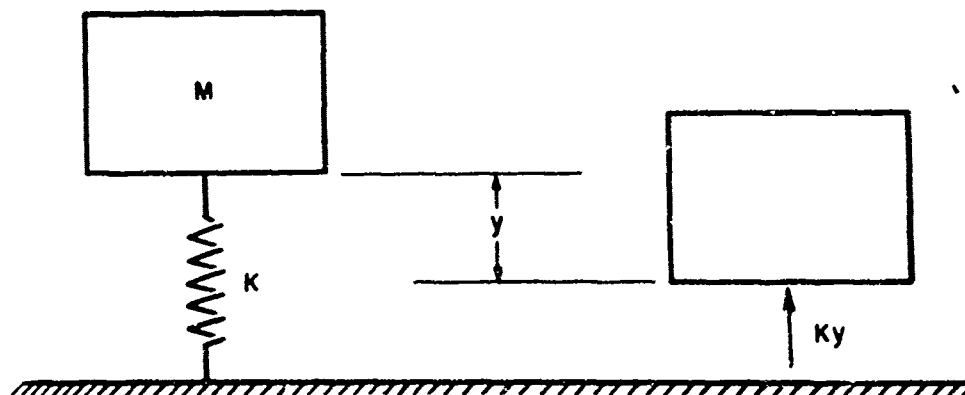


Figure A-1. Undamped Single-Degree-of-Freedom System

From equation A-2,

$$0 = \sqrt{C_1^2 - \omega^2 y_0^2},$$

$$C_1 = \omega y_0.$$

Substituting for  $C_1$  in equation A-3,

$$y = y_0 \sin(\omega t + C_2).$$

Solving for  $C_2$ ,

$$y_0 = y_0 \sin(0 + C_2),$$

$$C_2 = \frac{\pi}{2}.$$

Equation A-3 then becomes

$$y = y_0 \sin\left(\omega t + \frac{\pi}{2}\right),$$

$$y = y_0 \cos \omega t$$

A-4

**A-3. VISCOUSLY DAMPED SYSTEM.** The forces which act on the mass of the viscously damped system (shown schematically in figure A-2) are the spring force and the damping force which is proportional to the velocity and directed against it. Assume the downward direction is positive for displacement, velocity, and force; so, when the displacement and velocity are positive, the spring force and damping force are negative. The motion of the mass can be described by:

$$\begin{aligned} m\ddot{y} &= \Sigma F, \\ m\ddot{y} &= -ky - c\dot{y}, \\ m\ddot{y} + c\dot{y} + ky &= 0. \end{aligned}$$

A-5

If it is assumed that

$$\begin{aligned} y &= Ce^{st}, \\ \dot{y} &= sCe^{st}, \\ \ddot{y} &= s^2 Ce^{st}, \end{aligned}$$

equation A-5 may be written  $(ms^2 + cs + k) Ce^{st} = 0,$

A-6

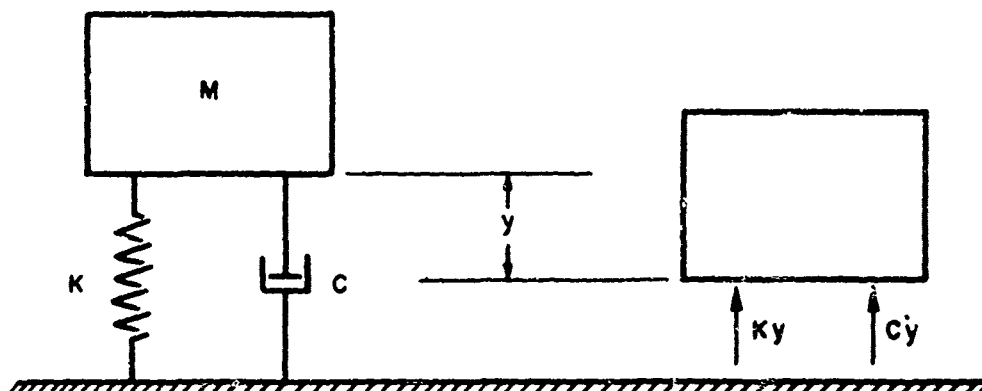


Figure A-2. Damped Single-Degree-of-Freedom System

and  $y = Ce^{st}$  is a solution of equation A-5 if

$$ms^2 + cs + k = 0.$$

A-7

If  $c^2$  is not equal to  $4km$ , there are two values of  $s$  which satisfy the auxiliary equation A-7,

$$s_1 = \frac{-c + \sqrt{c^2 - 4km}}{2m},$$

$$s_2 = \frac{-c - \sqrt{c^2 - 4km}}{2m},$$

and the general solution of equation A-5 is:

$$y = C_1 e^{s_1 t} + C_2 e^{s_2 t}.$$

A-8

A-4. If  $c^2 > 4km$ ,  $s_1$  and  $s_2$  are real numbers and are both negative since the value of the radical is less than  $c$ . The motion described by equation A-8 in this case is the sum of two decreasing exponential curves. There is no oscillation and the system is said to be overdamped.

A-5. If  $c^2 < 4km$ ,  $s_1$  and  $s_2$  are complex numbers and may be written:

$$s_1 = -\frac{c}{2m} + j \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}},$$

$$s_2 = -\frac{c}{2m} - j \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}.$$

The general solution A-8 now contains terms of the form  $e^{a+jb}$  and  $e^{a-jb}$ . Using the relationships

$$e^{jb} = \cos b + j \sin b,$$

$$e^{-jb} = \cos b - j \sin b,$$

and for simplicity let  $b = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$ ; then

$$s_1 = -\frac{c}{2m} + jb \quad \text{and} \quad s_2 = -\frac{c}{2m} - jb, \text{ and}$$

equation A-8 may be written

$$y = e^{-\frac{c}{2m}t} \left[ C_1 (\cos bt + j \sin bt) + C_2 (\cos bt - j \sin bt) \right]$$

or  $y = e^{-\frac{c}{2m}t} \left[ (C_1 + C_2) \cos bt + j(C_1 - C_2) \sin bt \right]$ .

If new constants,  $A = C_1 + C_2$  and

$B = j(C_1 - C_2)$  are chosen,

$$y = e^{-\frac{c}{2m}t} (A \cos bt + B \sin bt). \quad \text{A-9}$$

Multiplying and dividing by  $C = \sqrt{A^2 + B^2}$ ,

$$y = e^{-\frac{c}{2m}t} C \left( \frac{A}{C} \cos bt + \frac{B}{C} \sin bt \right),$$

$$\text{or} \quad y = e^{-\frac{c}{2m}t} C \cos (bt - \phi), \quad \text{A-10}$$

where  $\phi = \tan^{-1} \frac{B}{A}$ .

The motion described by equation A-10 is an oscillatory motion in which the peak amplitude,  $Ce^{-\frac{c}{2m}t}$ , decreases logarithmically with time.

A-6. Paragraphs A-4 and A-5 presented the solution of equation A-5 for the cases when  $c^2 \neq 4km$ . It was shown that when  $c^2 > 4km$  there is no free vibration, and when  $c^2 < 4km$  the motion is a logarithmically decaying harmonic motion. The value of  $c$  at which the transition takes place is  $c = 2\sqrt{km}$ . This is called the critical damping value and is denoted as  $c_c$ . Critical damping is the smallest amount of damping which will prevent free vibration. In solving equation A-5, when  $c = 2\sqrt{km}$ , the auxiliary equation A-7 does not have two roots because

$$s_1 = s_2 = -\frac{c}{2m},$$

and thus equation A-8 becomes:

$$y = (C_1 + C_2) e^{s_1 t}.$$

$(C_1 + C_2)$  is a constant and the solution, therefore, contains only one arbitrary constant instead of two. When  $c = c_c = 2\sqrt{km}$ , equation A-5 becomes

$$\ddot{y} + 2\sqrt{\frac{k}{m}} \dot{y} + \frac{k}{m} y = 0.$$

Choosing  $\omega = \sqrt{\frac{k}{m}},$

then  $\ddot{y} - 2(-\omega)\dot{y} + (-\omega)^2 y = 0.$

The solution to this equation is given in mathematical texts as

$$y = (C_1 + C_2 t) e^{-\omega t}, \quad \text{A-11}$$

and it can be verified by substitution.

A-7. The frequency of oscillation in equation A-10 is

$$\frac{b}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

and the period of oscillation is  $\frac{2\pi}{b}$ . The ratio of peak amplitudes for successive cycles is

$$\frac{C e^{-\frac{c}{2m} t}}{C e^{-\frac{c}{2m} (t + \frac{2\pi}{b})}} = e^{\frac{\pi c}{mb}} \quad \text{A-12}$$

The logarithm of this ratio is  $\frac{\pi c}{mb}$  and this quantity is known as the logarithmic decrement.

#### A-8. FORCED VIBRATION OF AN UNDAMPED, SINGLE-DEGREE-OF-FREEDOM SYSTEM.

A-9. Referring to figure A-3, let the support move vertically with a harmonic motion,  $x_0 \sin \Omega t$ . Using the previous convention of displacement and force as being positive downward, the spring force is positive when the spring is elongated ( $x > y$ ). The equation of motion from Newton's law is

$$m\ddot{y} = k(x - y).$$

Letting  $\omega = \sqrt{k/m}$  and substituting  $x_0 \sin \Omega t$  for  $x$ , then

$$\ddot{y} + \omega^2 y = \omega^2 x_0 \sin \Omega t. \quad \text{A-13}$$

By study it may be seen that  $y = y_0 \sin \Omega t$  will satisfy the equation if  $y_0$  is chosen properly. Substituting  $y = y_0 \sin \Omega t$  into equation A-13,

$$\begin{aligned} -y_0 \Omega^2 \sin \Omega t + y_0 \omega^2 \sin \Omega t &= \omega^2 x_0 \sin \Omega t, \\ y_0 (\omega^2 - \Omega^2) &= \omega^2 x_0, \\ y_0 &= \frac{x_0}{1 - (\frac{\Omega}{\omega})^2}. \end{aligned} \quad \text{A-14}$$

A solution of equation A-13, therefore, is

$$y = \frac{x_0}{1 - (\frac{\Omega}{\omega})^2} \sin \Omega t. \quad \text{A-15}$$

The response motion described by equation A-15 is a harmonic motion with a frequency equal to the excitation frequency. The ratio of response amplitude to excitation amplitude is

$$\frac{y_0}{x_0} = \frac{1}{1 - \left(\frac{\Omega}{\omega}\right)^2},$$

and is plotted in figure 1-3.

A-10. Although equation A-15 is a solution of equation A-13, it is a steady-state solution and does not contain constants of integration to account for initial conditions. The complete solution of equation A-13 consists of the solution to the equation with the right-hand member equal to zero (equation A-8) plus the particular solution A-15. The general solution is

$$y = C_1 e^{s_1 t} + C_2 e^{s_2 t} + \frac{x_0}{1 - \left(\frac{\Omega}{\omega}\right)^2} \sin \Omega t \quad A-16$$

which is the sum of the transient and steady-state motions. With a small but finite amount of damping, the transient motion eventually would disappear and the motion then would be described by equation A-15.

#### A-11. FORCED VIBRATION OF A VISCOUSLY DAMPED, SINGLE-DEGREE-OF-FREEDOM SYSTEM.

A-12. Two cases of forced vibration of a viscously damped system will be considered, the difference between them lying in the nature of the excitation. In the first case, the excitation consists of a varying force applied to the mass and, in the second case, the support is given a harmonic motion.

A-13. FORCE APPLIED TO MASS. Figure A-4 represents a viscously damped, single-degree-of-freedom system in which a varying force,  $P_0 \sin \Omega t$ , is applied to the mass. With displacement and force assumed to be positive downward, the equation of motion is

$$m\ddot{y} + c\dot{y} + ky = P_0 \sin \Omega t. \quad A-17$$

An assumption that  $y = y_0 \sin \Omega t$  is obviously incorrect in this case since, with this assumption, the term  $c\dot{y}$  results in a cosine term. It can be assumed, however, that

$$\begin{aligned} y &= D \sin \Omega t - E \cos \Omega t \\ &= y_0 \sin (\Omega t - \phi) \quad \text{where } y_0 = \sqrt{D^2 + E^2} \text{ and } \phi = \tan^{-1} \frac{E}{D} \\ \dot{y} &= D\Omega \cos \Omega t + E\Omega \sin \Omega t \\ \ddot{y} &= -D\Omega^2 \sin \Omega t + E\Omega^2 \cos \Omega t. \end{aligned}$$

Substituting these values into equation A-17 and equating the coefficients of the sine and cosine terms respectively of the resulting equation gives the following two equations:

$$-Dm\Omega^2 + Ec\Omega + Dk = P_0$$

$$Em\Omega^2 + Dc\Omega - Ek = 0.$$

The values of D and E can be determined from the above two equations and with these two values,  $y_0$  and  $\phi$  can be found as follows:

$$y_0 = \frac{P_0}{\sqrt{D^2 + E^2}} = \frac{P_0}{\sqrt{c^2\Omega^2 + (k - m\Omega^2)^2}} \quad A-18$$

$$\tan \phi = \frac{E}{D} = \frac{c\Omega}{k - m\Omega^2} \quad A-19$$

A solution of equation A-17, therefore, is

$$y = \frac{P_0}{\sqrt{c^2\Omega^2 + (k - m\Omega^2)^2}} \sin(\Omega t - \tan^{-1} \frac{c\Omega}{k - m\Omega^2}) \quad A-20^*$$

A-14. It is customary to express the amount of damping present in a system in terms of its ratio to the critical damping value,  $c_c = 2\sqrt{km}$ . Consequently, the maximum response amplitude,  $y_0$ , and the phase angle,  $\phi$ , are usually expressed in terms of this damping ratio. Substitution of the critical damping value into equations A-18 and A-19 results in the expressions:

$$y_0 = \frac{\frac{P_0}{k}}{\sqrt{\left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]^2 + \left(2\frac{c}{c_c} \cdot \frac{\Omega}{\omega}\right)^2}}, \quad A-22$$

$$\tan \phi = \frac{2\frac{c}{c_c} \cdot \frac{\Omega}{\omega}}{1 - \left(\frac{\Omega}{\omega}\right)^2}, \quad A-23$$

where  $\omega = \sqrt{\frac{k}{m}}$ , the natural frequency of the undamped system.

A-15. Paragraphs A-13 and A-14 considered the response of a system to a varying force,  $P_0 \sin \Omega t$ , in which the amplitude of the force is constant regardless of frequency. In many practical cases, such as when the force results from an unbalance in a rotating machine part, the force is not of the form  $P_0 \sin \Omega t$ , but rather of the form  $m r \Omega^2 \sin \Omega t$ . In this case the equation of motion is

\* As was pointed out in paragraph A-10 for the undamped system, equation A-20 is a steady-state solution of equation A-17. The complete solution consists of the sum of the transient and steady-state motions and, for the most common case when damping is less than critical, the complete solution (from equations A-9 and A-20) is:

$$y = e^{-\frac{c}{2m}t} (A \cos bt + B \sin bt) + \frac{P_0}{\sqrt{c^2\Omega^2 + (k - m\Omega^2)^2}} \sin(\Omega t - \tan^{-1} \frac{c\Omega}{k - m\Omega^2}) \quad A-21^*$$

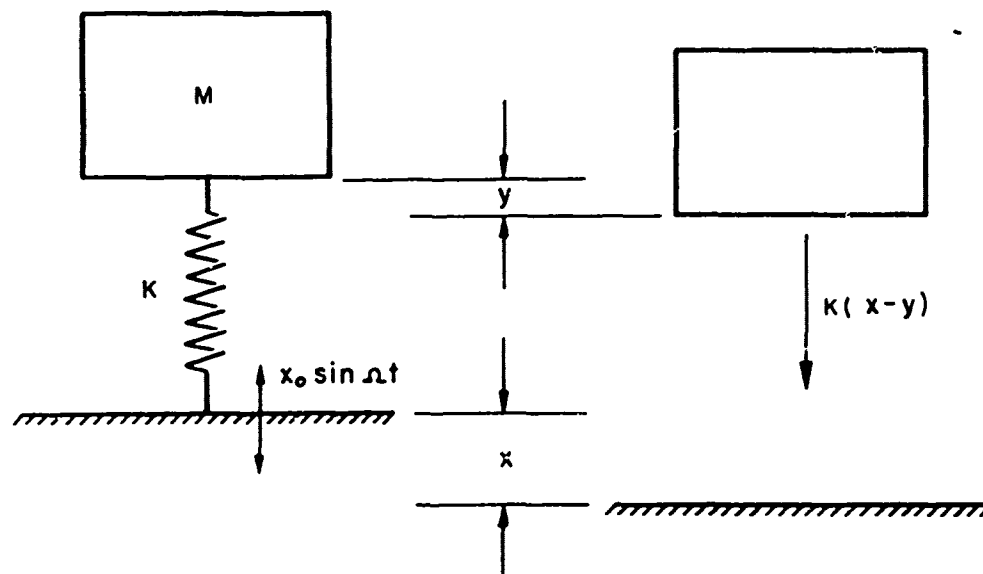


Figure A-3. Motion-Excited, Undamped, Single-Degree-of-Freedom System

$$m\ddot{y} + c\dot{y} + ky = m\Omega^2 \sin \Omega t, \quad A-24$$

and the solution is the same as for equation A-17, with the exception of the substitution of  $m\Omega^2$  for  $P_0$ . The expression for phase angle  $\phi$ , remains the same (equation A-19 or A-23) while the expression for response amplitude,  $y_0$ , (equation A-18 or A-22) becomes

$$y_0 = \frac{m\Omega^2}{k} \sqrt{\left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]^2 + \left(2 \frac{c}{c_c} \cdot \frac{\Omega}{\omega}\right)^2} \quad A-25$$

**A-16. HARMONIC MOTION OF THE SUPPORT.** The type of excitation of greatest interest to designers of airborne electronic equipment is movement of the support. Figure A-5 shows such a system and the forces which act on the mass. The equation of motion from Newton's law is

$$m\ddot{y} = k(x - y) + c(\dot{x} - \dot{y}),$$

$$m\ddot{y} + c\dot{y} + ky = c\dot{x} + kx = c\Omega x_0 \cos \Omega t + kx_0 \sin \Omega t. \quad A-26$$

This equation may be solved algebraically as was equation A-17 by assuming

$$y = D \sin \Omega t - E \cos \Omega t$$

$$\text{or } y = y_0 \cos \phi \sin \Omega t - y_0 \sin \phi \cos \Omega t = y_0 \sin (\Omega t - \phi).$$

This expression for  $y$  may be substituted into equation A-26 and the expressions for  $y_0$  and  $\phi$  can be determined. These expressions are

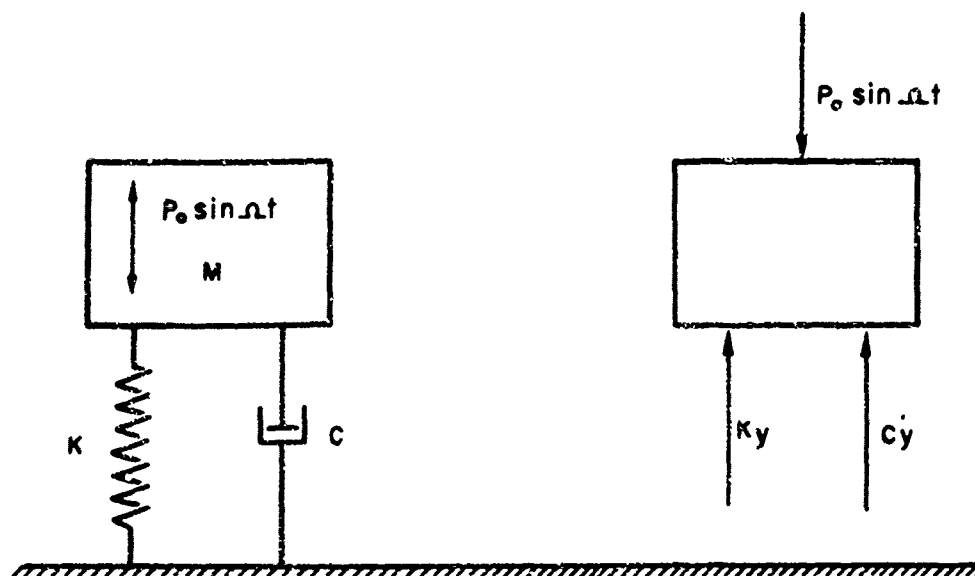


Figure A-4. Force-Excited, Damped, Single-Degree-of-Freedom System

$$y_0 = x_0 \sqrt{\frac{k^2 + c^2 \Omega^2}{(k - m\Omega^2)^2 + c^2 \Omega^2}}, \quad \text{A-27}$$

$$\phi = \tan^{-1} \frac{mc\Omega^2}{k^2 - m\Omega^2 k + c^2 \Omega^2}. \quad \text{A-28}$$

The steady-state solution of equation A-26 is

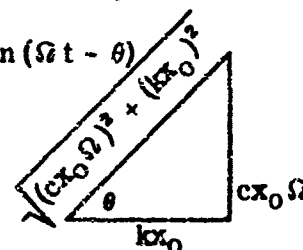
$$y = x_0 \sqrt{\frac{k^2 + c^2 \Omega^2}{(k - m\Omega^2)^2 + c^2 \Omega^2}} \sin\left(\Omega t - \tan^{-1} \frac{mc\Omega^2}{k^2 - m\Omega^2 k + c^2 \Omega^2}\right). \quad \text{A-29}$$

A-17. The expressions for  $y_0$  and  $\phi$  can also be obtained by writing equation A-26 in the form of equation A-17 and then substituting the appropriate expressions into equation A-20. Writing equation A-26 in the form of equation A-17,

$$m\ddot{y} + c\dot{y} + ky = \sqrt{(cx_0\Omega)^2 + (kx_0)^2} \sin(\Omega t - \theta)$$

where

$$\theta = \tan^{-1} \frac{c\Omega}{k}.$$



Substituting  $\sqrt{(cx_0\Omega)^2 + (kx_0)^2}$  for  $P_0$  and  $(\Omega t + \theta)$  for  $\Omega t$  in equation A-20 yields

$$y = x_0 \sqrt{\frac{k^2 + c^2 \Omega^2}{(k - m\Omega^2)^2 + c^2 \Omega^2}} \sin\left(\Omega t + \tan^{-1} \frac{c\Omega}{k} - \tan^{-1} \frac{c\Omega}{k - m\Omega^2}\right).$$

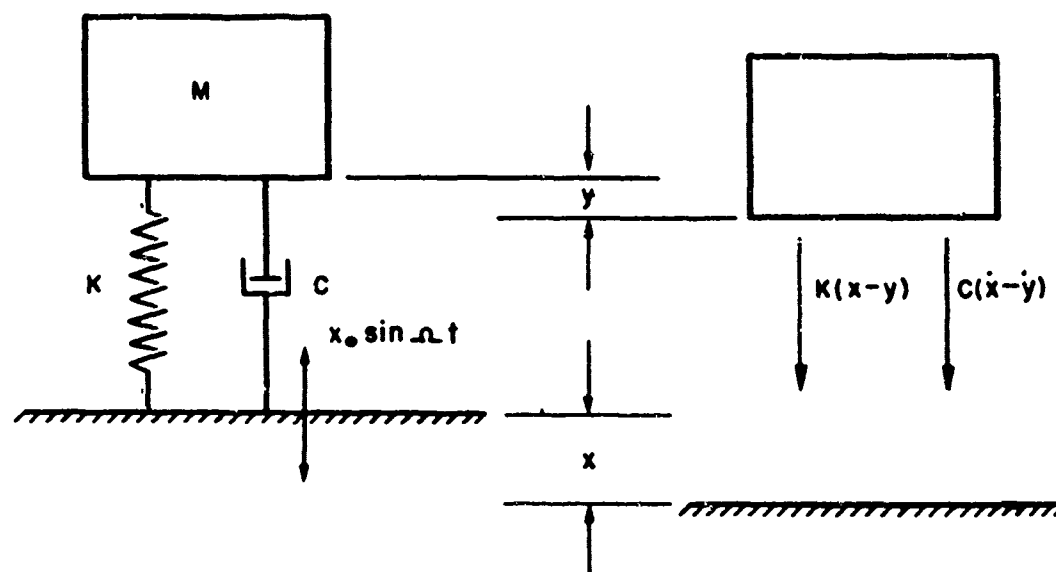


Figure A-5. Motion-Excited, Damped, Single-Degree-of-Freedom System

This is the equivalent of equation A-29 since it can be shown that

$$\tan^{-1} \frac{c\Omega}{k} - \tan^{-1} \frac{c\Omega}{k-m\Omega^2} = -\tan^{-1} \frac{mc\Omega^3}{k^2 - m\Omega^2 k + c^2\Omega^2}$$

A-18. It is customary to express the response amplitude and phase angle in terms of the damping ratio,  $\frac{c}{c_c}$ , and the frequency ratio,  $\frac{\Omega}{\omega}$ . Since  $c_c = 2\sqrt{km}$  and  $\omega = \sqrt{k/m}$ , equations A-27 and A-28 become

$$y_0 = x_0 \sqrt{\frac{1 + \left(2 \frac{c}{c_c} \cdot \frac{\Omega}{\omega}\right)^2}{\left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]^2 + \left(2 \frac{c}{c_c} \cdot \frac{\Omega}{\omega}\right)^2}}, \quad \text{A-30}$$

$$\phi = \tan^{-1} \frac{2 \frac{c}{c_c} \left(\frac{\Omega}{\omega}\right)^3}{1 - \frac{\Omega^2}{\omega^2} + \left(2 \frac{c}{c_c} \cdot \frac{\Omega}{\omega}\right)^2}. \quad \text{A-31}$$

A-19. The resonant frequency (the excitation frequency at which the maximum response occurs) is not identical with the natural frequency (the frequency of free vibration) for a viscously damped system. The resonant frequency of a viscously damped, single-degree-of-freedom system may be found from equation A-27 in the following manner. Starting with

$$T = \frac{y_0}{x_0} = \sqrt{\frac{k^2 + c^2\Omega^2}{(k-m\Omega^2)^2 + c^2\Omega^2}},$$

if this expression is differentiated and the terms collected, the following is obtained:

$$\frac{dt}{d\Omega} = \frac{2k^3 m \Omega - c^2 m^2 \Omega^5 - 2k^2 m^2 \Omega^3}{[(k - m\Omega^2)^2 + c^2 \Omega^2]^{3/2} [k^2 + c^2 \Omega^2]^{1/2}}.$$

At the maximum of the transmissibility curve,

$$\begin{aligned} \frac{dt}{d\Omega} &= 0 \quad \text{or} \quad 2k^3 m \Omega - c^2 m^2 \Omega^5 - 2k^2 m^2 \Omega^3 = 0, \\ (-m\Omega) (c^2 m \Omega^4 + 2k^2 m \Omega^2 - 2k^3) &= 0. \end{aligned}$$

The result  $\Omega = 0$  obtained from the first factor is expected since the transmissibility curve is horizontal at  $\Omega = 0$ . From the second factor, the exciting frequency at which peak transmissibility occurs:

$$\Omega = \sqrt{\frac{-k^2 m + \sqrt{k^4 m^2 + 2c^2 k m}}{c^2 m}} = \omega_r. \quad \text{A-32}$$

This is the resonant frequency as compared to the natural frequency which was found (in paragraph A-5) to be

$$\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}.$$

A-20. The transmissibility at resonance for a viscously damped single-degree-of-freedom system may be expressed in terms of the damping ratio  $\frac{c}{c_c}$  by expressing the resonant frequency in terms of the damping ratio and substituting this expression into the expression for transmissibility. From equation A-32,

$$\omega_r = \omega \sqrt{\frac{-1 + \sqrt{1 + 8 \left(\frac{c}{c_c}\right)^2}}{4 \left(\frac{c}{c_c}\right)^2}}.$$

Substituting this expression for the exciting frequency  $\Omega$  in equation A-30 yields

$$T_{\max} = \frac{4 \left(\frac{c}{c_c}\right)^2}{\sqrt{16 \left(\frac{c}{c_c}\right)^4 - 8 \left(\frac{c}{c_c}\right)^2 - 2 + 2 \sqrt{1 + 8 \left(\frac{c}{c_c}\right)^2}}}$$

which is given as equation 5-9 and is plotted as figure 5-12.

**A-21. THE EFFECT OF SUPPORT FLEXIBILITY IN A MOTION-EXCITED SYSTEM.** Paragraphs A-16 and A-17 deal with a system in which the motion of the support is unaffected by the reactive load of the equipment. The immediate support structure, particularly in aircraft, may have considerable flexibility in which case the motion of that structure, which is the environment for the equipment, will be affected by the equipment response. Figure A-6 represents an equipment of mass  $m$  mounted on a shelf of mass  $m_1$  and spring constant  $k_1$ . With all motion restricted to the vertical direction, a summation of forces for the equipment and shelf respectively leads to the following two equations:

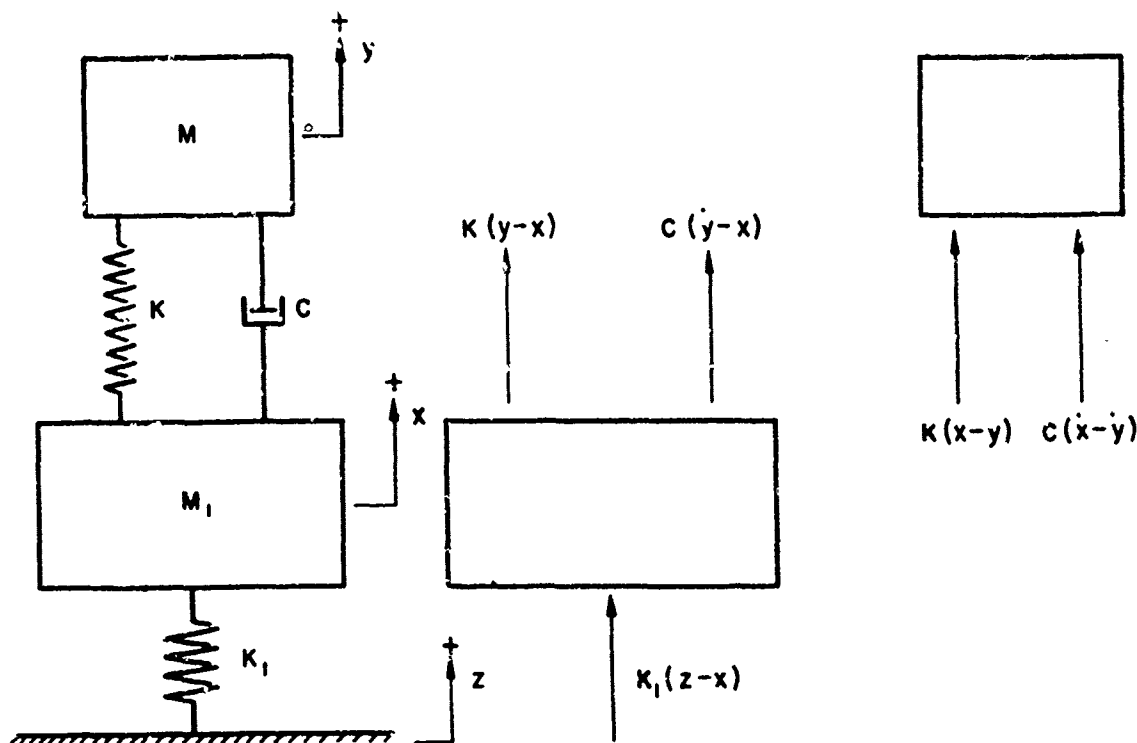


Figure A-6. Two-Degree-of-Freedom System

$$m\ddot{y} + c\dot{y} + ky - c\dot{x} - kx = 0, \quad \text{A-33}$$

$$m_1\ddot{x} + c\dot{x} + kx + k_1x - c\dot{y} - ky = k_1z. \quad \text{A-34}$$

If the motion  $z$  is of the form  $z_0 \sin \Omega t$ , the steady-state motions  $x$  and  $y$  (neglecting starting transients which will be damped out) will be harmonic motions of the same frequency but not necessarily in phase.

A-22. Each term of equations A-33 and A-34 represents a force which varies sinusoidally, and which, therefore, may be represented by a rotating vector. Writing the vectors as complex numbers and grouping terms, the equations become

$$(k - m\Omega^2 + jc\Omega)y - (k + jc\Omega)x = 0, \quad \text{A-35}$$

$$(k + k_1 - m_1\Omega^2 + jc\Omega)x - (k + jc\Omega)y = k_1z, \quad \text{A-36}$$

where  $x$ ,  $y$ , and  $z$  are complex numbers which represent displacement vectors. The fact that they are complex numbers accounts for the phase angles between them. From equation A-35,

$$\frac{y}{x} = \frac{k + jc\Omega}{k - m\Omega^2 + jc\Omega}. \quad \text{A-37}$$

A-23. Before proceeding, an examination of the meaning of equation A-37 is in order. Both  $y$  and  $x$  are vectors represented by complex numbers and the ratio of

these complex numbers is given by this equation. The maximum displacements  $y_0$  and  $x_0$  are represented by the length of the vectors  $y$  and  $x$ , and the length of a vector is equal to the square root of the sum of the squares of the real and imaginary components. From equation A-37, therefore, the ratio of peak displacements  $y_0$  and  $x_0$  may be written as

$$\frac{y_0}{x_0} = \sqrt{\frac{k^2 + c^2 \Omega^2}{(k - m\Omega^2)^2 + c^2 \Omega^2}} \quad \text{A-38}$$

This result agrees with equation A-27. The phase angle between the  $x$  and  $y$  motions may also be obtained from equation A-37. Considering the numerator to be of the form,  $y_0 \cos \alpha + jy_0 \sin \alpha$ , and the denominator to be of the form,  $x_0 \cos \beta + jx_0 \sin \beta$ , the phase angle between the  $x$  and  $y$  vectors would be equal to the difference between their respective angles. This difference is

$$\tan^{-1} \frac{\sin \beta}{\cos \beta} - \tan^{-1} \frac{\sin \alpha}{\cos \alpha} = \tan^{-1} \frac{c\Omega}{k - m\Omega^2} - \tan^{-1} \frac{c\Omega}{k} = \tan^{-1} \frac{mc\Omega^3}{k^2 - m\Omega^2 k + c^2 \Omega^2}$$

which is in agreement with equation A-28.

A-24. Returning to the solution of equations A-33 and A-34, substituting the value of  $y$  obtainable from A-37 or A-35 into A-36 results in the expression

$$\frac{x}{z} = \frac{k_1 (k - m\Omega^2 + jc\Omega)}{(k_1 - m_1 \Omega^2 + k + jc\Omega) (k - m\Omega^2 + jc\Omega) - (k + jc\Omega)^2}$$

After multiplying and collecting terms, dividing numerator and denominator by  $kk_1$ , and substituting  $\omega_1^2$  for  $\frac{k_1}{m_1}$ ,  $\omega^2$  for  $\frac{k}{m}$ ,  $\frac{c}{2}$  for  $k$ , and  $m_1 \omega_1^2$  for  $k_1$ , the expression becomes

$$\frac{x}{z} = \frac{1 - (\frac{\Omega}{\omega})^2 + j2 \frac{c}{c_c} \cdot \frac{\Omega}{\omega}}{1 - (\frac{\Omega}{\omega})^2 - (\frac{\Omega}{\omega_1})^2 + (\frac{\Omega}{\omega})^2 (\frac{\Omega}{\omega_1})^2 - \frac{m}{m_1} (\frac{\Omega}{\omega_1})^2 + j2 \frac{c}{c_c} (\frac{\Omega}{\omega} - \frac{m\Omega\Omega^2}{m_1\omega\omega_1^2} - \frac{\Omega\Omega^2}{\omega\omega_1^2})}$$

Substituting  $g = \frac{\Omega}{\omega_1}$ ,  $h = \frac{\Omega}{\omega}$ , and  $\mu = \frac{m}{m_1}$ ,

$$\frac{x}{z} = \frac{1 - h^2 + j2 \frac{c}{c_c} h}{(1 - h^2)(1 - g^2) - \mu g^2 + j2 \frac{c}{c_c} h (1 - \mu g^2 - g^2)} \quad \text{A-39}$$

Using the reasoning of paragraph A-20,

$$\frac{x_0}{z_0} = \sqrt{\frac{(1 - h^2)^2 + (2 \frac{c}{c_c} h)^2}{[(1 - h^2)(1 - g^2) - \mu g^2]^2 + [2 \frac{c}{c_c} h (1 - \mu g^2 - g^2)]^2}} \quad \text{A-40}$$

This, then, is the expression for the ratio of the amplitude of the shelf to the amplitude of the support when the shelf is loaded with the spring-mounted equipment.

A-25. The motion  $y$  of the equipment  $m$  in terms of the motion  $z$  may be found from equations A-37 and A-39. Writing A-37 in terms of the ratio  $h = \frac{\Omega}{\omega}$ , we obtain

$$\frac{y}{x} = \frac{1 + j 2 \frac{c}{c_c} h}{1 - h^2 + j 2 \frac{c}{c_c} h} \quad \text{A-41}$$

From equations A-41 and A-39,

$$\frac{y}{z} = \frac{y}{x} \cdot \frac{x}{z} = \frac{1 + j 2 \frac{c}{c_c} h}{(1-g^2)(1-h^2) - \mu g^2 + j 2 \frac{c}{c_c} h (1-\mu g^2 - g^2)}, \quad \text{A-42}$$

$$\frac{y_0}{z_0} = \sqrt{\frac{1 + (2 \frac{c}{c_c} h)^2}{\left[(1-g^2)(1-h^2) - \mu g^2\right]^2 + \left[2 \frac{c}{c_c} h (1-\mu g^2 - g^2)\right]^2}} \quad \text{A-43}$$

A-26. If the equipment of mass  $m$  is mounted rigidly to the shelf of mass  $m_1$ ,  $k$  is infinitely large, the ratio  $h = \frac{\Omega}{\omega}$  becomes zero, and equation A-43 reduces to

$$\frac{y_0}{z_0} = \frac{1}{1 - \mu g^2 - g^2} \cdot \frac{k_1}{k_1 - (m + m_1) \Omega^2}$$

A-27. The difference between the equipment motion when the shelf motion is not affected by the loading of the equipment, and the equipment motion when the shelf is a compliant system, may be seen by comparison of equations A-38 and A-43.

#### A-28. MULTI-DEGREE-OF-FREEDOM SYSTEM.

A-29. A six-degree-of-freedom system is shown schematically in figure A-7. The equations of motion for free vibration, which are summations of forces in a principal direction or summations of moments about a principal axis, are:

$$\begin{aligned} m\ddot{x} = & -(k_{1x} + k_{2x} + k_{3x} + \cdots + k_{6x})x + b_1(k_{1x} + k_{4x} + k_{5x} + k_{6x})\alpha_3 \\ & - b_2(k_{2x} + k_{3x} + k_{6x} + k_{7x})\alpha_3 + a_1(k_{1x} + k_{2x} + k_{3x} + k_{4x})\alpha_2 \\ & - a_2(k_{5x} + k_{6x} + k_{7x} + k_{8x})\alpha_2 \end{aligned} \quad \text{A-44a}$$

$$\begin{aligned}
m\ddot{y} = & -(k_{1y} + k_{2y} + \dots + k_{8y}) y - b_1 (k_{1y} + k_{4y} + k_{5y} + k_{8y}) \alpha_1 \\
& + b_2 (k_{2y} + k_{3y} + k_{6y} + k_{7y}) \alpha_1 + c_1 (k_{1y} + k_{2y} + k_{5y} + k_{8y}) \alpha_2 \\
& - c_2 (k_{3y} + k_{4y} + k_{7y} + k_{8y}) \alpha_2
\end{aligned}
\tag{A-44b}$$

$$\begin{aligned}
m\ddot{z} = & -(k_{1z} + k_{2z} + \dots + k_{8z}) z - a_1 (k_{1z} + k_{2z} + k_{7z} + k_{8z}) \alpha_1 \\
& + a_2 (k_{5z} + k_{6z} + k_{7z} + k_{8z}) \alpha_1 - c_1 (k_{1z} + k_{2z} + k_{5z} + k_{8z}) \alpha_3 \\
& + c_2 (k_{3z} + k_{4z} + k_{7z} + k_{8z}) \alpha_3
\end{aligned}
\tag{A-44c}$$

$$\begin{aligned}
I_X \ddot{\alpha}_1 = & -b_1^2 (k_{1y} + k_{5y} + k_{4y} + k_{8y}) \alpha_1 - a_1^2 (k_{1z} + k_{2z} + k_{4z} + k_{3z}) \alpha_1 \\
& - b_2^2 (k_{2y} + k_{6y} + k_{3y} + k_{7y}) \alpha_1 - a_2^2 (k_{5z} + k_{6z} + k_{7z} + k_{8z}) \alpha_1 \\
& + b_1 c_1 (k_{1y} + k_{5y}) \alpha_2 - b_2 c_1 (k_{2y} + k_{6y}) \alpha_2 - b_1 c_2 (k_{4y} + k_{8y}) \alpha_2 \\
& + b_2 c_2 (k_{3y} + k_{7y}) \alpha_2 - a_1 c_1 (k_{1z} + k_{2z}) \alpha_3 + c_1 a_2 (k_{5z} + k_{6z}) \alpha_3 \\
& + a_1 c_2 (k_{4z} + k_{3z}) \alpha_3 - a_2 c_2 (k_{7z} + k_{8z}) \alpha_3 - b_1 (k_{1y} + k_{5y} + k_{4y} + k_{3y}) z \\
& + b_2 (k_{2y} + k_{6y} + k_{3y} + k_{7y}) y - a_1 (k_{1z} + k_{2z} + k_{5z} + k_{4z}) z \\
& + a_2 (k_{5z} + k_{6z} + k_{7z} + k_{8z}) z
\end{aligned}
\tag{A-44d}$$

$$\begin{aligned}
I_Y \ddot{\alpha}_3 = & -b_1^2 (k_{1x} + k_{2x} + k_{5x} + k_{8x}) \alpha_3 - b_2^2 (k_{2x} + k_{3x} + k_{6x} + k_{7x}) \alpha_3 \\
& - c_1^2 (k_{1z} + k_{2z} + k_{5z} + k_{8z}) \alpha_3 - c_2^2 (k_{3z} + k_{4z} + k_{7z} + k_{8z}) \alpha_3 \\
& - a_1 b_1 (k_{1x} + k_{4x}) \alpha_2 + a_1 b_2 (k_{2x} + k_{3x}) \alpha_2 - a_2 b_2 (k_{6x} + k_{7x}) \alpha_2 \\
& + a_2 b_1 (k_{5x} + k_{8x}) \alpha_2 - a_1 c_1 (k_{1z} + k_{2z}) \alpha_1 + a_1 c_2 (k_{3z} + k_{4z}) \alpha_1 \\
& + a_2 c_1 (k_{5z} + k_{6z}) \alpha_1 - a_2 c_2 (k_{7z} + k_{8z}) \alpha_1 + b_1 (k_{1x} + k_{4x} + k_{5x} + k_{8x}) x \\
& - b_2 (k_{2x} + k_{3x} + k_{6x} + k_{7x}) x - c_1 (k_{1z} + k_{2z} + k_{5z} + k_{8z}) z \\
& + c_2 (k_{3z} + k_{4z} + k_{7z} + k_{8z}) z
\end{aligned}
\tag{A-44e}$$

$$\begin{aligned}
I_Z \ddot{\alpha}_2 = & -c_1^2 (k_{1y} + k_{2y} + k_{5y} + k_{8y}) \alpha_2 - c_2^2 (k_{3y} + k_{4y} + k_{7y} + k_{8y}) \alpha_2 \\
& - a_1^2 (k_{1x} + k_{2x} + k_{5x} + k_{8x}) \alpha_2 - a_2^2 (k_{3x} + k_{6x} + k_{7x} + k_{8x}) \alpha_2 \\
& - b_2 c_1 (k_{2y} + k_{6y}) \alpha_1 + b_1 c_1 (k_{1y} + k_{5y}) \alpha_1 - b_1 c_2 (k_{4y} + k_{8y}) \alpha_1 \\
& + b_2 c_2 (k_{3y} + k_{7y}) \alpha_1 - a_1 b_1 (k_{1x} + k_{4x}) \alpha_3 + a_1 b_2 (k_{2x} + k_{3x}) \alpha_3 \\
& + a_2 b_1 (k_{5x} + k_{8x}) \alpha_3 - a_2 b_2 (k_{6x} + k_{7x}) \alpha_3 - a_2 (k_{5x} + k_{6x} + k_{7x} + k_{8x}) x \\
& + a_1 (k_{1x} + k_{2x} + k_{5x} + k_{4x}) x - c_1 (k_{1y} + k_{2y} + k_{5y} + k_{8y}) y \\
& + c_2 (k_{3y} + k_{4y} + k_{7y} + k_{8y}) y
\end{aligned}
\tag{A-44f}$$

The equations as given are somewhat lengthier than necessary for the case shown in figure A-7. Forces which are collinear may be combined and, therefore, spring constants with coincident action lines may be combined. This may be seen from the equations in which  $k_{1y}$  and  $k_{5y}$  always appear together as a sum; likewise,  $k_{1x}$  and  $k_{4x}$  always appear together as a sum, etc. Replacing these sums by single spring constants would reduce the number of spring constants from 24 to 12. This simplification is minor and it is obvious that the solution of these simultaneous equations in order to find natural frequencies is still a formidable task. The addition of damping to the system would further complicate the equations.

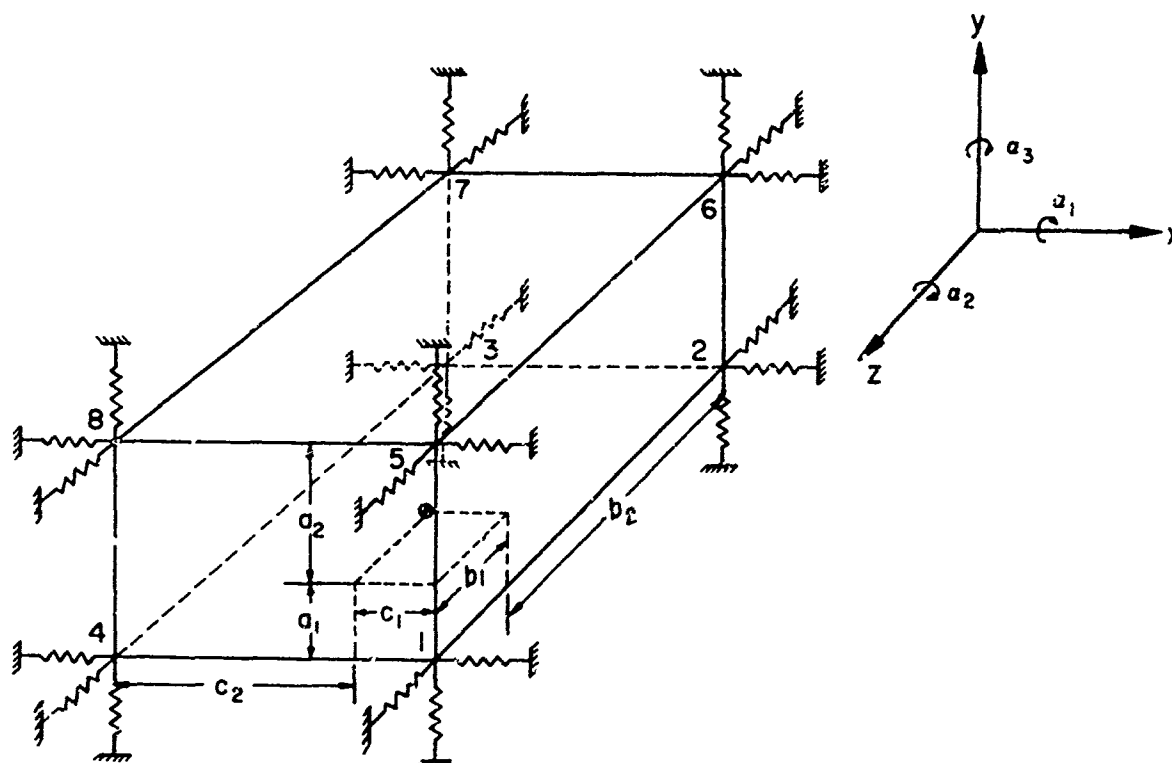


Figure A-7. Six-Degree-of-Freedom System

A-30. Inspection of these equations, however, can lead to certain conclusions concerning possible simpler systems. For the general case of figure A-7, motion in any one of the six primary modes is dependent upon motion in one or more of the other primary modes. This is indicated by the presence of terms involving motion in these other modes in the equation containing an acceleration term for the mode being considered. For example, from equation A-44a, it can be seen that the motions  $\alpha_2$  and  $\alpha_3$  may couple with the motion  $x$ , and from equation A-44b, the motion  $\alpha_3$  may couple with the motion  $z$ . If the system is completely unsymmetrical, motion in any one mode will be accompanied by motion in each of the other five modes.

A-31. The requirements for decoupling may be seen from the general equations. For example, if  $b_1 (k_{1x} + k_{4x} + k_{5x} + k_{6x}) = b_2 (k_{2x} + k_{3x} + k_{6x} + k_{7x})$  there is no coupling between motions  $\alpha$  and  $x$ . Complete decoupling will occur for the system in figure A-7 when

$$\frac{k_{1y} + k_{3y}}{k_{2y} + k_{6y}} = \frac{k_{4y} + k_{5y}}{k_{3y} + k_{7y}} = \frac{k_{1x} + k_{4x}}{k_{2x} + k_{3x}} = \frac{k_{5x} + k_{6x}}{k_{6x} + k_{7x}} = \frac{b_2}{b_1} ,$$

$$\frac{k_{1y} + k_{3y}}{k_{4y} + k_{5y}} = \frac{k_{2y} + k_{6y}}{k_{3y} + k_{7y}} = \frac{k_{1z} + k_{2z}}{k_{3z} + k_{4z}} = \frac{k_{5z} + k_{6z}}{k_{7z} + k_{8z}} = \frac{c_2}{c_1} ,$$

and

$$\frac{k_{1X} + k_{4X}}{k_{5X} + k_{8X}} = \frac{k_{2X} + k_{3X}}{k_{6X} + k_{7X}} = \frac{k_{1Z} + k_{2Z}}{k_{3Z} + k_{6Z}} = \frac{k_{3Z} + k_{4Z}}{k_{7Z} + k_{8Z}} = \frac{a_2}{a_1}$$

#### A-32. RANDOM VIBRATION.

A-33. The preceding portions of this section were concerned with the response of simple mechanical systems to harmonic vibratory excitation which is describable in terms of frequency and amplitude. Although actual environments are seldom completely periodic, many of them may, for practical purposes, be treated as such. In some cases, however, particularly in missiles, there are few discernible periods, thus discouraging analysis on the basis of periodicity.

A-34. NORMAL DISTRIBUTION OF INSTANTANEOUS ACCELERATIONS. A prerequisite to the analysis of the response of a system to an environment is the definition of that environment in a usable form.\* Attempts to define vibration without a discernible period resulted in the use of probability statistics which cover random occurrences, although it has not been definitely established that the environmental vibrations encountered are completely random.

A-35. Random vibration, if the distribution of the instantaneous values of acceleration is in accordance with a normal or Gaussian probability density function and if statistical equilibrium exists (the probability density function does not change with time), may be defined in terms of the probability density function. The probability that the instantaneous acceleration at any time will have a value between  $\ddot{x}$  and  $\ddot{x} + d\ddot{x}$  is equal to  $p(\ddot{x}) d\ddot{x}$ . The probability density function,  $p(\ddot{x})$ , for a normal (or Gaussian) distribution is

$$p(\ddot{x}) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ddot{x} - \bar{\ddot{x}}}{\sigma} \right)^2} \quad \text{A-45}$$

where

- $p(\ddot{x})$  = the probability density function,
- $\ddot{x}$  = the instantaneous value of acceleration,
- $\bar{\ddot{x}}$  = the mean value of instantaneous acceleration,
- $\sigma$  = the standard deviation = the root mean square of the deviations of the instantaneous values from the mean,  $\bar{x}$ .

A-36. The general form of the normal curve which is defined by equation A-45 is shown in figure A-8. In the case of random vibration, the mean value of instantaneous acceleration is zero and the root mean square of the deviations of the instantaneous values of acceleration from the mean is the rms acceleration. The distribution curve for instantaneous acceleration for random vibration, then, is centered about a zero mean and if the instantaneous acceleration is expressed as a multiple of the rms acceleration, the distribution is as shown in figure A-9.

\* Any vibration or shock environment can be completely defined by giving the time-history of either acceleration, velocity, or displacement. This time-history, however, is not amenable to use in an analysis of system response.

A-37. As previously stated, the probability of occurrence of an acceleration between  $\ddot{x}$  and  $\ddot{x} + d\ddot{x}$  is equal to  $p(\ddot{x}) d\ddot{x}$ . The probability that the acceleration at any time lies between  $\ddot{x}_1$  and  $\ddot{x}_2$  is

$$\int_{\ddot{x}_1}^{\ddot{x}_2} p(\ddot{x}) d\ddot{x}$$

which is the area under the curve in the range  $\ddot{x}_1$  to  $\ddot{x}_2$ . The total area under the curve is one, which it obviously should be since it is a certainty that some value of acceleration must exist when  $\pm \infty$  and zero are considered to be values.

A-38. Figure A-10 shows a cumulative distribution curve for a normal distribution. The curve indicates the probability of not exceeding (which is equal to the fraction of the data which falls below) any absolute value of acceleration. It is obtained by evaluating the area under the normal curve between the limits of plus and minus  $x$ . The "probability of not exceeding" can be expressed mathematically as:

$$\int_{-\ddot{x}}^{+\ddot{x}} p(\ddot{x}) d\ddot{x} = 2 \int_0^{+\ddot{x}} p(\ddot{x}) d\ddot{x} \quad \text{A-46}$$

The curve shows that, with a normal probability density, the instantaneous acceleration is less than the rms acceleration 68.26% of the time, is less than twice the rms acceleration 95.44% of the time, and is less than three times the rms acceleration 99.74% of the time.

A-39. RAYLEIGH DISTRIBUTION OF PEAK ACCELERATIONS. From the physical damage standpoint, the instantaneous accelerations are of less interest than

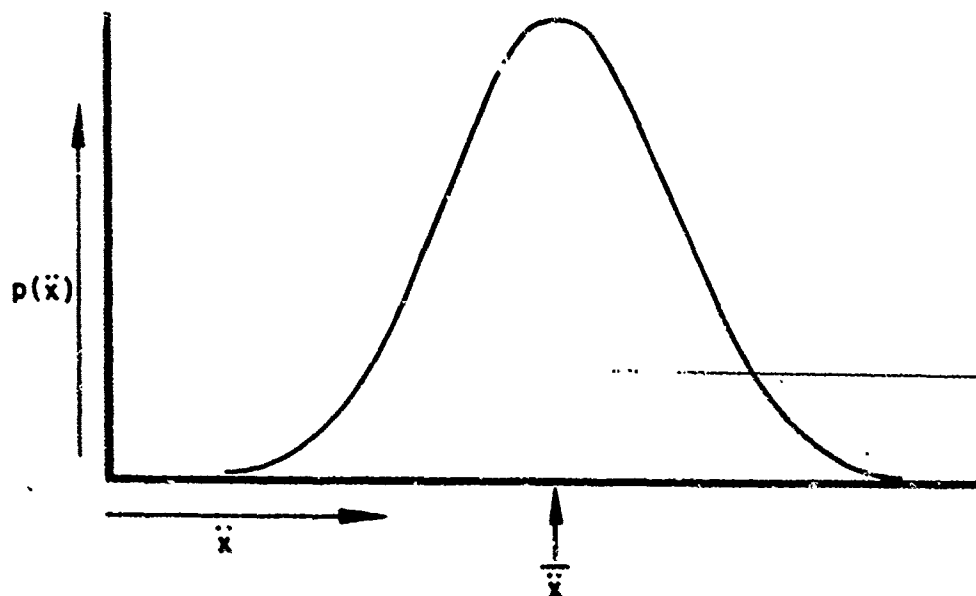


Figure A-8. Normal Distribution Curve

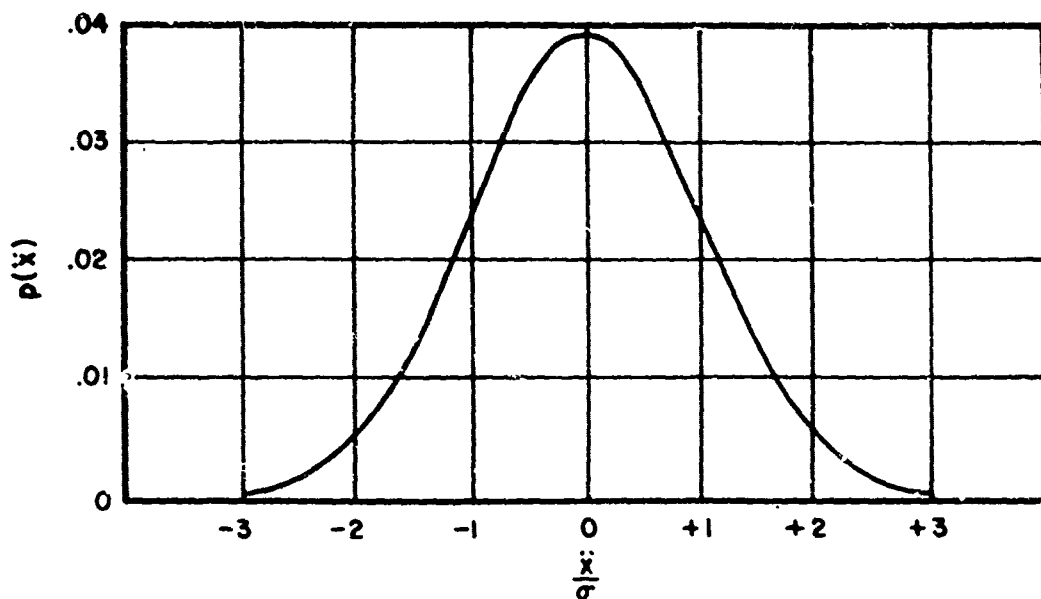


Figure A-9. Normal Distribution Curve for Zero Mean with Variable Expressed in Multiples of the Standard Deviation

the peak accelerations. Studies have shown that for a narrow-frequency-band random signal in which the instantaneous values are normally distributed, the probability density distribution of the envelope of the peak values is

$$p(\ddot{x}_p) = \frac{\ddot{x}_p}{\sigma^2} e^{-1/2 \left( \frac{\ddot{x}_p}{\sigma} \right)^2} \quad A-47$$

where

- $p(\ddot{x}_p)$  = the probability that  $\ddot{x}_p$  falls between  $\ddot{x}_p$  and  $\ddot{x}_p + d\ddot{x}_p$ ,
- $\ddot{x}_p$  = the instantaneous value of the envelope of the acceleration peaks for a narrow (filtered) frequency band,
- $\sigma$  = the rms of the values of the instantaneous acceleration = the standard deviation.

This is known as a Rayleigh probability density function. Figure A-11 is a plot of this function. While the probability density for instantaneous acceleration is a maximum at zero acceleration (because the instantaneous acceleration reaches zero after each half cycle), the probability density for peak acceleration approaches zero as the peak acceleration approaches zero. The probability density for peak acceleration reaches a maximum at the rms acceleration.

A-40. The cumulative distribution curve for a Rayleigh probability density is shown in figure A-12. As with instantaneous acceleration with a normal probability density, the probability that  $\ddot{x}_p$  falls between  $\ddot{x}_p$  and  $d\ddot{x}_p$  is the definite integral of the probability density function, and the probability that the peak acceleration does not exceed  $\ddot{x}_p$  for any one cycle is:

$$\text{probability of not exceeding} = \int_0^{\ddot{x}_p} \frac{\ddot{x}_p}{\sigma^2} e^{-1/2 \left( \frac{\ddot{x}_p}{\sigma} \right)^2} d\ddot{x}_p \quad \text{A-48}$$

The peak acceleration exceeds the rms acceleration 60.5% of the time, exceeds twice the rms acceleration 13.5% of the time, and exceeds three times the rms acceleration 1.1% of the time.

**A-41. MEAN SQUARE ACCELERATION DENSITY (POWER SPECTRAL DENSITY).** If an electrical analog of a complex periodic vibration is played through a narrow bandpass filter or wave analyzer, the output consists of a sinusoidal signal with voltage peaks which are identical from cycle to cycle. Provided the bandwidth is sufficiently narrow so as to include only one component frequency, and is properly tuned to that frequency, narrowing the bandwidth will not change the peak or rms voltage output. Within limits, the output is independent of filter bandwidth. If an electrical analog of a random vibration is played through a bandpass filter, the peak value of the output will vary randomly from cycle to cycle. If the random vibration is continuous (in that it contains energy at each frequency in a broad frequency range), narrowing the bandwidth of the filter will decrease the rms voltage output. The rms voltage will be approximately proportional to the square root of the bandwidth.

**A-42.** Since the output of a bandpass filter or wave analyzer for a random signal input does not consist of consecutive peaks of equal magnitude, the peak value of the output cannot be used to describe this signal. It is fortunate that the rms output not only is convenient to measure but, in addition, has statistical meaning. Either the rms or its square (mean square) may be used. The preferred instrument for measuring the filter output is a slow-acting hot wire or thermocouple voltmeter which will give true rms values, regardless of waveform or frequency, over a broad frequency range.

**A-43.** Since, with continuous random vibration, the rms voltage output of the bandpass filter varies with the filter bandwidth, this rms output is meaningless unless the filter bandwidth is known. (In addition, the selectivity of the analyzing filter affects the rms output.) To overcome the obstacle presented by this dependency of output on bandwidth, it has become customary to express the output on a "per cycle" basis by dividing the filter response by the filter bandwidth. If the indicated response is expressed in rms units, it is divided by the square root of the bandwidth giving units of  $g/\sqrt{\text{cps}}$ . If the indicated response is expressed in mean square units it is divided by the bandwidth giving units of  $g^2/\text{cps}$ . The indi-

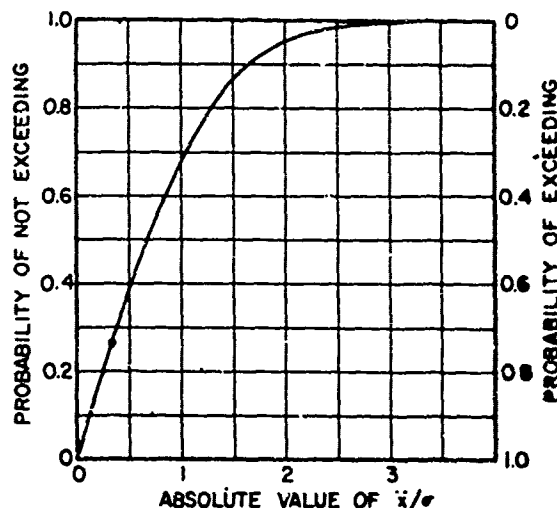


Figure A-10. Cumulative Frequency Distribution for a Normal Probability Density Function

cated response is taken here to be the voltage indication multiplied by the system sensitivity in either volts per rms g or volts per  $g^2$ . The effective bandwidth of the filter must be determined in relation to the shape of the filter selectivity curve, and is not necessarily a simple matter. For discrete random vibration, conversion to mean square acceleration density results only in a change of units.

A-44. The common form in which analyzed random vibration is presented is a plot of mean square acceleration density ( $g^2/cps$ ) versus frequency (center frequency of the bandpass filter). The mean square acceleration density at any one frequency,  $f$ , represents the contribution of frequency components between  $f$  and  $df$  to the overall mean square acceleration. The mean square acceleration which exists in a frequency band,  $f_1$  to  $f_2$ , may be found by integration of the mean square acceleration density between these frequencies:

$$\ddot{x}_0^2 = \int_{f_1}^{f_2} \ddot{x}_0^2(f) df$$

where  $\ddot{x}_0^2$  = mean square acceleration ( $\ddot{x}_0$  = rms acceleration),  
 $\ddot{x}_0^2(f)$  = mean square acceleration density as a function of frequency.

A-45. The use of mean square acceleration as a method of describing random vibration is predicated on the assumption that the values of instantaneous acceleration are distributed in accordance with a normal probability, and that the probability density does not change with time. The mean square is, statistically, the second moment of the area under the probability density curve (analogous to the moment of inertia of the area about the mean). The mean square or rms suffices to describe a zero mean distribution if it is additionally known that the distribution is normal. If the distribution is not normal, higher order moments are necessary to describe the curve. Only the mean square, however, is used in describing random vibration, assuming that the distribution closely approximates normality.

A-46. RESPONSE OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM TO RANDOM EXCITATION. The response of a single-degree-of-freedom system to sinusoidal excitation of the support is given by equation A-30. The ratio of the maximum response amplitude to the maximum excitation amplitude is called the transmissibility,  $T$ ; thus

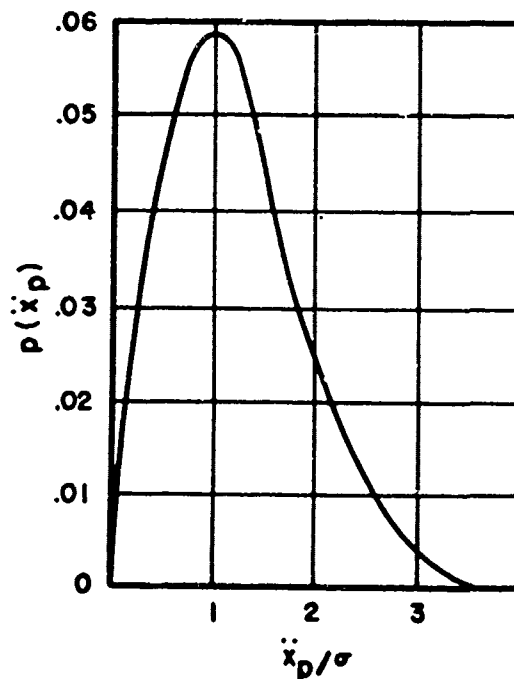


Figure A-11. Rayleigh Distribution Curve

$$T = \sqrt{\frac{1 + \left(2 \frac{\Omega}{\omega} \frac{c}{c_c}\right)^2}{\left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]^2 + \left(2 \frac{\Omega}{\omega} \frac{c}{c_c}\right)^2}}$$

If a damped single-degree-of-freedom system is subjected to random excitation of the support at a single frequency, the rms response acceleration is related to the rms excitation acceleration by the same transmissibility: thus

$$\ddot{y}_0 = \ddot{x}_0 T,$$

where  $\ddot{y}_0$  = the rms response acceleration at a single frequency,  
 $\ddot{x}_0$  = the rms excitation acceleration at a single frequency.

In terms of mean square acceleration,

$$\ddot{y}_0^2 = \ddot{x}_0^2 T^2$$

where  $\ddot{y}_0^2$  = the mean square response acceleration at a single frequency,  
 $\ddot{x}_0^2$  = the mean square excitation acceleration at a single frequency  
 (= mean square acceleration density since the bandwidth is unity).

A-47. When the random excitation of the support is not at a discrete frequency, but rather is continuous random vibration, the overall mean square response acceleration of the system is the sum of its responses to the mean square excitation which exists at each frequency in the spectrum.

$$\ddot{y}_0^2 = \int_{f=0}^{f=\infty} T^2 \ddot{x}_0^2(f) df \quad \text{A-48}$$

In order to evaluate this integral, the mean square acceleration density,  $\ddot{x}_0^2(f)$ , must be known as a function of frequency.

A-48. If the excitation spectrum is assumed to have a constant mean square acceleration density over a frequency range of zero to infinity, and if the damping in the system is so small that an approximate T may be used, equation A-48 becomes

$$\ddot{y}_0^2 = \ddot{x}_0^2(f) \int_0^{\infty} \frac{df}{\left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]^2 + \left(2 \frac{\Omega}{\omega} \frac{c}{c_c}\right)^2} \quad \text{A-49}$$

If the integral of equation A-49 is evaluated, the expression for mean square response acceleration becomes:

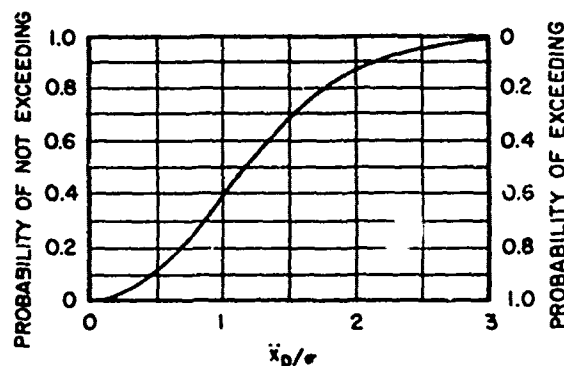


Figure A-12. Cumulative Distribution for a Rayleigh Probability Density Function

$$\ddot{y}_0^2 = \frac{\omega}{8 \frac{c}{c_c}} \ddot{x}_0^2(f).$$

The rms response acceleration is:

$$\ddot{y}_0 = \ddot{x}_0(f) \sqrt{\frac{\omega}{8 \frac{c}{c_c}}}.$$

Substituting  $2\pi f_n$  for  $\omega$ , and  $Q$  for  $\frac{1}{2 \frac{c}{c_c}}$ :

$$\ddot{y}_0 = \ddot{x}_0(f) \sqrt{\frac{\pi}{2} f_n Q} = \sqrt{\frac{\pi}{2} f_n Q \ddot{x}_0^2(f)}, \quad \text{A-50}$$

where

- $\ddot{y}_0$  = the rms response of the lightly damped single-degree-of-freedom system,
- $f_n$  = the natural frequency of the system,
- $Q$  = the magnification at resonance of the system,
- $\ddot{x}_0^2(f)$  = the mean square acceleration density of a white noise (flat spectrum) excitation.

A-49. The response of a simple system as given by equation A-50 is the rms response to an environment with a flat mean square acceleration density spectrum. If it were desired to sinusoidally excite the system at its natural frequency to produce the same rms response, the excitation level required, in units of rms g's would be:

$$\text{rms g's input} = \frac{1}{Q} \sqrt{\frac{\pi}{2} f_n Q \ddot{x}_0^2(f)}.$$

The peak acceleration of a sinusoidal excitation necessary to produce this rms acceleration level would be:

$$\text{peak g input} = \frac{1.414}{Q} \sqrt{\frac{\pi}{2} f_n Q \ddot{x}_0^2(f)}.$$

For design purposes, the peak response acceleration may also be of interest. Equation A-50 gives the rms response only, and the probability of a given peak response for a given rms response depends upon the probability density function. If it is assumed that the distribution is normal or Gaussian, the probability of a given peak acceleration may be found from figure A-12.

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